

- 1 The weights of the contents of packets of table salt are known to be normally distributed with standard deviation 6 g. The mean weight of the contents of a random sample of 4 packets was found to be 748 g. Calculate a 99% confidence interval for the mean weight of the contents of all packets. *(4 marks)*

Question Number and part	Solution	Marks	Total marks	Comments
1	$748 \pm 2.5758 \times 6 / \sqrt{4}$ 748 ± 7.73 $740.3 \sim 755.7$	B1 M1 M1 A1	4	2.5758 (2.57 ~ 2.58) attempt to use $6/\sqrt{4}$ completely correct method 748 ± 7.73 (7.72 ~ 7.74) or 740.3 (740 ~ 740.3) and 755.7 (755.7 ~ 756)
Total			4	

- 3 A health food co-operative imports a large quantity of dates and packs them into plastic bags labelled 500 grams. Georgina, a Consumer Protection Officer, checked a random sample of 95 bags and found the contents had a mean weight of 498.6 grams, and a standard deviation of 9.3 grams.

(a) Assuming that weights follow a normal distribution, calculate, for the mean weight of contents of all the bags:

(i) a 95% confidence interval; (4 marks)

(ii) an 80% confidence interval. (2 marks)

(b) The health food co-operative also imports raisins. Georgina intends to take a random sample of 500 gram packets of raisins, weigh the contents and use the results to calculate an 80% and a 95% confidence interval for the mean weight, μ , of the contents of all the co-operative's packets of raisins.

(i) Find the probability that:

(A) the 80% confidence interval contains μ ; (1 mark)

(B) the 95% confidence interval contains μ but the 80% confidence interval does not. (2 marks)

(ii) Instead of calculating both confidence intervals from the same sample, Georgina now decides to calculate the 95% confidence interval from one sample and the 80% confidence interval from a second independent random sample. Find the probability that the 95% confidence interval contains μ but the 80% confidence interval does not. (2 marks)

Q	Solution	Marks	Total	Comments
3(a)	95% confidence interval	B1 M1		1.96 cao
(i)	$498.6 \pm 1.96 \times \frac{9.3}{\sqrt{95}}$			Use of $\frac{9.3}{\sqrt{95}}$
	498.6 ± 1.87	m1		Correct method - their z
	$496.7 - 500.5$	A1	4	496.7 (496.7–496.8) and 500.5 (500.4–500.5) or 498.6 ± 1.87 (1.86–1.88) Allow 497 and 500
(ii)	80% confidence interval			
	$498.6 \pm 1.2816 \times \frac{9.3}{\sqrt{95}}$	M1	2	Method - their z
	498.6 ± 1.22			
	$497.4 - 499.8$	A1		497.4 (497.3–497.4) and 499.8 (499.8–499.9) or 498.6 ± 1.22 (1.22–1.23) Allow 497 and 500 but penalise if answers to (i) and (ii) are identical
(b)(i)	(A) 0.8 (B) 0.15	B1 M1 A1	3	0.8 cao Method 0.15 cao
(ii)	$0.95 \times 0.2 = 0.19$	M1 A1	2	Method 0.19 cao
Total			11	

- 6 Applicants to join a police force are tested for physical fitness. Based on their performance, a physical fitness score is calculated for each applicant. Assume that the distribution of scores is normal.

- (a) The scores for a random sample of ten applicants were

55 23 44 69 22 45 54 72 34 66

Experience suggests that the standard deviation of scores is 14.8.

Calculate a 99% confidence interval for the mean score of all applicants. *(5 marks)*

- (b) The scores of a further random sample of 110 applicants had a mean of 49.5 and a standard deviation of 16.5.

Use the data from this second sample to calculate:

- (i) a 95% confidence interval for the mean score of all applicants; *(3 marks)*
- (ii) an interval within which the score of approximately 95% of applicants will lie. *(2 marks)*
- (c) By interpreting your results in parts (b)(i) and (b)(ii), comment on the ability of the applicants to achieve a score of 25. *(3 marks)*
- (d) Give **two** reasons why a confidence interval based on a sample of size 110 would be preferable to one based on a sample of size 10. *(2 marks)*
- (e) It is suggested that a much better estimate of the mean physical fitness of all recruits could be made by combining the two samples before calculating a confidence interval. Comment on this suggestion. *(3 marks)*

Question Number and part	Solution	Marks	Total	Comments
6(a)	$\bar{x} = 48.4$ 99% confidence interval $48.4 \pm 2.5758 \times \frac{14.8}{\sqrt{10}}$	B1 B1 M1		48.4 cao 2.5758 (2.57 – 2.58) Use of $\frac{14.8}{\sqrt{10}}$ or $\frac{5}{\sqrt{10}}$
	ie 48.4 ± 12.1 $36.3 - 60.5$	m1 A1	5	Completely correct method – candidate's z 36.3 (36.3 – 36.4) and 60.5 (60.4 – 60.5) or 48.4 and 12.1 (12–12.1)
(b)(i)	95% confidence interval $49.5 \pm 1.96 \times \frac{16.5}{\sqrt{110}}$ 49.5 ± 3.08 $46.42 - 52.58$	B1 M1 A1	3	1.96 Method – their z 49.5 ± 3.08 (3.08 – 3.09) or 46.42 (46.41– 46.42) and 52.58 (52.58 – 52.59) Allow 46.4 and 52.6
(ii)	$49.5 \pm 1.96 \times 16.5$ $17.2 - 81.8$	M1 A1	2	Method – their z 49.5 ± 32.34 (32.3 – 32.4) or ± 17.2 (17.1 – 17.2) and 81.8 (81.8 – 81.9)
(c)	Mean greater than 25 but many individuals will be less than 25	E1 E1 E1	3	Mean \neq 25 Mean > 25 Some individuals < 25
(d)	Interval narrower No need for normal assumption	E1 E1	2	Interval narrower Normal not needed
(e)	Both samples random so valid suggestion. Would be better but not much ($\frac{49.5}{\sqrt{110}}$ very similar to $\frac{49.5}{\sqrt{120}}$)	E1 E1 E1	3	Valid Better Not much Any three points
	Total		18	