exampro

GCSE Mathematics (8300)

Name: Mr Ladak

Class:

Higher Upper
Test 1a

Date: 01/02/2017

Time: **1 hour**

Marks: **46**

Comments:

I have included some comments in green to help explain steps.

Please let me know if you need any help.

Q1.

Bag X contains 9 blue balls and 18 red balls. 27 balls

Bag Y contains 7 blue balls and 14 red balls. 21 balls

Liz picks a ball at random from bag X.

She puts the ball into bag Y.

Mike now picks a ball at random from bag Y.

Show that

P (Liz picks a blue ball) = P (Mike picks a blue ball)

This is dependent on Liz's pick so there are two possible outcomes

$P(\text{liz picks blue}) = \frac{9}{27} = \frac{1}{3}$

$P(\text{Mike picks blue}) = P(\text{Liz blue, mike blue}) + P(\text{Liz red, mike blue})$

$= \frac{9}{27} \times \frac{8}{22} + \frac{18}{27} \times \frac{7}{22}$

now 22 balls in Y and 8 are blue

now 22 balls in Y and still 7 are blue

$= \frac{72}{594} + \frac{126}{594}$

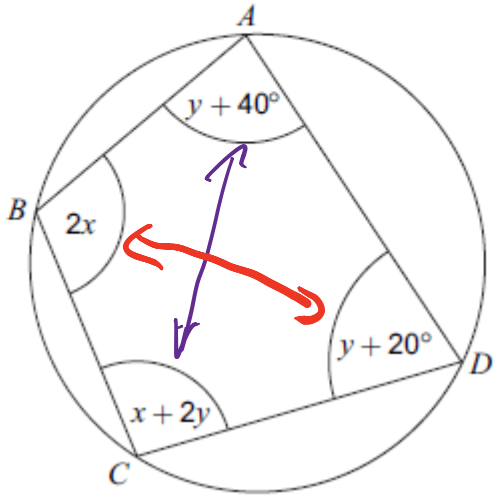
$= \frac{198}{594} = \frac{1}{3}$

$\therefore$ P(Liz picks a blue ball) = P(Mike picks a blue ball)

(Total 4 marks)

Q2.

$ABCD$ is a cyclic quadrilateral.



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Opposite angles in a cyclic quadrilateral sum to 180°

Work out the values of x and y.

$$x + 2y + y + 40 = 180$$
$$x + 3y = 140$$

$$2x + y + 20 = 180$$
$$2x + y = 160$$

$x + 3y = 140$ ---- ①

$2x + y = 160$ --- ②

$2x + 6y = 280$ ---- ① × 2 = ③

$5y = 120$ ---- ③ − ②

$\boxed{y = 24}$

$2x + 24 = 160$ ---- sub ②

$2x = 136$

$\boxed{x = 68}$

x = ...68°.......... , y = ...24°..........

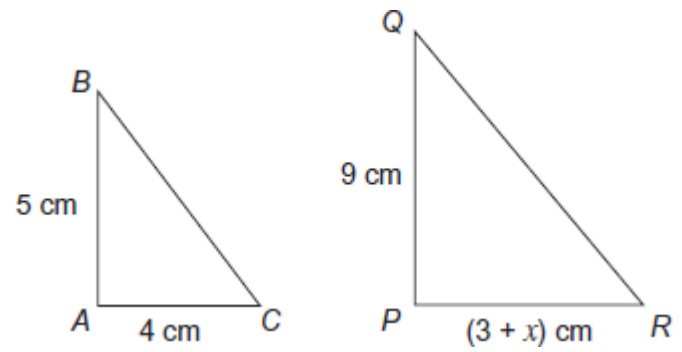
(Total 5 marks)

Check it works and makes the opposite angles sum to 180°

✓ It does! ☺

Q3. *ABC* and *PQR* are similar triangles.

Not drawn accurately



(a) Which **one** of the following equations is correct for these triangles?
Circle your answer.

