4 A smooth sphere, A, has mass $3m$ and velocity $7\mathbf{i} - 8\mathbf{j}$. It collides with a second smooth sphere, B, which has mass m and velocity $2\mathbf{i} + 5\mathbf{j}$. The two spheres have the same radius. After the collision, the velocity of B is $5\mathbf{i} - 4\mathbf{j}$.

(a) Find the velocity of A after the collision. *(4 marks)*

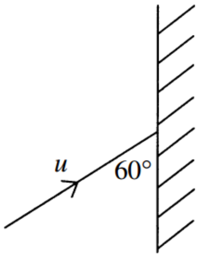
(b) Find the change in momentum of B. *(2 marks)*

(c) Find, as a vector, the direction of the line of centres of the spheres during the collision. Give a reason for your answer. *(2 marks)*

Question number and part	Solution	Marks	Total marks	Comments
4(a)	Using conservation of momentum	M1		
	$3m\begin{pmatrix}7\\-8\end{pmatrix} + m\begin{pmatrix}2\\5\end{pmatrix} = m\begin{pmatrix}5\\-4\end{pmatrix} + 3m\,\mathbf{v}$	A1		
	$\begin{pmatrix}21\\-24\end{pmatrix} + \begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix} + 3\,\mathbf{v}$	M1		
	$3\,\mathbf{v} = \begin{pmatrix}18\\-15\end{pmatrix}$			
	$\mathbf{v} = \begin{pmatrix}6\\-5\end{pmatrix}$	A1	4	
(b)	Change in momentum =			
	$m\begin{pmatrix}5\\-4\end{pmatrix} - m\begin{pmatrix}2\\5\end{pmatrix}$	M1		M1 for $-3m\mathbf{i} + 9m\mathbf{j}$
	$= 3m\mathbf{i} - 9m\mathbf{j}$	A1	2	**sc** 1 for $3\mathbf{i} - 9\mathbf{j}$
(c)	Direction is $\mathbf{i} - 3\mathbf{j}$ oe	B1√		ft from (b)
	Line of centres is parallel to the change in momentum	B1	2	
	Total		**8**	

5 A sphere of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall. Just before it hits the wall, the sphere is moving at an angle of $60°$ to the wall with velocity u.

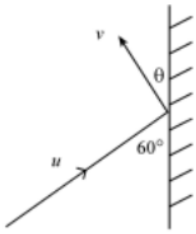
The diagram shows the view from above.



The coefficient of restitution between the wall and the sphere is $\frac{3}{4}$.

(a) Modelling the sphere as a particle, find the angle through which the direction of motion of the sphere is changed. *(6 marks)*

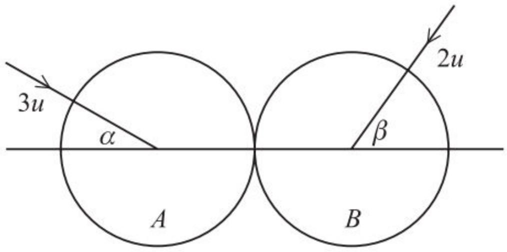
(b) The impulse exerted by the wall on the sphere acts on the sphere for 0.05 seconds. Given that $m = 0.3\,\text{kg}$, and $u = 5\,\text{m}\,\text{s}^{-1}$, find the average impulsive force acting on the sphere. *(7 marks)*

5(a)				
	Velocity // wall unaltered	M1		
	$u \cos 60 = v \cos \theta$	A1		
	Velocity perp to wall			
	$e\,u \sin 60 = v \sin \theta$	M1A1		
	Dividing: $\tan\theta = e \tan 60$			
	$= \frac{3}{4}\sqrt{3}$			
	$\therefore\ \theta = 52.4°$	A1		
	$\therefore$ Direction of motion is changed by 112.4°	A1	6	
(b)	Impulse is change in momentum perp to wall	M1		
	$= mu \sin 60 + mv \sin\theta$	M1		
	$= mu \sin 60\,(1 + e)$	A1		
	$= 0.3 \times 5 \times \frac{\sqrt{3}}{2} \times 1.75$			
	$= \frac{21}{16}\sqrt{3}$	A1		
	Time × Impulse = change in momentum	M1		
	$\therefore$ Impulse $= 20 \times \frac{21\sqrt{3}}{16}$	A1		
	$= 45.5$	A1	7	
	Total		**13**	

