FP4 Determinants Challenge

Challenge 1

The matrix $\boldsymbol{A}$ is $\begin{bmatrix} 4 & k \\ 3 & 6 \end{bmatrix}$.

(a) Find the determinant of $\boldsymbol{A}$. *(1 mark)*

(b) Find the value of k for which the inverse of $\boldsymbol{A}$ does not exist. *(2 marks)*

(c) The transformation $\mathbf{T}$ is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

(i) A triangle has area 5 square units.

Find the area of its image under $\mathbf{T}$ in the case when $k = 8$. *(1 mark)*

(ii) Find the possible values of k in order that, under $\mathbf{T}$, a triangle with area 5 square units would be mapped onto a triangle with area 15 square units. *(3 marks)*

Challenge 2

(a) Show, using row operations, that $(a+b+c)$ is a factor of

$$\Delta = \begin{vmatrix} ab & bc & ca \\ a+b & b+c & c+a \\ c & a & b \end{vmatrix}$$ *(2 marks)*

(b) Factorise Δ completely into linear factors. *(4 marks)*

Challenge 3

(a) (i) Find, in terms of k, the determinant of the matrix $\boldsymbol{A} = \begin{bmatrix} 4 & -2 \\ 5 & k \end{bmatrix}$. *(1 mark)*

(ii) Find the value of k for which $\boldsymbol{A}$ is singular. *(2 marks)*

(b) (i) Find the inverse of the matrix $\boldsymbol{B} = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$. *(3 marks)*

(ii) The transformation $\mathbf{T}$ is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} x \\ y \end{bmatrix}$$

Find the coordinates of the point which is mapped onto $(6, 7)$ under $\mathbf{T}$. *(3 marks)*

Final Challenge

(a) Factorise $\begin{vmatrix} x^2 & x & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix}$. *(4 marks)*

(b) It is given that

$$\mathbf{A} = \begin{bmatrix} x & 0 & 2 \\ 0 & x & 9 \\ 0 & 1 & x \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} x^2 & x & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}.$$

Using your result in part (a), or otherwise, express det(**AB**) in factorised form. *(4 marks)*