$\frac{3+x}{4} = \frac{5}{9}$ $\frac{3+x}{9} = \frac{5}{4}$ $\frac{3+x}{5} = \frac{9}{4}$ $\frac{3+x}{4} = \frac{9}{5}$

(1)

(b) Solve the equation you circled to work out the value of x.

$$\frac{3+x}{4} = \frac{9}{5}$$

$$5(3+x) = 36$$

$$15+5x = 36$$

$$5x = 21$$

$$x = 4.2$$

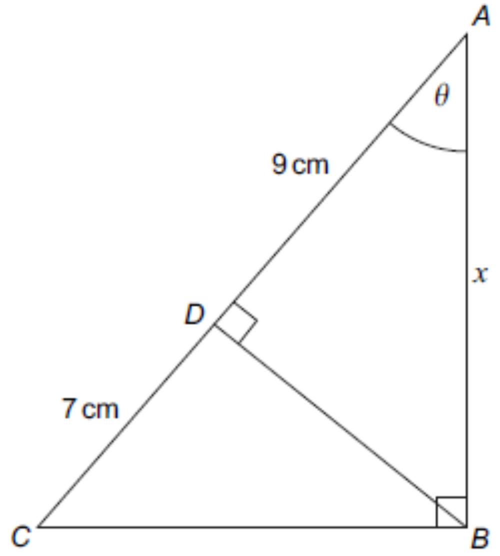
Multiply RHS by "4" and LHS by "5"

x = 4.2

(4)
(Total 5 marks)

Q4.

ABC is a right-angled triangle.
D is a point on *AC*.
BD is perpendicular to *AC*.



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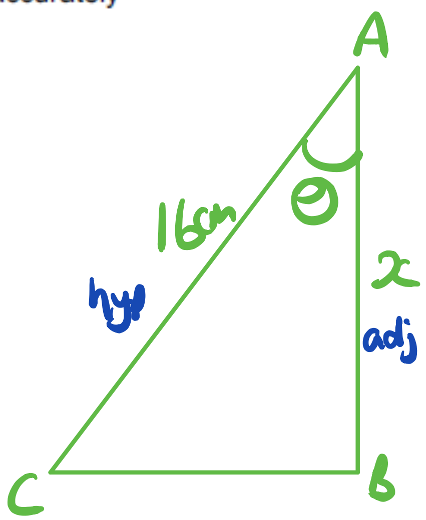


(a) Use triangle *ABC* to write cos θ in terms of x

$\cos\theta = \frac{adj}{hyp}$

cos θ = $\frac{x}{16}$

(1)

(b) By writing another expression for cos θ in terms of x, or otherwise, work out the value of x.

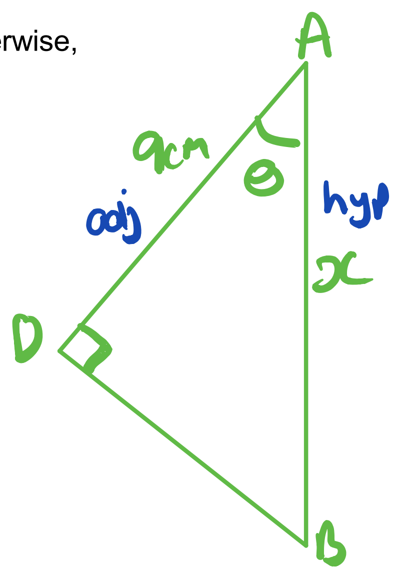
$\cos\theta = \frac{9}{x}$

$\therefore \frac{x}{16} = \frac{9}{x}$

Multiply LHS by "x" and RHS by "16"

$x^2 = 144$

$x = 12$



x = 12cm cm

(2)
(Total 3 marks)

Q5.

