exampro

GCSE Mathematics (8300)

Name:

Class:

Higher Upper

1C

Author: Mr Ladak

Date:

Time: **1 hour**

Marks: 47 Marks

Comments:

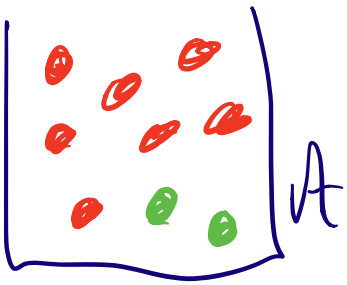
Q1.

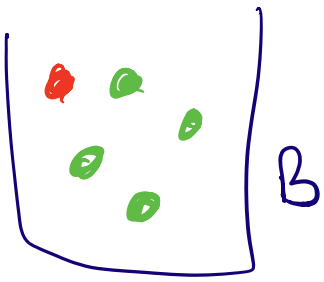
Two bags, A and B, contain beads.

Bag A contains 7 red beads and 2 yellow beads.
Bag B contains 1 red bead and 4 yellow beads.

One bead is taken at random from bag A and put into bag B.
One bead is then taken at random from bag B and put into Bag A.

Work out the probability that bag A still contains exactly 7 red beads.





2 possibilities for successful outcome.

① Red is taken out of 'A' and put into 'B' followed by a red out of 'B' and put into 'A'

② Yellow is taken out of 'A' and put into 'B' followed by a yellow out of 'B' and into 'A'

$\therefore$ P(Still 7 reds in A) = P(R, R) + P(Y, Y)

$= \frac{7}{9} \times \frac{2}{6} + \frac{2}{9} \times \frac{5}{6}$

Notice that there are now 6 beads in 'B' with 2 reds

$= \frac{14}{54} + \frac{10}{54}$

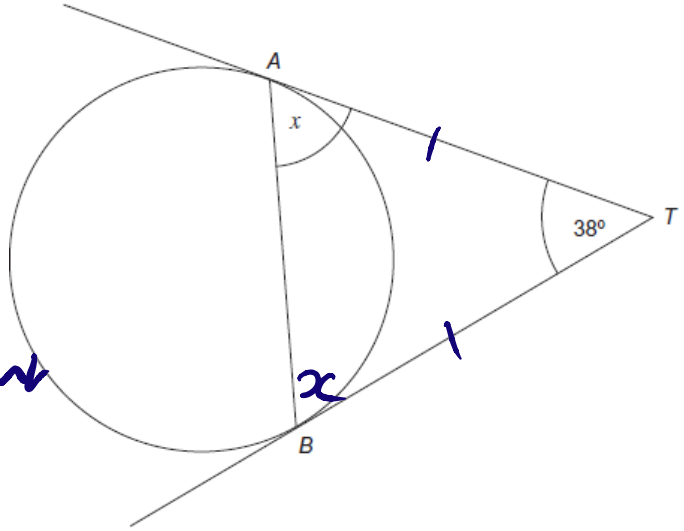
$= \frac{24}{54}$

$= \frac{4}{9}$

(Total 4 marks)

Q2.

(a) *AB* is a chord of the circle.
TA and *TB* are tangents to the circle.



Work out the size of angle x.

Two tangents that meet at a point are equidistant

$\therefore$ Δ ATB is isosceles

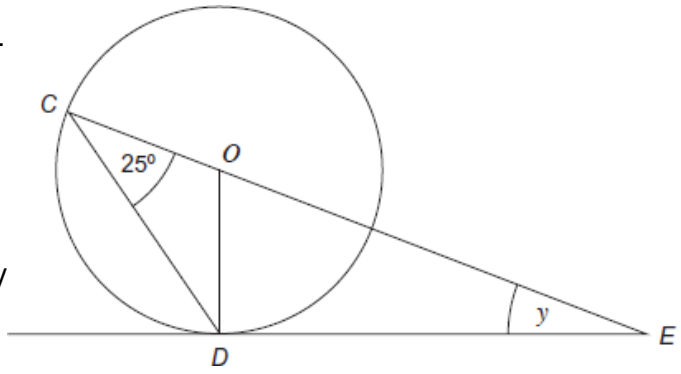
$2x + 38 = 180$

$2x = 142$

$x = 71°$

(2)

(b) The diagram shows a circle, centre O.
C and *D* are points on the circumference.
COE is a straight line.
DE is a tangent.



