2D Collisions Challenge

Challenge 1

A smooth sphere, A, has mass $3m$ and velocity $7\mathbf{i} - 8\mathbf{j}$. It collides with a second smooth sphere, B, which has mass m and velocity $2\mathbf{i} + 5\mathbf{j}$. The two spheres have the same radius. After the collision, the velocity of B is $5\mathbf{i} - 4\mathbf{j}$.

(a) Find the velocity of A after the collision. *(4 marks)*

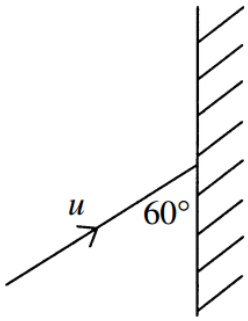
(b) Find the change in momentum of B. *(2 marks)*

(c) Find, as a vector, the direction of the line of centres of the spheres during the collision. Give a reason for your answer. *(2 marks)*

Challenge 2

A sphere of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall. Just before it hits the wall, the sphere is moving at an angle of 60° to the wall with velocity u.

The diagram shows the view from above.



The coefficient of restitution between the wall and the sphere is $\frac{3}{4}$.

(a) Modelling the sphere as a particle, find the angle through which the direction of motion of the sphere is changed. *(6 marks)*

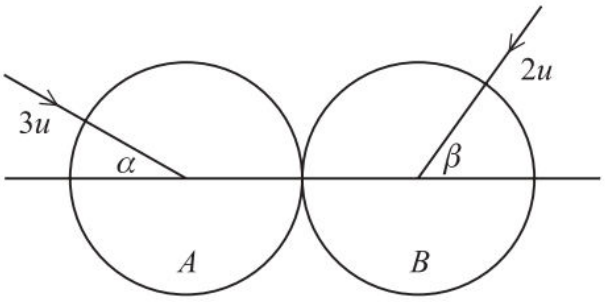
(b) The impulse exerted by the wall on the sphere acts on the sphere for 0.05 seconds. Given that $m = 0.3\,\text{kg}$, and $u = 5\,\text{m}\,\text{s}^{-1}$, find the average impulsive force acting on the sphere. *(7 marks)*

Challenge 3

Two smooth spheres, A and B, of equal radius and masses m and M respectively, are moving on a horizontal plane. Sphere A has speed $3u$, and sphere B has speed $2u$ and is approaching sphere A. The spheres collide and the velocities of the spheres before impact make acute angles α and β with the line of centres, as shown in the diagram.

$$\tan \alpha = \frac{3}{4} \text{ and } \tan \beta = \frac{12}{5}$$

The coefficient of restitution between the spheres is e.



After the collision, the velocity of B is in the direction of the velocity of A before the impact.

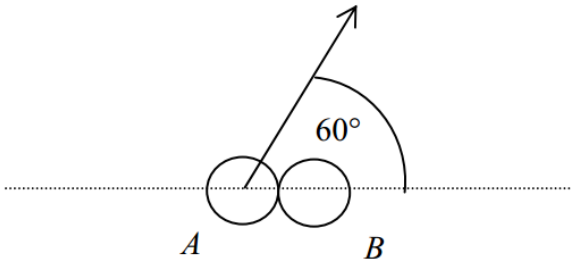
Show that

$$e = \frac{2m + 105M}{103m}$$

(9 marks)

Final Challenge

Two smooth spheres, A and B, have mass m and $2m$ respectively. Sphere A is moving with a constant velocity of 5 m s^{-1} when it collides with sphere B, which was at rest. The velocity of A was at an angle of 60° to the line of centres of the sphere when the collision took place. The coefficient of restitution between the two sphere is $\frac{1}{2}$.



(a) Show that the speed of B after the collision is $\frac{5}{4}$ m s^{-1}. *(7 marks)*

(b) Find the speed of A after the collision. *(4 marks)*