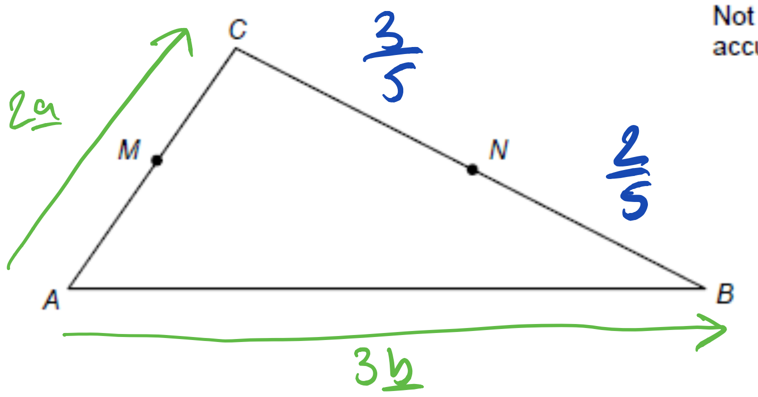
In triangle *ABC*

M is the midpoint of *AC*

N is the point on *BC* where *BN* : *NC* = 2 : 3

$\overrightarrow{AC} = 2\mathbf{a}$

$\overrightarrow{AB} = 3\mathbf{b}$



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(a) Work out $\overrightarrow{MN}$ in terms of **a** and **b**.

Give your answer in its simplest form.

$\overrightarrow{MC} = \underline{a}$

$\overrightarrow{CN} = \frac{3}{5}\overrightarrow{CB}$

$\overrightarrow{CB} = 3\underline{b} - 2\underline{a}$

$\overrightarrow{CN} = \frac{3}{5}(3\underline{b} - 2\underline{a})$

$\overrightarrow{CN} = \frac{9}{5}\underline{b} - \frac{6}{5}\underline{a}$

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$

$\overrightarrow{MN} = \underline{a} + \frac{9}{5}\underline{b} - \frac{6}{5}\underline{a}$

$\overrightarrow{MN} = -\frac{1}{5}\underline{a} + \frac{9}{5}\underline{b}$

$= \frac{1}{5}(-\underline{a} + 9\underline{b})$

Answer $\frac{1}{5}(9\underline{b} - \underline{a})$

(3)

(b) Use your answer to part (a) to explain why *MN* is **not** parallel to *AB*.

$\overrightarrow{MN}$ is not a scalar multiple of $\overrightarrow{AB}$

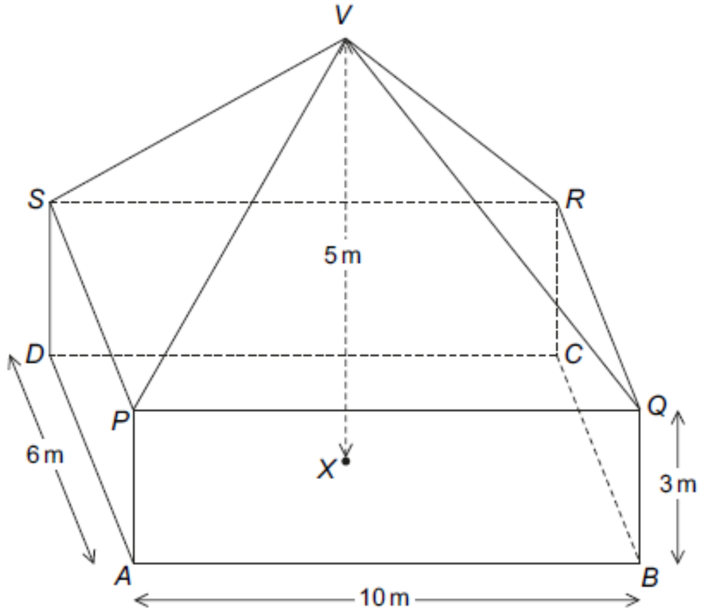
∴ not parallel

(1)

(Total 4 marks)

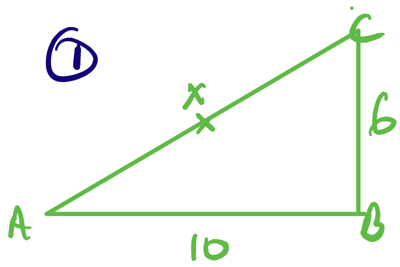
Q6.

The diagram shows a cuboid *ABCDPQRS* and a pyramid *PQRSV*.
V is directly above the centre, *X*, of *ABCD*.



The total height, *VX*, is 5 metres.

(a) Work out the angle between the line *VA* and the plane *ABCD*.