Work out the size of angle y.
You **must** show your working, which may

ΔOCD is isosceles as OC and OD are radii

$\therefore$ $C\hat{O}D = 180° - 25° - 25° = 130°$

COE is a straight line $\therefore$ $DOE = 180° - 130° = 50°$

Tangent and radius are perpendicular

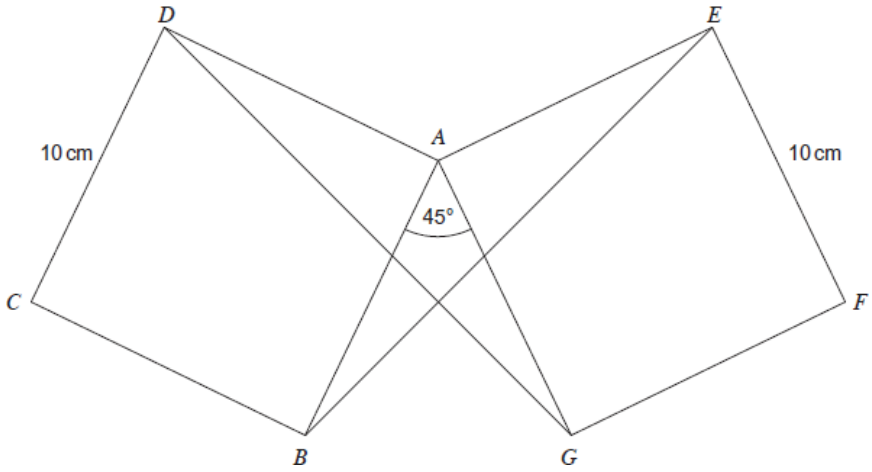
$\therefore$ $ODE = 90°$

$\therefore$ $50 + 90 + y = 180°$

$y = 40°$

(3)
(Total 5 marks)

Q3. $ABCD$ and $AEFG$ are identical squares.
$CD = EF = 10$ cm
Angle $BAG = 45°$



Prove that triangles AGD and ABE are congruent.

"$\cong$" means "congruent to"

"$\because$" means "because"

$AD \cong AE$ $\because$ identical squares

$AG \cong AB$ $\because$ identical squares

$DAB \cong EAG$ $\because$ angles in squares are 90°

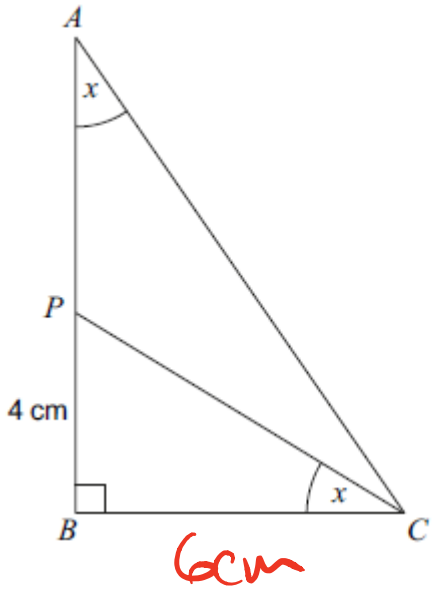
$DAG \cong EAB$ $\because$ $90° + 45° = 135°$

$\Delta DAG \cong \Delta EAB$ $\because$ SAS ← A triangle with 2 fixed sides and a fixed angle in between is unique.

(Total 4 marks)

Q4.

ABC is a right-angled triangle.
P is a point on AB.



Not drawn accurately

$BP = 4cm$ and $\tan x = \frac{2}{3}$

opp/adj

(a) Work out the length of BC.

$\tan x = \frac{opp}{adj} = \frac{BP}{BC}$

$\frac{2}{3} = \frac{4}{BC}$

$2BC = 12$

$BC = 6$

Answer 6 cm

(2)

(b) Work out the length of AP.

$\tan x = \frac{opp}{adj} = \frac{BC}{AB}$

$\frac{2}{3} = \frac{6}{AB}$

$2AB = 18$

$AB = 9$

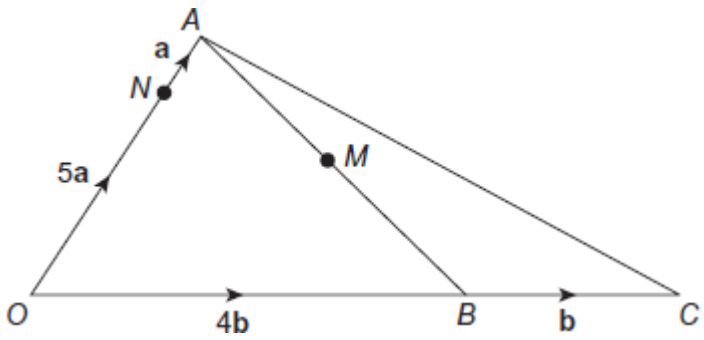
$\therefore AP = 9 - 4 = 5$

Answer 5 cm

(3)

(Total 5 marks)

Q5.



$\overrightarrow{ON} = 5\mathbf{a}$ $\overrightarrow{NA} = \mathbf{a}$

$\overrightarrow{OB} = 4\mathbf{b}$ $\overrightarrow{BC} = \mathbf{b}$

M is the midpoint of *AB*.

