4 The matrix $\boldsymbol{A}$ is $\begin{bmatrix} 4 & k \\ 3 & 6 \end{bmatrix}$.

(a) Find the determinant of $\boldsymbol{A}$. *(1 mark)*

(b) Find the value of k for which the inverse of $\boldsymbol{A}$ does not exist. *(2 marks)*

(c) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

(i) A triangle has area 5 square units.

Find the area of its image under **T** in the case when $k = 8$. *(1 mark)*

(ii) Find the possible values of k in order that, under **T**, a triangle with area 5 square units would be mapped onto a triangle with area 15 square units. *(3 marks)*

4(a)	$24 - 3k$	B1	1	
(b)	Det = 0 $\Rightarrow$ $k = 8$	M1√ A1√	2	ft (a)
(c)(i)	Area = 0	B1√	1	ft $5 \times$ cand's Det with $k = 8$
(ii)	Det = 3 and / or -3 $\Rightarrow k = 7$ $\Rightarrow k = 9$	M1 A1√ A1	3	ft cand's "$24 - 3k = 3$" cao
	Total		**7**	

2 (a) Show, using row operations, that $(a+b+c)$ is a factor of

$$\Delta = \begin{vmatrix} ab & bc & ca \\ a+b & b+c & c+a \\ c & a & b \end{vmatrix}$$ *(2 marks)*

(b) Factorise Δ completely into linear factors. *(4 marks)*

2 (a)	$R_2' = R_2 + R_3$ (e.g.) $\Delta = \begin{vmatrix} ab & bc & ca \\ a+b+c & b+c+a & c+a+b \\ c & a & b \end{vmatrix}$	M1		
	$= (a+b+c)\begin{vmatrix} ab & bc & ca \\ 1 & 1 & 1 \\ c & a & b \end{vmatrix}$	A1	2	
(b)	$C_1' = C_1 - C_3$ $C_2' = C_2 - C_3$ $\Delta = (a+b+c)\begin{vmatrix} a(b-c) & c(b-a) & ca \\ 0 & 0 & 1 \\ c-b & a-b & b \end{vmatrix}$	M1		Good attempt at 2nd factor
	$= (a+b+c)(b-c)(b-a)\begin{vmatrix} a & c & ca \\ 0 & 0 & 1 \\ -1 & -1 & b \end{vmatrix}$	A1		At least one further factor found
	Full factorisation attempt (possibly by multiplying out)	M1		
	$= (a+b+c)(a-b)(b-c)(c-a)$	A1	4	cao, any form
	Total		**6**	

2 (a) (i) Find, in terms of k, the determinant of the matrix $\boldsymbol{A} = \begin{bmatrix} 4 & -2 \\ 5 & k \end{bmatrix}$. *(1 mark)*

(ii) Find the value of k for which $\boldsymbol{A}$ is singular. *(2 marks)*

(b) (i) Find the inverse of the matrix $\boldsymbol{B} = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$. *(3 marks)*

(ii) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} x \\ y \end{bmatrix}$$

Find the coordinates of the point which is mapped onto $(6, 7)$ under **T**. *(3 marks)*

2(a)(i)	$\det \boldsymbol{A} = 4k + 10$	B1	1	
(ii)	Singular $\Rightarrow \det \boldsymbol{A} = 0$ $\Rightarrow k = -2.5$	M1 A1√	2	oe ft candidate's determinant
(b)(i)	$\det \boldsymbol{B} = -2$	B1		
	$\boldsymbol{B}^{-1} = \frac{1}{\det \boldsymbol{B}} \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix}$	M1		Condone one slip in matrix, multiplication by det $\boldsymbol{B}$, or omission of det $\boldsymbol{B}$
	$= \frac{1}{-2} \begin{bmatrix} -3 & 2 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ 2.5 & -2 \end{bmatrix}$	A1	3	Any equivalent
(ii)	$\boldsymbol{B}^{-1} \begin{bmatrix} 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$	M1		Must premultiply column by inverse and attempt to multiply to get column vector. **Or** setting up and solving sim eqns.
	$x = 2$ $y = 1$	A1√ A1√	3	May leave as column or coordinates **sc** B3 correct answer without working
	Total		**9**	

5 (a) Factorise $\begin{vmatrix} x^2 & x & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix}$. *(4 marks)*

(b) It is given that

$$\mathbf{A} = \begin{bmatrix} x & 0 & 2 \\ 0 & x & 9 \\ 0 & 1 & x \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} x^2 & x & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}.$$

Using your result in part (a), or otherwise, express det(**AB**) in factorised form. *(4 marks)*

5 (a)	$x^2(1-2)-x(1-4)+1(2-4)$	M1A1		
	$-x^2+3x-2$	A1		
	$-(x-1)(x-2)$	A1F	4	
(b)	det **A** det **B** used	M1		
	$\det \mathbf{B} = x^3-9x$	M1A1		
	$\det \mathbf{AB} = -x(x-3)(x+3)(x-1)(x-2)$	A1F	4	If any other method used, it must be complete
	Total		**8**	