1 $AC = \sqrt{10^2 + 6^2} = 2\sqrt{34}$

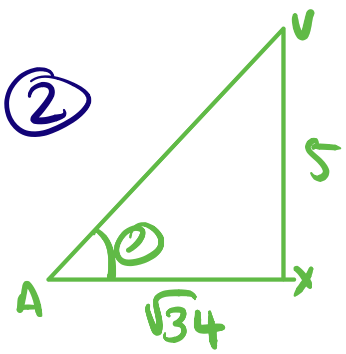
$\therefore AX = \sqrt{34}$

2 $\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{5}{\sqrt{34}}$

$\theta = \tan^{-1}\left(\frac{5}{\sqrt{34}}\right)$

$\theta = 40.6°$



Answer ....40.6.... degrees

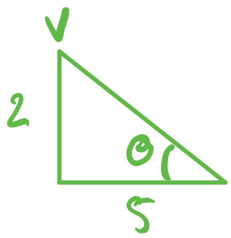
(4)

(b) Work out the angle between the planes *VQR* and *PQRS*.

$\tan\theta = \frac{2}{5}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta = 21.8°$



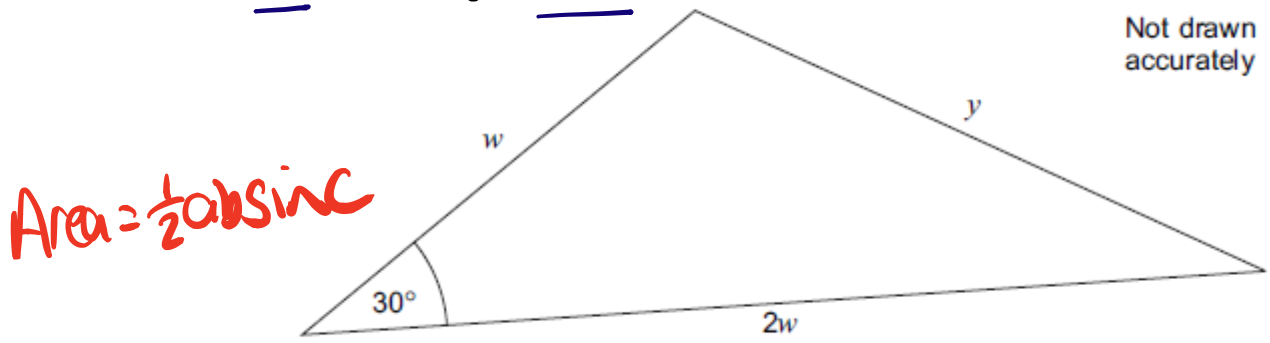
Answer ....21.8.... degrees

(2)
(Total 6 marks)

You should know everything in red

Q7.

The area of this triangle is 18cm²



Work out y.

$\frac{1}{2}ab\sin C = 18$

$\frac{1}{2}w \times 2w \sin 30° = 18$

$\frac{1}{2} \times 2w^2 \times \frac{1}{2} = 18$

$w^2 = 36$

$\boxed{w = 6}$

$\boxed{\sin 30° = \frac{1}{2}}$

Use cosine rule to work out y

$a^2 = b^2 + c^2 - 2bc\cos A$

$y^2 = 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos 30°$

$y^2 = 36 + 144 - 144 \times \frac{\sqrt{3}}{2}$

$\boxed{\cos 30° = \frac{\sqrt{3}}{2}}$

$y^2 = 180 - 72\sqrt{3}$

$y^2 = 55.2923 \ldots$

$y = 7.44\text{cm}$

$y =$ ......7.44...... cm

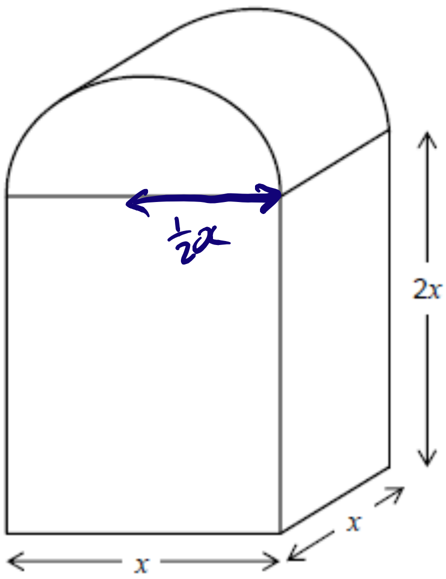
(Total 5 marks)

Q8.

