Calculus

The function f is defined for $x \geqslant 0$ by

$$f(x) = x^{\frac{1}{2}} + 2.$$

(a) (i) Find $f'(x)$. *(2 marks)*

(ii) Hence find the gradient of the curve $y = f(x)$ at the point for which $x = 4$. *(1 mark)*

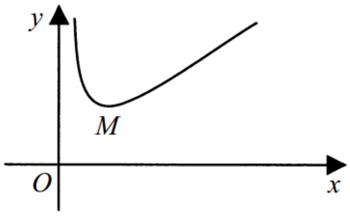
(b) (i) Find $\int f(x)\,dx$. *(3 marks)*

(ii) Hence show that $\int_0^4 f(x)\,dx = \frac{40}{3}$. *(2 marks)*

(c) Show that $f^{-1}(x) = (x-2)^2$. *(2 marks)*

Q	Solution	Marks	Total	Comments
6 (a)(i)	$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$	M1A1	2	M1 if coefficient or index correct
(ii)	Gradient at $x = 4$ is $\frac{1}{4}$	A1F	1	ft wrong coeff
(b)(i)	$\int f(x)dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \ldots$	M1A1		M1 for $kx^{\frac{3}{2}}$
	$\ldots + 2x\ (+c)$	B1	3	
(ii)	Substituting $x = 4$	M1		In c's integral (not f(x) or $f'(x)$)
	$\int_0^4 f(x)dx = \frac{40}{3}$	A1	2	Convincingly found (AG)
(c)	$y = x^{\frac{1}{2}} + 2 \Rightarrow x^{\frac{1}{2}} = y - 2$	M1		OE
	$\ldots \Rightarrow x = (y-2)^2$, hence result	A1	2	Convincingly shown (AG)

The curve with equation $y = 2x + \frac{27}{x^2} - 7$ is defined for $x > 0$, and is sketched below.



(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

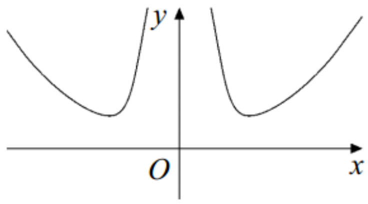
(ii) The curve has a minimum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\int \left(2x + \frac{27}{x^2} - 7\right) dx$. *(3 marks)*

(ii) Hence determine the area of the region bounded by the curve, the lines $x = 1$, $x = 2$ and the x-axis. *(2 marks)*

Question Number and part	Solution	Marks	Total marks	Comments
6(a)(i)	$\frac{dy}{dx} = 2 - \frac{54}{x^3}$	M1 A1 A1	3	Clearly attempting to differentiate One term correct but NOT $2 + f(x) - 7$ All correct (withhold if +c in answer)
(ii)	Putting " their " $\frac{dy}{dx} = 0$	M1		
	$x^3 = 27$	m1		Attempt at $x^3 = \ldots$ or $x = \ldots$ etc
	$x = 3$	A1	3	
(b)(i)	$x^2 - \frac{27}{x} - 7x \quad (+c)$	M1 A1 A1	3	Clearly attempting to integrate Two terms correct All correct (ignore omission of $+c$)
(ii)	$[4 - 13.5 - 14] - [1 - 27 - 7]$	M1		Evaluating limits at 1 and 2
	$= 9.5$	A1	2	
	Total		**11**	

A curve has equation $y = x^2 + \frac{81}{x^2}$. Its graph is sketched below.



(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the stationary points of the curve occur when $x^4 = 81$. *(2 marks)*

(iii) Hence find the x-coordinates of the stationary points. *(2 marks)*

(iv) Find the value of the y-coordinate at each stationary point. *(1 mark)*

(b) (i) Find $\int \left(x^2 + \frac{81}{x^2}\right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve, the lines $x = 1$, $x = 3$ and the x-axis. *(2 marks)*

7(a)(i)	$\frac{dy}{dx} = 2x$	B1		
	$-\frac{162}{x^3}$	M1		Power x^{-3}
		A1	3	
(ii)	$2x - \frac{162}{x^3} = 0$	M1		Putting candidate's $\frac{dy}{dx} = 0$
	$\Rightarrow x^4 = 81$ **ag**	A1	2	M1 only for verification $x = \pm 3$
(iii)	$x^2 = 9$ or $x = \sqrt[4]{81}$	M1		Or $x = 3$ as only value given
	$x = \pm 3$	A1	2	
(iv)	$y = 18$	B1	1	No need to show both equal 18 B0 if 2 different y values given
(b)(i)	$\frac{x^3}{3}$ $-\frac{81}{x}$ $(+C)$	B1 M1 A1	3	x^3 term x^{-1} power correct second term
(ii)	$\left[\frac{27}{3} - \frac{81}{3}\right] - \left[\frac{1}{3} - 81\right]$	M1		Correct use of limits 1 and 3 substituted into answer for part (b)(i)
	$62\frac{2}{3}$	A1	2	Accept 62.7 or better, condone 62.66 etc
	Total		**13**	

The curve C has the equation

$$y = 3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}, \quad x > 0.$$

(a) Find the coordinates of the points where C crosses the x-axis. **(4)**

(b) Find the exact coordinates of the stationary point of C. **(5)**

(c) Determine the nature of the stationary point. **(2)**

(d) Sketch the curve C. **(2)**

(a) $3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = 0, \quad 3x^{\frac{1}{2}} - x - 2 = 0$ M1

$x - 3x^{\frac{1}{2}} + 2 = 0, \quad (x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 2) = 0$ M1

$x^{\frac{1}{2}} = 1, 2$ A1

$x = 1, 4 \;\therefore (1, 0), (4, 0)$ A1

(b) $\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$ M1 A1

for minimum, $-\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}} = 0$ M1

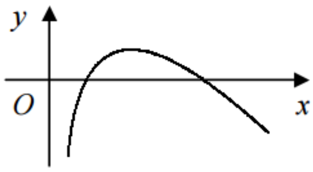
$-\frac{1}{2}x^{-\frac{3}{2}}(x - 2) = 0$

$x = 2, \; y = 3 - \sqrt{2} - \frac{2}{\sqrt{2}} \;\therefore (2, 3 - 2\sqrt{2})$ A2

(c) $\frac{d^2y}{dx^2} = \frac{1}{4}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$ M1

when $x = 2$, $\frac{d^2y}{dx^2} = \frac{1}{8\sqrt{2}} - \frac{3}{8\sqrt{2}} = -\frac{1}{4\sqrt{2}}, \; \frac{d^2y}{dx^2} < 0 \;\therefore$ maximum A1

(d)



B2

(13)