2 Sara is using the "vena contractor phenomenon" to measure the rate of flow of liquid out of an inverted cone of semi-vertical angle α.

The standard formula for the rate of flow is:

$$R = \frac{8}{15} C_D \tan\alpha \sqrt{2gh^5},$$

where C_D is the coefficient of discharge which is a dimensionless constant and h is the height of liquid in the inverted cone.

By using dimensional analysis, show that the dimension of R is a rate of flow. *(4 marks)*

2	Dimensions of h is L Dimensions of g is LT^{-2} $\therefore$ dimensions of $\frac{8}{15} C_D \tan\alpha \sqrt{2gh^5}$ is dimension of $\left(gh^5\right)^{\frac{1}{2}}$ $= \left(LT^{-2}.L^5\right)^{\frac{1}{2}}$ $= \left(L^6T^{-2}\right)^{\frac{1}{2}}$ $= L^3T^{-1}$ = volume/sec which is a rate of flow	B1 M1 A1 A1		Ability to handle powers
	Total		**4**	

3 The gravitational force acting between two bodies of mass m_1 and m_2 in deep space is

$$\frac{km_1m_2}{d^2}$$

where d is the distance between the bodies.

The dimensional constant, k, is of the form $M^{\alpha}L^{\beta}T^{\gamma}$.

By considering dimensions find α, β and γ. *(4 marks)*

Question	Solution	Marks	Total	Comments
3	Force $= \frac{km_1m_2}{d^2}$ Dimensions of force is MLT^{-2} $k = \text{Force} \times \frac{d^2}{m_1m_2}$ $= MLT^{-2} \cdot \frac{L^2}{M^2}$ $= M^{-1}L^3T^{-2}$	 B1 M1 A1 A1	 **(4)**	
		TOTAL	**(4)**	

3 The gravitational force of the sun, which has mass m_1, on a planet, of mass m_2, is an attractive force directed along the line joining them and of magnitude $\frac{Gm_1m_2}{d^2}$ where d is the distance between their centres and G is the universal gravitational constant.

Use dimensional analysis to find the dimensions of G. *(4 marks)*

Q	Solution	Marks	Total	Comments
3	Force $= G\frac{m_1\ m_2}{d^2}$ or $G = \frac{d^2 \times \text{force}}{m_1\ m_2}$ Dimensions of a force (*ma*) is M L T^{-2}	B1		
	$\therefore$ Dimensions of G are $\frac{L^2.M\,L\,T^{-2}}{M^2}$	M1		
	$= L^3\ T^{-2}M^{-1}$	A1A1	4	
	Total		**4**	

2 The acceleration, a, of a body falling with speed v and subject to air resistance may be modelled by the equation

$$a = g - \lambda v^2$$

where λ is constant.

Find the dimensions of λ in order that the equation is dimensionally consistent. *(4 marks)*

2	Dimensions of a and g are LT^{-2} Dimension of v is LT^{-1} $\lambda = \frac{LT^{-2}}{(LT^{-1})^2}$ $= L^{-1}$	B1 B1 M1 A1	4	
	Total		**4**	

4 John believes that a possible formula is

$$Q = 2\pi\sqrt{\frac{l}{g}}$$

By considering dimensions, find the dimensions of Q. *(4 marks)*

4	Considering dimensions $2, \pi$ have no dimension, l has length, L g is m/s^2 i.e. $\frac{L}{T^2}$ $\therefore 2\pi\sqrt{\frac{l}{g}}$ is $\sqrt{\frac{L}{\frac{L}{T^2}}}$ $=\sqrt{T^2}$ $=T$ $\therefore$ dimension of result is time	M1 A1 M1 A1		
	Total		**4**	