M2 Differential Equations Challenge

Challenge 1

A particle, of mass m, moves in a straight line on a smooth horizontal surface. As it moves it experiences a resistance force of magnitude kv^2, where k is a constant and v is the speed of the particle, at time t. The particle moves with speed U at time $t = 0$.

Show that $v = \dfrac{mU}{Ukt + m}$. *(6 marks)*

Challenge 2

A parachutist, of mass 80 kg, is falling vertically. When his speed is 30 ms^{-1}, his parachute opens. He then experiences an air resistance force of magnitude $196v$ N, where v ms^{-1} is his speed.

(a) Show that at time t seconds after the parachute is opened, the speed of the parachutist is given by

$$v = 4 + 26e^{-2.45t}.$$ *(6 marks)*

(b) Sketch a graph to show how the parachutist's speed varies with time. *(2 marks)*

Challenge 3

A particle of mass m is moving along a straight horizontal line. At time t the particle has speed v. Initially the particle is at the origin and has speed U. As it moves the particle is subject to a resistance force of magnitude mkv^3.

(a) Show that $v^2 = \dfrac{U^2}{2kU^2t + 1}$. *(6 marks)*

(b) What happens to v as t increases? *(1 mark)*

Final Challenge

A car accelerates from rest along a straight horizontal road. It experiences a constant horizontal forward force of magnitude 2000 newtons and a resistance force. The resistance force has magnitude $40v$ newtons, when the speed of the car is $v\,\mathrm{m\,s^{-1}}$. The mass of the car is 1000 kg.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{50 - v}{25}$$

(2 marks)

(b) Find the velocity of the car at time t. *(5 marks)*