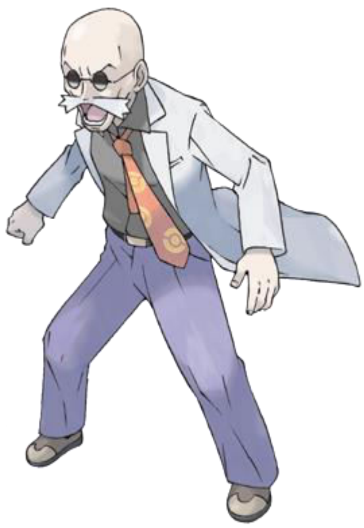
C3 Differentiation challenge

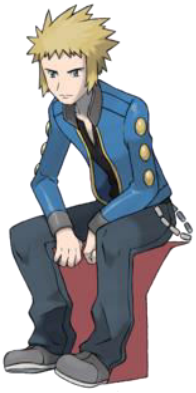
Challenge 1

Find the equation of the tangent to the curve $y = \frac{2 + x}{\cos x}$ at the point on the curve where $x = 0$.

(6 marks)



Challenge 2



(a) Differentiate:

(i) $2x^{\frac{1}{2}}$;

(ii) $\ln(x + 1)$. *(3 marks)*

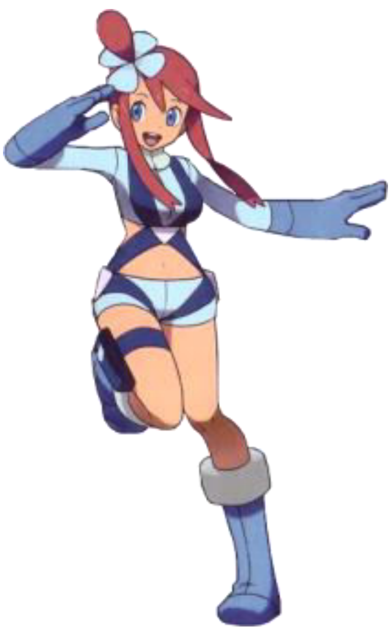
(b) Hence show that $\int_1^4 \left(x^{-\frac{1}{2}} + \frac{1}{x+1} \right) \mathrm{d}x = 2 + \ln \frac{5}{2}$. *(5 marks)*

Challenge 3

(a) By using the chain rule, or otherwise, find $\frac{dy}{dx}$ when $y = \ln(x^2 + 9)$. *(3 marks)*

(b) Hence show that $\int_0^3 \frac{x}{x^2 + 9} dx = \frac{1}{2} \ln 2$. *(3 marks)*

(c) Show that $\int_0^3 \frac{x + 1}{x^2 + 9} dx = \frac{1}{2} \ln 2 + \frac{\pi}{12}$. *(4 marks)*



Final Challenge

A curve has equation

$$y = e^{2x} - 4x .$$

(a) Show that the x-coordinate of the stationary point on the curve is $\frac{1}{2}\ln 2$. Find the corresponding y-coordinate in the form $a + b\ln 2$, where a and b are integers to be determined. *(6 marks)*

(b) Find an expression for $\frac{d^2y}{dx^2}$ and hence determine the nature of the stationary point. *(3 marks)*