8 A particle, of mass m, moves in a straight line on a smooth horizontal surface. As it moves it experiences a resistance force of magnitude kv^2, where k is a constant and v is the speed of the particle, at time t. The particle moves with speed U at time $t = 0$.

Show that $v = \dfrac{mU}{Ukt + m}$. *(6 marks)*

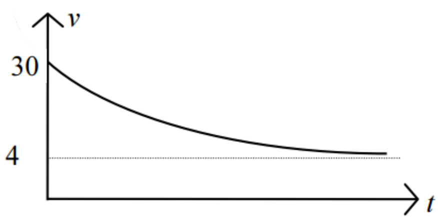
8	$m\dfrac{dv}{dt} = -kv^2$	M1		Forming an equation of motion with $\dfrac{dv}{dt}$ (2 terms)
	$\displaystyle\int \frac{1}{v^2}\,dv = \int -\frac{k}{m}\,dt$			
	$-\dfrac{1}{v} = -\dfrac{k}{m}t + c$	M1		Integration with $\dfrac{1}{v}$ term (constant not needed)
	$\dfrac{1}{v} = \dfrac{kt - mc}{m}$	A1		Correct expression
	$v = \dfrac{m}{kt - mc}$			
	$t = 0, v = U \Rightarrow c = -\dfrac{1}{U}$	M1 A1		Finding c Correct c
	$v = \dfrac{m}{kt + \dfrac{m}{U}} = \dfrac{Um}{Ukt + m}$	A1	6	Complete correct solution **ag**
	Total		**6**	

8 A parachutist, of mass 80 kg, is falling vertically. When his speed is 30 ms^{-1}, his parachute opens. He then experiences an air resistance force of magnitude $196v$ N, where v ms^{-1} is his speed.

(a) Show that at time t seconds after the parachute is opened, the speed of the parachutist is given by

$$v = 4 + 26e^{-2.45t}.$$ *(6 marks)*

(b) Sketch a graph to show how the parachutist's speed varies with time. *(2 marks)*

Question	Solution	Marks	Total	Comments
8 (a)	$80\frac{dv}{dt} = 80 \times 9.8 - 196v$	M1		
	$\frac{dv}{dt} = 9.8 - 2.45v$	A1		A1: 3 term equation of motion with $a = \frac{dv}{dt}$
	$\frac{1}{v-4}\frac{dv}{dt} = -2.45$			
	$\int \frac{1}{v-4} dv = \int -2.45 dt$	M1A1		M1: Integrating to get ln term
	$\ln\lvert v-4 \rvert = -2.45t + c$			
	$v - 4 = Ae^{-2.45t}$			M1: Use of initial conditions
	$v = 4 + Ae^{-2.45t}$	M1		
	$v = 30, t = 0 \Rightarrow A = 26$	A1	**(6)**	
	$v = 4 + 26e^{-2.45t}$			
8 (b)		B1 B1	(2)	B1: Group with non-zero asymptote B1: Values

8 A particle of mass m is moving along a straight horizontal line. At time t the particle has speed v. Initially the particle is at the origin and has speed U. As it moves the particle is subject to a resistance force of magnitude mkv^3.

(a) Show that $v^2 = \dfrac{U^2}{2kU^2t + 1}$. *(6 marks)*

(b) What happens to v as t increases? *(1 mark)*

8(a)	$m\dfrac{dv}{dt} = -mkv^3$	M1		Forming a differential equation
	$\int \dfrac{1}{v^3}dv = -\int k dt$			
	$-\dfrac{1}{2v^2} = -kt + c$	m1		Integrating to get a $\dfrac{1}{v^2}$ term
		A1		Correct Integral including c
	$v = U,\ t = 0 \Rightarrow c = -\dfrac{1}{2U^2}$	m1 A1		Finding c Correct c
	$\dfrac{1}{2v^2} = kt + \dfrac{1}{2U^2} = \dfrac{2ktU^2 + 1}{2U^2}$			
	$v^2 = \dfrac{U^2}{2ktU^2 + 1}$	A1	6	Correct final answer from correct working
(b)	v tends to zero	B1	1	Allow decreases
	Total		**7**	

8 A car accelerates from rest along a straight horizontal road. It experiences a constant horizontal forward force of magnitude 2000 newtons and a resistance force. The resistance force has magnitude $40v$ newtons, when the speed of the car is $v\,\text{m s}^{-1}$. The mass of the car is 1000 kg.

(a) Show that

$$\frac{dv}{dt} = \frac{50 - v}{25}$$ *(2 marks)*

(b) Find the velocity of the car at time t. *(5 marks)*

8(a)	$1000\frac{dv}{dt} = 2000 - 40v$	M1		Use of Newton's 2nd law
	$\frac{dv}{dt} = \frac{2000-40v}{1000} = \frac{50-v}{25}$	A1	2	Correct expression from correct working
(b)	$\int \frac{1}{50-v}dv = \int \frac{1}{25}dt$ $-\ln\lvert 50-v\rvert = \frac{t}{25} + c$	M1 A1		Integration to obtain t and 1n term Correct integration
	$50 - v = Ae^{\frac{t}{25}}$ $v = 50 - Ae^{-\frac{t}{25}}$ $v = 0, t = 0 \Rightarrow A = 50$	m1 m1		Solving for v Finding constant of integration
	$v = 50\left(1 - e^{-\frac{t}{25}}\right)$	A1	5	Correct final answer
	Total		**7**	