1 The weights of the contents of packets of table salt are known to be normally distributed with standard deviation 6 g. The mean weight of the contents of a random sample of 4 packets was found to be 748 g. Calculate a 99% confidence interval for the mean weight of the contents of all packets.

(4 marks)

Question Number and part	Solution	Marks	Total marks	Comments
1	748±2.5758×6/√4	B1 M1 M1		2.5758 (2.57 $\sim$ 2.58) attempt to use $6/\sqrt{4}$ completely correct method
	748 ± 7.73 740.3 ~ 755.7	<b>A</b> 1	4	$748 \pm 7.73 (7.72 \sim 7.74)$ or $740.3 (740 \sim 740.3)$ and $755.7 (755.7 \sim 756)$
	Total		4	

2 A manufacturer makes batteries for use in bicycle lights. The working lives, in hours, of the batteries are known to be normally distributed with a standard deviation of 1.8. A random sample of batteries was tested and their working lives were as follows:

- (a) Calculate a 95% confidence interval for the mean working life of the batteries. (6 marks)
- (b) State the width of the confidence interval you have calculated. (1 mark)
- (c) What percentage would be associated with a confidence interval of width 2 hours calculated from the given data? (5 marks)
- (d) A further random sample of size seven is to be taken and used to calculate a confidence interval of width 2 hours. State the probability that this confidence interval will **not** contain the mean working life.

  (1 mark)

	ne mean working me.			(1 mark)
2(a)	$\bar{x} = 48.114$	B1		(48.1 - 48.12)
	95% confidence interval for mean	B1		1.96
	$48.114 \pm 1.96 \times \frac{1.8}{\sqrt{7}}$	M1		for use of $\frac{1.8}{\sqrt{7}}$
		m2		<b>V</b> /
	48.114±1.333			completely correct method, 'their' $\bar{x}$ , z
	46.8 – 49.4	<b>A</b> 1	6	48.1 (48.1 - 48.12 ) and 1.33 (1.33 - 1.34)
				or 46.8 (46.7 - 46.8) and 49.4 (49.4 - 49.5)
(b)	2.67	<b>B</b> 1	1	(2.66 - 2.68) cao
(c)	$\frac{2z\times1.8}{}=2$	M1		for any correct equation - allow small
	$\sqrt{7}$			slip e.g. omission of first 2
	z = 1.470 1-0.92922	m1		any completely correct equation
	1-0.92922	m1		method of solution - requires M mark
	% confidence = $100[1 - 2(1 - 0.92922)]$	m1		method for % confidence - requires M mark
	= 85.8%	<b>A</b> 1	5	(85 - 86)
(d)	0.142	B1√	1	'their' answer to (c)
	Total		13	

- 3 A health food co-operative imports a large quantity of dates and packs them into plastic bags labelled 500 grams. Georgina, a Consumer Protection Officer, checked a random sample of 95 bags and found the contents had a mean weight of 498.6 grams, and a standard deviation of 9.3 grams.
  - (a) Assuming that weights follow a normal distribution, calculate, for the mean weight of contents of all the bags:

(i) a 95% confidence interval; (4 marks)

(ii) an 80% confidence interval. (2 marks)

- (b) The health food co-operative also imports raisins. Georgina intends to take a random sample of 500 gram packets of raisins, weigh the contents and use the results to calculate an 80% and a 95% confidence interval for the mean weight, μ, of the contents of all the co-operative's packets of raisins.
  - (i) Find the probability that:
    - (A) the 80% confidence interval contains  $\mu$ ; (1 mark)
    - (B) the 95% confidence interval contains  $\mu$  but the 80% confidence interval does not. (2 marks)
  - (ii) Instead of calculating both confidence intervals from the same sample, Georgina now decides to calculate the 95% confidence interval from one sample and the 80% confidence interval from a second independent random sample. Find the probability that the 95% confidence interval contains  $\mu$  but the 80% confidence interval does not. (2 marks)