7 Two smooth spheres, A and B, of equal radius and masses m and M respectively, are moving on a horizontal plane. Sphere A has speed $3u$, and sphere B has speed $2u$ and is approaching sphere A. The spheres collide and the velocities of the spheres before impact make acute angles α and β with the line of centres, as shown in the diagram.

$$\tan\,\alpha = \tfrac{3}{4} \text{ and } \tan\,\beta = \tfrac{12}{5}$$

The coefficient of restitution between the spheres is e.



After the collision, the velocity of B is in the direction of the velocity of A before the impact.

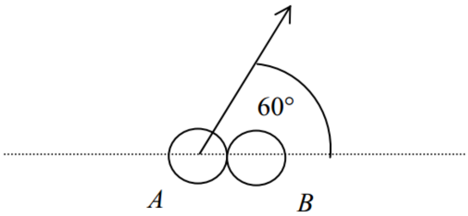
Show that

$$e = \frac{2m + 105M}{103m}$$

(9 marks)

7	Let v_1 and v_2 be components of velocities of A and B along the line of centres after collision Let v_3 and v_4 be components of velocities of A and B perpendicular to the line of centres after collision Velocities perpendicular to line of centres are unaltered			
	$\therefore v_4 = 2u\sin\beta = \frac{24}{13}u$	B1		using $\tan\beta = 12/5$
	$3um\cos\alpha - 2uM\cos\beta = mv_1 + Mv_2$ **(1)**	M1 A1		using conservation of momentum along line of centres
	$e(3u\cos\alpha + 2u\cos\beta) = v_2 - v_1$	M1 A1		using restitution equation
	$e\left(3.\frac{4}{5} + 2.\frac{5}{13}\right)u = v_2 - v_1$			
	$\frac{206}{65}eu = v_2 - v_1$ **(2)**			
	$(m+M)v_2 = \frac{206}{65}meu + \frac{12}{5}um - \frac{10}{13}uM$ $= \frac{(206me + 156m - 50M)\,u}{65}$			from m **(2)** + **(1)**
	$v_2 = \frac{(206me + 156m - 50M)u}{65(m+M)}$	M1 A1		
	$\tan\alpha = \frac{v_4}{v_2} \Rightarrow 3v_2 = 4v_4$	M1		since direction at B is now old direction of A
	$\frac{96}{13}u = \frac{3(206me + 156m - 50M)u}{65(m+M)}$ $480m + 480M = 618me + 468m - 150M$ $12m + 630M = 618me$ $e = \frac{2m + 105M}{103m}$	A1		
	Total		**9**	

7 Two smooth spheres, A and B, have mass m and $2m$ respectively. Sphere A is moving with a constant velocity of 5 m s^{-1} when it collides with sphere B, which was at rest. The velocity of A was at an angle of 60° to the line of centres of the sphere when the collision took place. The coefficient of restitution between the two sphere is $\frac{1}{2}$.



(a) Show that the speed of B after the collision is $\frac{5}{4}$ m s^{-1}. *(7 marks)*

(b) Find the speed of A after the collision. *(4 marks)*

7(a)	$5m\cos 60° = m \times v_A \cos\alpha + 2mv_B$	M1		
	$\frac{5}{2} = v_A \cos\alpha + 2v_B$	A1		
	$v_A \cos\alpha - v_B = -\frac{1}{2}(5\cos 60°)$	M1A1		
	$v_A \cos\alpha = v_B - \frac{5}{4}$			
	$\frac{5}{2} = 3v_B - \frac{5}{4}$	M1A1		
	$v_B = \frac{15}{12} = \frac{5}{4}$	A1	7	
(b)	$v_A \sin\alpha = 5\sin 30° = \frac{5\sqrt{3}}{2}$	B1		
	$v_A \cos\alpha = \frac{5}{4} - \frac{5}{4} = 0$	M1		
	$v_A = \frac{5\sqrt{3}}{2} = 4.33$ m s^{-1}	M1A1	4	
	Total		**11**	