Core 2 - Calculus

Challenge 1

The function f is defined for $x \geqslant 0$ by

$$\mathrm{f}(x) = x^{\frac{1}{2}} + 2\,.$$

(a) (i) Find $\mathrm{f}'(x)$. *(2 marks)*

(ii) Hence find the gradient of the curve $y = \mathrm{f}(x)$ at the point for which $x = 4$. *(1 mark)*

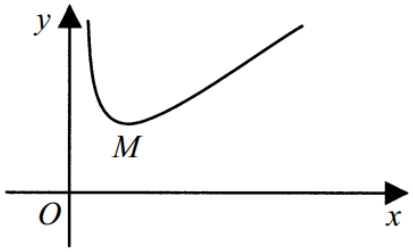
(b) (i) Find $\int \mathrm{f}(x)\,\mathrm{d}x$. *(3 marks)*

(ii) Hence show that $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x = \frac{40}{3}$. *(2 marks)*

(c) Show that $\mathrm{f}^{-1}(x) = (x-2)^2$. *(2 marks)*

Challenge 2

The curve with equation $y = 2x + \frac{27}{x^2} - 7$ is defined for $x > 0$, and is sketched below.



(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

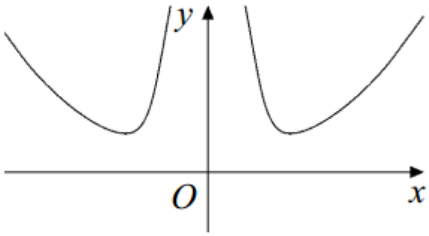
(ii) The curve has a minimum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\int \left(2x + \frac{27}{x^2} - 7\right) dx$. *(3 marks)*

(ii) Hence determine the area of the region bounded by the curve, the lines $x = 1$, $x = 2$ and the x-axis. *(2 marks)*

Challenge 3

A curve has equation $y = x^2 + \frac{81}{x^2}$. Its graph is sketched below.



(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the stationary points of the curve occur when $x^4 = 81$. *(2 marks)*

(iii) Hence find the x-coordinates of the stationary points. *(2 marks)*

(iv) Find the value of the y-coordinate at each stationary point. *(1 mark)*

(b) (i) Find $\int \left(x^2 + \frac{81}{x^2}\right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve, the lines $x = 1$, $x = 3$ and the x-axis. *(2 marks)*

Final Challenge

The curve C has the equation

$$y = 3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}, \quad x > 0.$$

(a) Find the coordinates of the points where C crosses the x-axis. **(4)**

(b) Find the exact coordinates of the stationary point of C. **(5)**

(c) Determine the nature of the stationary point. **(2)**

(d) Sketch the curve C. **(2)**