3 At time t seconds, a force  $(4t + 3 e^{-2t})$  N acts on a particle of mass 3 kg.

The particle can only move in the direction of the force.

When t = 0, the particle has velocity 5 m s<sup>-1</sup>.

Find the velocity of the particle after 4 seconds.

(5 marks)

| 3 | $F = 4t + 3e^{-2t}$                                                |            |   |     |  |
|---|--------------------------------------------------------------------|------------|---|-----|--|
|   | Using change in momentum = $\int F dt$                             | M1         |   |     |  |
|   | change in momentum = $\int (4t + 3e^{-2t}) dt$                     |            |   |     |  |
|   | $=2t^2 - \frac{3}{2}e^{-2t} + c$                                   | M1         |   |     |  |
|   | When $t = 0, c = \frac{3}{2}$                                      | M1         |   |     |  |
|   | New momentum $mv = mu + 2t^2 - \frac{3}{2}e^{-2t} + \frac{3}{2}$   |            |   |     |  |
|   | $3v = 3 \times 5 + 2 \times 4^2 - \frac{3}{2}e^{-8} + \frac{3}{2}$ | <b>A</b> 1 |   |     |  |
|   | $v = \frac{97}{6} - \frac{1}{2}e^{-8}$ m/s                         | <b>A</b> 1 |   |     |  |
|   | Total                                                              |            | 5 | 1.0 |  |

4 A shell lying at rest on the ground explodes into two parts, one of mass 8m and the other of mass m. Both parts move horizontally.

The relative velocity of the two parts immediately after the explosion is 36u.

Find the speeds of the two parts immediately after the explosion.

(6 marks)

| 4 | $v \leftarrow 8m \pmod{m} \rightarrow V$          |            |     |  |
|---|---------------------------------------------------|------------|-----|--|
|   | C of Momentum $8m.v - mV = 0$                     | M1 A1      |     |  |
|   | Relative velocity $v + V = 36u$                   | M1 A1      |     |  |
|   | v = 4u  V = 32u                                   | <b>A</b> 1 |     |  |
|   | Parts move in opposite directions:                |            |     |  |
|   | shown by diagram                                  |            |     |  |
|   | OR by correcting – sign in velocity to give speed | B1         | (6) |  |
|   |                                                   | TOTAL      | (6) |  |

- A squash ball, of mass 40 grams, is travelling horizontally at  $12 \,\mathrm{m\,s^{-1}}$ . It strikes a wall which is perpendicular to its velocity and rebounds directly with speed  $8 \,\mathrm{m\,s^{-1}}$ .
  - (a) Calculate the impulse acting on the ball.

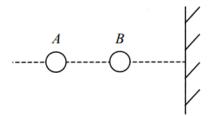
(4 marks)

(b) The ball is in contact with the wall for 0.05 seconds. Find the magnitude of the average force acting on the ball during the time that it is in contact with the wall. (2 marks)

| Question | Solution                          | Marks | Total | Comments         |
|----------|-----------------------------------|-------|-------|------------------|
| number   |                                   |       | marks |                  |
| and part |                                   |       |       |                  |
| 1(a)     | Change in momentum is             |       |       |                  |
|          | $0.04 \times 12 - 0.04 \times -8$ | B1    |       | Conversion to kg |
|          |                                   | M1    |       |                  |
|          |                                   | B1    |       | Correct signs    |
|          | Impulse is 0.8 Ns                 | A1    | 4     | - 0.8 B2 M1      |
| (b)      | Using Force × time = impulse      |       |       |                  |
|          | $Force = \frac{0.8}{0.05}$        | M1    |       |                  |
|          | = 16  N                           | A1√   | 2     | ft               |
| ·        | Total                             | ·     | 6     |                  |

5 Two spheres, A of mass 2m and B of mass m, are at rest on a smooth horizontal surface.

The centres of the spheres lie on a straight line which is perpendicular to a fixed smooth vertical wall. All motion takes place in the direction of this line.



Sphere A is hit by a blow of impulse J and moves with a speed of 3u directly towards the wall. Model the spheres as particles.

(a) Find J in terms of m and u. (2 marks)

(b) Sphere A collides directly with sphere B which is at rest.

The coefficient of restitution in this collision is  $\frac{3}{5}$ .

Find the speeds of A and B after this impact in terms of u. (6 marks)

(c) Sphere B now hits the wall.

The coefficient of restitution between the sphere and the wall is  $\frac{1}{2}$ .

Find the speed of B after it strikes the wall. (2 marks)

(d) Initially sphere B is a distance 4a from the wall.

Find the distance of B from the wall when B and A collide again. (7 marks)

| Question<br>Number<br>and part | Solution                                                                           | Marks | Total | Comments        |
|--------------------------------|------------------------------------------------------------------------------------|-------|-------|-----------------|
| 5(a)                           | Impulse = change in momentum                                                       | M1    |       |                 |
|                                | $J = 2m \times 3\mu$                                                               |       |       |                 |
|                                | J = 6mu                                                                            | A1    | 2     |                 |
| (b)                            | Conservation of momentum                                                           | M1    |       |                 |
|                                | $6mu = 2mv_a + mv_b$                                                               | A1    |       |                 |
|                                | Restitution                                                                        | M1    |       |                 |
|                                | $3u \times e = 3u \times \frac{3}{5} = v_a - v_a$                                  | A1    |       |                 |
|                                | $6u = 2v_a + v_b$                                                                  |       |       |                 |
|                                | Subtracting $\frac{21}{5}u = 3v_a$                                                 |       |       |                 |
|                                | $\frac{7}{5}u = v_a$                                                               | A1    |       |                 |
|                                | $6u + \frac{18}{5}u = 3v_b$                                                        |       |       |                 |
|                                | $\mathbf{v}_b = \frac{16}{5}u$                                                     | A1    | 6     |                 |
| (c)                            | Velocity after impact is $e \times \text{velocity}$<br>before impact               | M1    |       |                 |
|                                | $= \frac{1}{2} \times \frac{16}{5} u$ $= \frac{8}{5} u$                            | A1√   | 2     | ft [from (b)]   |
| (d)                            | Time taken for B to strike wall after collision is $\frac{4a}{16u}$                | M1    |       |                 |
|                                | $= \frac{5a}{4u}$ In this time A has travelled $\frac{7}{5}u \times \frac{5a}{4u}$ | Al√   |       | ft [from (b)]   |
|                                | $=\frac{7}{4}a$                                                                    | B1    |       |                 |
|                                | Two spheres are $\frac{9}{4a}$ apart                                               | B1√   |       | ft [from above] |
|                                | Speeds of spheres are (A) $\frac{7}{5}u$ and (B) $\frac{8}{5}u$                    | A1    |       |                 |
|                                | Distance from the wall is $\frac{8}{15} \times \frac{9}{4} a$                      | M1    |       |                 |
|                                | $=\frac{6}{5}a$                                                                    | A1    | 7     |                 |
|                                | Total                                                                              |       | 17    |                 |