(a) Show that $\overrightarrow{NM} = 2(\mathbf{b} - \mathbf{a})$

$\overrightarrow{NM} = \overrightarrow{NA} + \overrightarrow{AM}$

$\overrightarrow{NM} = \overrightarrow{NA} + \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OB}$

$\overrightarrow{AB} = 4\underline{b} - 6\underline{a}$

$\overrightarrow{NM} = \underline{a} + \frac{1}{2}(4\underline{b} - 6\underline{a})$

$\overrightarrow{NM} = \underline{a} + 2\underline{b} - 3\underline{a}$

$\overrightarrow{NM} = 2\underline{b} - 2\underline{a}$

$\overrightarrow{NM} = 2(\underline{b} - \underline{a})$

(2)

(b) Work out the ratio *NM* : *NC*

$\overrightarrow{NC} = \overrightarrow{NO} + \overrightarrow{OC}$

$\overrightarrow{NC} = 5\underline{b} - 5\underline{a}$

$\overrightarrow{NC} = 5(\underline{b} - \underline{a})$

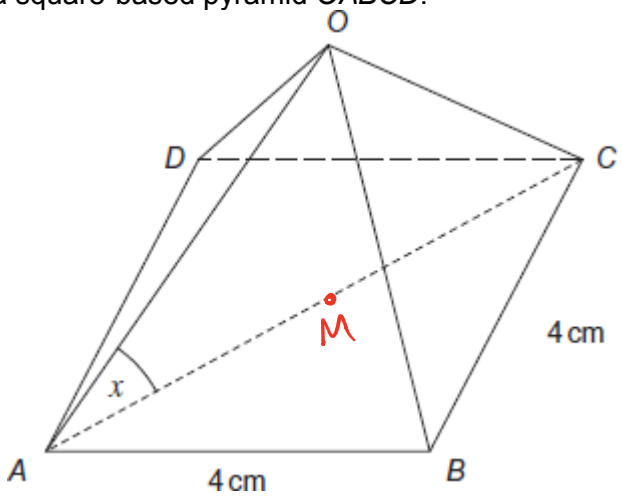
$\therefore$ NM : NC = 2 : 5

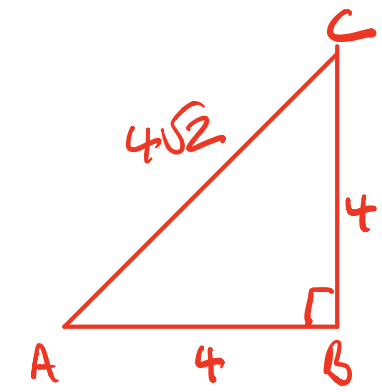
Answer ..... : .....

(2)
(Total 4 marks)

Q6.

The diagram shows a square-based pyramid *OABCD*.





$AC = \sqrt{4^2+4^2}$

$AC = 4\sqrt{2}$

OA = *OB* = *OC* = *OD* = 6 cm
AB = *BC* = 4 cm

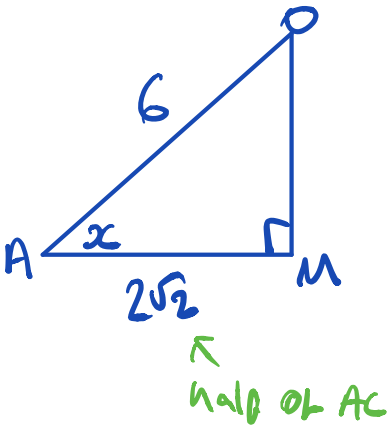
Work out the size of the angle between *OA* and the base *ABCD*, marked x on the diagram.

$\cos x = \frac{adj}{hyp}$

$\cos x = \frac{2\sqrt{2}}{6}$

$x = \cos^{-1}\left(\frac{2\sqrt{2}}{6}\right)$

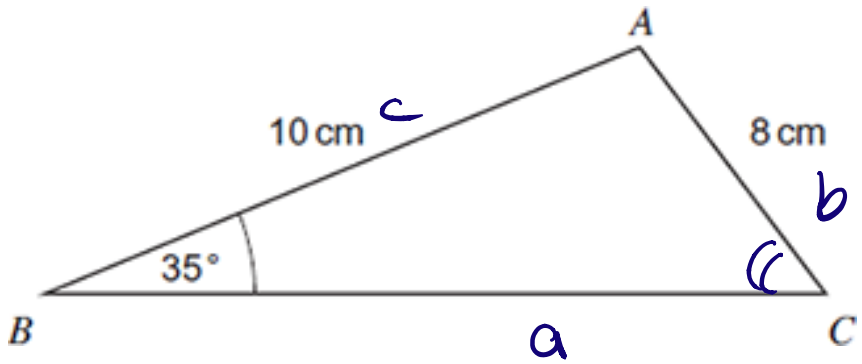
$x = 61.9°$



Answer ..... degrees

(Total 4 marks)

Q7. In the diagram, angle A is obtuse.



