# FP2 – Complex Numbers

## Challenge 1

(a) Sketch on an Argand diagram the circle with equation

$$|z - 1 - i| = \sqrt{2}.$$ *(2 marks)*

(b) The point $P$ lies on the circle and represents the complex number $z$. Show on your Argand diagram the position of $P$ when the value of $|z + 1 + i|$ is as large as possible, and determine this largest value. *(3 marks)*

# Challenge 2

(a) Draw an Argand diagram to show the points $A$ and $B$ which represent the complex numbers $1 - 3i$ and $5 - i$ respectively. *(1 mark)*

(b) (i) The circle $C$ has $AB$ as a diameter. Find its radius and the coordinates of its centre. *(4 marks)*

(ii) Write down the equation of $C$ in the form

$$|z - z_0| = k.$$ *(2 marks)*

# Challenge 3

The complex numbers $z_1$ and $z_2$ are given by

$$z_1 = 1 + \sqrt{3}\,\mathrm{i} \qquad \text{and} \qquad z_2 = \mathrm{i}z_1.$$

(a) (i) Express $z_2$ in the form $a + \mathrm{i}b$. *(1 mark)*

(ii) Find the modulus and argument of $z_2$. *(2 marks)*

(b) Label the points representing $z_1$ and $z_2$ on an Argand diagram. *(1 mark)*

(c) On the **same** Argand diagram, sketch the locus of points $z$ satisfying:

(i) $|z - z_1| = |z - z_2|$; *(2 marks)*

(ii) $\arg(z - z_1) = \arg z_2$. *(2 marks)*

# Final Challenge

(a) Shade, on an Argand diagram, the region in which

$$|z - 2i| \leqslant 1.$$ *(4 marks)*

(b) Find the greatest and least values of the argument of complex numbers $z$ satisfying

$$|z - 2i| \leqslant 1,$$

giving your answers in terms of $\pi$. *(4 marks)*