M3 Dimensional Analysis Challenge

Challenge 1

Sara is using the "vena contractor phenomenon" to measure the rate of flow of liquid out of an inverted cone of semi-vertical angle α.

The standard formula for the rate of flow is:

$$R = \frac{8}{15} C_D \tan\alpha \sqrt{2gh^5},$$

where C_D is the coefficient of discharge which is a dimensionless constant and h is the height of liquid in the inverted cone.

By using dimensional analysis, show that the dimension of R is a rate of flow. *(4 marks)*

Challenge 2

The gravitational force acting between two bodies of mass m_1 and m_2 in deep space is

$$\frac{km_1m_2}{d^2}$$

where d is the distance between the bodies.

The dimensional constant, k, is of the form $M^{\alpha}L^{\beta}T^{\gamma}$.

By considering dimensions find α, β and γ. *(4 marks)*

Challenge 3

The gravitational force of the sun, which has mass m_1, on a planet, of mass m_2, is an attractive force directed along the line joining them and of magnitude $\frac{Gm_1m_2}{d^2}$ where d is the distance between their centres and G is the universal gravitational constant.

Use dimensional analysis to find the dimensions of G. *(4 marks)*

Final Challenge

The acceleration, a, of a body falling with speed v and subject to air resistance may be modelled by the equation

$$a = g - \lambda v^2$$

where λ is constant.

Find the dimensions of λ in order that the equation is dimensionally consistent. *(4 marks)*

John believes that a possible formula is

$$Q = 2\pi\sqrt{\frac{l}{g}}$$

By considering dimensions, find the dimensions of Q. *(4 marks)*