Work out the size of angle A.

$$\frac{\sin C}{10} = \frac{\sin 35^\circ}{8}$$

$$\sin C = \frac{10 \sin 35^\circ}{8}$$

$$\sin C = 0.71697\ldots$$

$$C = 45.8^\circ$$

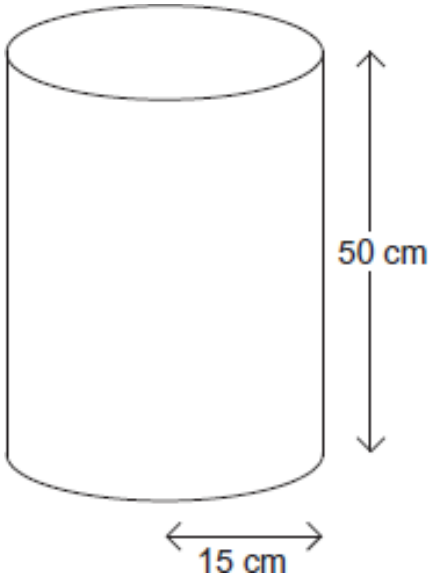
$$A = 180 - 35 - 45.8^\circ$$

$$A = 99.2^\circ$$

Answer 99.2 degrees

(Total 4 marks)

Q8.A tank is in the shape of a cylinder of radius 15 cm and height 50 cm



(a) Work out the volume of the tank.

$V = \pi r^2 h$

$V = \pi \times 15^2 \times 50$

Answer 11250π cm³

(3)

(b) The volume of another tank is 33 000 cm³

The tank is empty.
The tank is filled at the rate of 0.22 litres a second.

How many **minutes** will it take to fill the tank?

33000 cm³ = 33 litres

$\frac{33}{0.22} = 150$ seconds

2 minutes 30 seconds.

Answer 2.5 minutes

(4)
(Total 7 marks)

Q9. Two integers have a difference of 3. n and $n+3$
The difference between the squares of the two integers is three times the sum of the integers.

For example, $13 - 10 = 3$, $13^2 - 10^2 = 169 - 100 = 69$

and $3 \times (13 + 10) = 3 \times 23 = 69$

Prove this result algebraically.

$(n+3)^2 - n^2$

$\equiv n^2 + 6n + 9 - n^2$

$\equiv 6n + 9$

$= 3(2n+3)$

$3(n + n+3)$

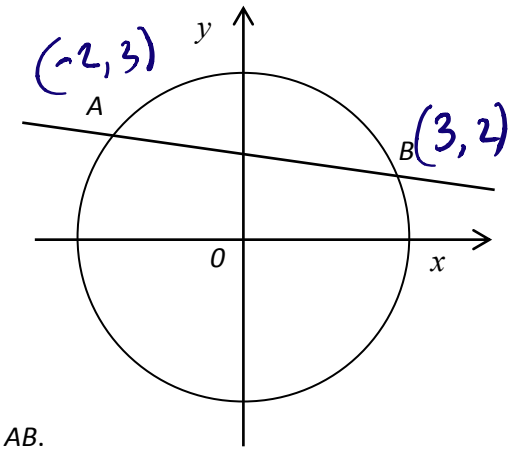
$\equiv 3(2n+3)$

$\therefore$ the result is true for all integers

(Total 4 marks)

Q10

The circle $x^2 + y^2 = 13$ and the line $5y + x = 13$ intersect at points *A* and *B*.



Work out the length of the chord *AB*.

$x = 13 - 5y$

$x^2 + y^2 = 13$

$(13 - 5y)^2 + y^2 = 13$

$169 - 130y + 25y^2 + y^2 = 13$

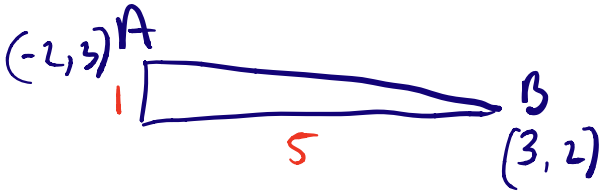
$26y^2 - 130y + 156 = 0$

$y^2 - 5y + 6 = 0$

$(y - 2)(y - 3) = 0$

$y = 2$ $\quad x = 13 - 5 \times 2 = 3$ $\quad (3, 2)$

$y = 3$ $\quad x = 13 - 5 \times 3 = -2$ $\quad (-2, 3)$



$AB = \sqrt{1^2 + 5^2}$

$AB = \sqrt{26}$

[6 marks]