In this question all dimensions are in centimetres.

A solid has uniform cross section.

The cross section is a rectangle and a semicircle joined together.



Work out an expression, in cm³, for the **total** volume of the solid.

Write your expression in the form $ax^3 + \frac{1}{b}\pi x^3$ where a and b are integers

Volume of cuboid = length × width × height

$= x \times x \times 2x = 2x^3$

Volume of cylinder $= \pi r^2 h$ ← you need to half this as it is a semi-cylinder

$= \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 \times x$

$= \frac{1}{2}\pi \times \frac{1}{4}x^2 \times x$

$= \frac{1}{8}\pi x^3$

Total volume $= 2x^3 + \frac{1}{8}\pi x^3$

Answer $2x^3 + \frac{1}{8}\pi x^3$ cm³

(Total 4 marks)

Q9. n is an integer.

$$S = \frac{1}{2}n(n+1)$$

Prove that $8S + 1$ is an odd square number.

$S = \frac{1}{2}n^2 + \frac{1}{2}n$

$8S = 8(\frac{1}{2}n^2 + \frac{1}{2}n)$

$= 4n^2 + 4n$

$8S + 1 = 4n^2 + 4n + 1$

$= (2n+1)(2n+1)$

$= (2n+1)^2$

$2n+1$ is odd

odd × odd = odd

$\therefore (2n+1)^2$ is an odd square number

(mic drop)

(Total 5 marks)

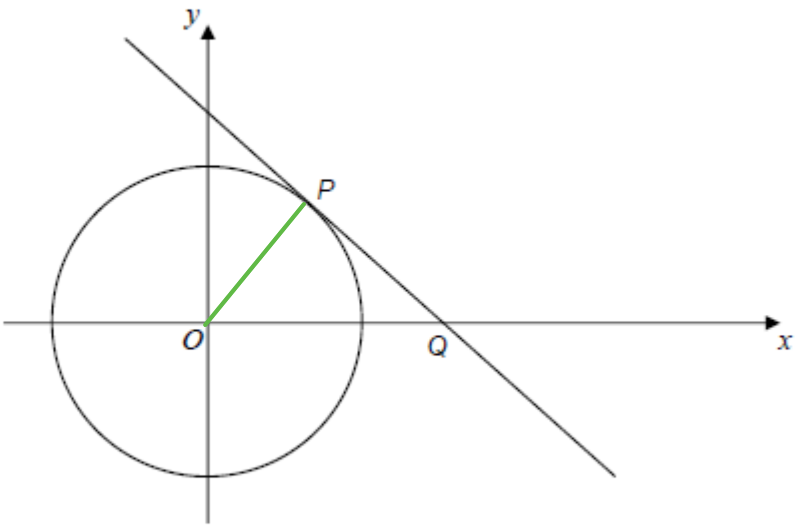
Q10.

The diagram shows the circle $x^2 + y^2 + 10$

P lies on the circle and has x-coordinate 1

The tangent at P intersects the x-axis at Q.

Not drawn accurately



Work out the coordinates of Q.

If $x = 1$ then $1^2 + y^2 = 10$
$y^2 = 9$
$y = 3$

$\therefore$ gradient of line OP = 3
$\therefore$ gradient of line PQ = $-\frac{1}{3}$

So $y = -\frac{1}{3}x + c$
Since this line passes through (1, 3)
$3 = -\frac{1}{3}(1) + c$
$c = \frac{10}{3}$
$\therefore y = -\frac{1}{3}x + \frac{10}{3}$

At point Q, $y = 0$
$\therefore 0 = -\frac{1}{3}x + \frac{10}{3}$
$0 = -x + 10$
$x = 10$

Answer (10, 0)

(Total 5 marks)