Differentiation

The function f is defined for all real values of x by

$$f(x) = (x^2 + 4)(2x - 1) .$$

(a) Prove that the curve with equation $y = f(x)$ crosses the x-axis at only one point and state the x-coordinate of this point. *(2 marks)*

(b) (i) Differentiate $f(x)$ with respect to x to obtain $f'(x)$. *(4 marks)*

(ii) Hence show that the gradient of the curve $y = f(x)$ is 12 at the point where $x = 1$. *(2 marks)*

(iii) Prove that the curve $y = f(x)$ has no stationary point. *(2 marks)*

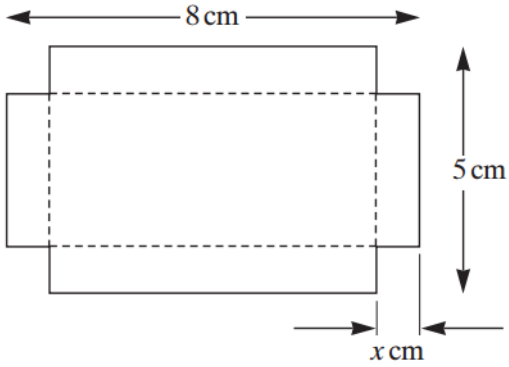
(c) The curve $y = f(x)$ intersects the line $y = x$ at only one point B.

(i) Show that the x-coordinate of B satisfies the equation

$$2x^3 - x^2 + 7x - 4 = 0 .$$ *(1 mark)*

Question	Solution	Marks	Total	Comments
7 (a)	$x^2 + 4 \neq 0$ for real x oe	B1		
	$y = 0$ when $2x-1=0$ ie $x = \frac{1}{2}$	B1	**(2)**	
(b)(i)	$f(x) = 2x^3 - x^2 + 8x - 4$ $f'(x) = 6x^2 - 2x + 8$	M1 A1 m1 A1 ft	**(4)**	M1: for attempt to expand brackets > 2 terms m1: for obvious attempt to differentiate A1 ft: only ft if equivalent difficulty
(ii)	$f'(1) = 6(1)^2 - 2(1) + 8$ $\ldots = 6 - 2 + 8 = 12$	M1 A1 cso	**(2)**	Attempt to find $f'(1)$ AG obtained convincingly
(iii)	For st. pt $f'(x)=0 \Rightarrow 6x^2 - 2x + 8 = 0$ $b^2 - 4ac = 4 - 192$	M1		Consideration of $b^2 - 4ac$ (any form)
	$-188 < 0$	A1	**(2)**	A1: for a statement about the 'correct' value of '$b^2 - 4ac$'
(c)(i)	At pt of intersection $2x^3 - x^2 + 8x - 4 = x$ x-coord. of B satisfies, $2x^3 - x^2 + 7x - 4 = 0$	B1	**(1)**	AG obtained convincingly

Small trays are to be made from rectangular pieces of card. Each piece of card is 8 cm by 5 cm and the tray is formed by removing squares of side x cm from each corner and folding the remaining card along the dashed lines, as shown in the diagram, to form an open-topped box.



(a) Explain why $0 < x < 2.5$. *(1 mark)*

(b) Show that the volume, V cm^3, of a tray is given by

$$V = 4x^3 - 26x^2 + 40x.$$ *(3 marks)*

(c) Find the value of x for which $\frac{dV}{dx} = 0$. *(5 marks)*

(d) Calculate the greatest possible volume of a tray. *(1 mark)*

Q	Solution	Marks	Total	Comments
5 (a)	$(0<)2x<5 \Rightarrow (0<)x<2.5$	B1	1	in effect, $5 \div 2$
(b)	$V = x(5-2x)(8-2x)$	M1		
	$= x(4x^2 - 26x + 40)$	M1		expanding sensible quadratic
	$= 4x^3 - 26x^2 + 40x$	A1	3	AG
(c)	$\frac{dV}{dx} = 12x^2 - 52x + 40$	M1A1		M1 for 2 correct
	$12x^2 - 52x + 40 = 0$			
	$x = 1 \left(\text{or } \frac{10}{3}\right)$ false argument M0	M1A2	5	M1 for solving quadratic allow A1 for $\frac{10}{3}$ only or 1, $-\frac{10}{3}$
(d)	18 (cm^3)	B1	1	
	Total		**10**	

An office worker can leave home at any time between 6.00 am and 10.00 am each morning. When he leaves home x **hours** after 6.00 am ($0 \leqslant x \leqslant 4$), his journey time to the office is y **minutes**, where

$$y = x^4 - 8x^3 + 16x^2 + 8.$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Find the **three** values of x for which $\dfrac{dy}{dx} = 0$. *(4 marks)*

(c) Show that y has a maximum value when $x = 2$. *(3 marks)*

(d) Find the time at which the office worker arrives at the office on a day when his journey time is a maximum. *(2 marks)*

8	**(a)**	$y' = 4x^3 - 24x^2 + 32x$	M1A2,1	3	M1 if at least one term correct; -1 EE
	(b)	$y' = 0$ for $x = 0$	B1		Condone factors instead of values in (b)
		... and when $x^2 - 6x + 8 = 0$	M1		OE method leading to 2 non-zero values
		ie for $x = 2, 4$	A2,1	4	-1 EE NMS 1/3 for $x = 2$ or $x = 4$, 2/3 for both, 4/4 for all three correct values
	(c)	Values of y for $x < 2$, $x = 2$, $x > 2$	M1A1		or of y' for $x < 2$, $x > 2$ or of y'' for $x = 2$
		Conclusion drawn	E1	3	AG
	(d)	Arrival time is 8.24 am	B1B1	2	
		Total		**12**	

The size of a population, P, of birds on an island is modelled by

$$P = 59 + 117t + 57t^2 - t^3,$$

where t is the time in years after 1970.

(a) Find $\frac{dP}{dt}$. *(2 marks)*

(b) (i) Find the positive value of t for which P has a stationary value. *(3 marks)*

(ii) Determine whether this stationary value is a maximum or a minimum. *(2 marks)*

(c) (i) State the year when the model predicts that the population will reach its maximum value. *(1 mark)*

(ii) Determine what the model predicts will happen in the year 2029. *(1 mark)*

Q	Solution	Marks	Total	Comments
4 (a)	$\frac{dP}{dt} = -3t^2 + 114t + 117$	M1A1	2	M1 if 2 correct terms
(b)(i)	$3t^2 - 114t - 117 = 0$	M1		set $\frac{dP}{dt} = 0$ quadratic only.
	$t = 39 (\text{or } -1)$	m1A1	3	allow answer only
(ii)	e.g. $\frac{dy}{dx}$ changes from $+ve$ to $-ve$	M1		allow sketch or values at T point
	Maximum	A1	2	S.R. B1 if not justified S.R. B1 if $\frac{d^2P}{dt^2} = 114 - 6t$ only
(c)(i)	2009√	B1√	1	√ on $t > 0$ from (b)(i)
(ii)	Extinction	B1	1	allow $P = 0$ do not allow "minimum"
	Total		**9**	