Q	Solution	Marks	Total	Comments
3(a)	95% confidence interval	B1 M1		1.96 cao
(i)	$498.6 \pm 1.96 \times \frac{9.3}{\sqrt{95}}$			Use of $\frac{9.3}{\sqrt{95}}$
	$498.6 \pm 1.87$	m1		Correct method - their z
	496.7 – 500.5	A1	4	496.7 (496.7–496.8) and 500.5 (500.4 –500.5) or 498.6± 1.87 (1.86–1.88) Allow 497 and 500
(ii)	80% confidence interval $498.6 \pm 1.2816 \times \frac{9.3}{\sqrt{95}}$	M1	2	Method - their z
	498.6 ± 1.22 497.4 – 499.8	A1		497.4 (497.3–497.4) and 499.8 (499.8 –499.9) or 498.6±1.22 (1.22 –1.23) Allow 497 and 500 but penalise if
(b)(i)	(A) 0.8 (B) 0.15	B1 M1 A1	3	answers to (i) and (ii) are identical  0.8 cao Method  0.15 cao
(ii)	$0.95 \times 0.2 = 0.19$	M1 A1	2	Method 0.19 cao
	Total		11	

- 6 Applicants to join a police force are tested for physical fitness. Based on their performance, a physical fitness score is calculated for each applicant. Assume that the distribution of scores is normal.
  - (a) The scores for a random sample of ten applicants were

55 23 44 69 22 45 54 72 34 66

Experience suggests that the standard deviation of scores is 14.8.

Calculate a 99% confidence interval for the mean score of all applicants. (5 marks)

(b) The scores of a further random sample of 110 applicants had a mean of 49.5 and a standard deviation of 16.5.

Use the data from this second sample to calculate:

- (i) a 95% confidence interval for the mean score of all applicants; (3 marks)
- (ii) an interval within which the score of approximately 95% of applicants will lie. (2 marks)
- (c) By interpreting your results in parts (b)(i) and (b)(ii), comment on the ability of the applicants to achieve a score of 25. (3 marks)
- (d) Give **two** reasons why a confidence interval based on a sample of size 110 would be preferable to one based on a sample of size 10. (2 marks)
- (e) It is suggested that a much better estimate of the mean physical fitness of all recruits could be made by combining the two samples before calculating a confidence interval. Comment on this suggestion.

  (3 marks)

Question Number and part	Solution	Marks	Total	Comments
6(a)	$\overline{x} = 48.4$	B1		48.4 cao
	99% confidence interval			101 0200000
	18.41.2.575014.8	B1		2.5758 (2.57 – 2.58)
	$48.4 \pm 2.5758 \times \frac{14.8}{\sqrt{10}}$	M1		14.8 5
	V10			Use of $\frac{14.8}{\sqrt{10}}$ or $\frac{5}{\sqrt{10}}$
	ie 48.4±12.1	m1		Completely correct method – candidate's z
	36.3 – 60.5	A1	5	36.3 (36.3 – 36.4) and 60.5 (60.4 – 60.5)
		100000	53384	or 48.4 and 12.1 (12–12.1)
(b)(i)	95% confidence interval			
	16.5			
	$49.5 \pm 1.96 \times \frac{16.5}{\sqrt{110}}$	B1		1.96
	A STATE OF THE STA	M1	-500	Method – their z
	$49.5 \pm 3.08$	A1	3	$49.5 \pm 3.08 (3.08 - 3.09)$
	46.42 - 52.58			or 46.42 (46.41–46.42)
	THE PROPERTY OF THE PROPERTY O			and 52.58 (52.58 – 52.59)
				Allow 46.4 and 52.6
(ii)	49.5±1.96×16.5	M1		Method – their z
100	17.2 – 81.8	A1	2	$49.5 \pm 32.34 (32.3 - 32.4)$
				or $\pm 17.2 (17.1 - 17.2)$
				and 81.8 (81.8 – 81.9)
(c)		E1		Mean ≠25
	individuals will be less than 25	E1		Mean > 25
		E1	3	Some individuals < 25
(d)	Interval narrower	E1		Interval narrower
(4)	No need for normal assumption	E1	2	Normal not needed
	Parameter is the	CHRONICLE .	90 <del>000</del>	50 A
(e)	Both samples random so valid suggestion.	E1		Valid
	Would be better but not much ( $\frac{49.5}{}$	г.		D. Walley
	Would be better but not much ( $\frac{49.5}{\sqrt{110}}$	E1		Better
	very similar to $\frac{49.5}{\sqrt{1-3}}$ )	E1	3	Not much
	$\sqrt{120}$			Any three points
	Total		18	