Core 4: First Order Differential Equations

Past Paper Questions
2006 - 2013

Name:

January 2006

8 (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

to find t in terms of x, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time t minutes, the depth of fuel in the tank is x cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

June 2006

7 Solve the differential equation

$$\frac{dy}{dx} = 6xy^2$$

given that $y = 1$ when $x = 2$. Give your answer in the form $y = f(x)$. *(6 marks)*

January 2007

8 (a) (i) Solve the differential equation $\frac{dy}{dt} = y\sin t$ to obtain y in terms of t. *(4 marks)*

(ii) Given that $y = 50$ when $t = \pi$, show that $y = 50e^{-(1+\cos t)}$. *(3 marks)*

(b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation

$$\frac{dy}{dt} = y\sin t$$

(i) Given that this height is 50 cm after π seconds, find, to the nearest centimetre, the height of the water after 6 seconds. *(2 marks)*

(ii) Find $\frac{d^2y}{dt^2}$ and hence verify that the water reaches a maximum height after π seconds. *(4 marks)*

June 2007

8 (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1+2y}}{x^2}$$

given that $y = 4$ when $x = 1$. *(6 marks)*

(b) Show that the solution can be written as $y = \frac{1}{2}\left(15 - \frac{8}{x} + \frac{1}{x^2}\right)$. *(2 marks)*

January 2008

8 Solve the differential equation

$$\frac{dy}{dx} = \frac{3\cos 3x}{y}$$

given that $y = 2$ when $x = \frac{\pi}{2}$. Give your answer in the form $y^2 = f(x)$. *(5 marks)*

January 2009

7 (a) A differential equation is given by $\frac{dx}{dt} = -kte^{\frac{1}{2}x}$, where k is a positive constant.

(i) Solve the differential equation. *(3 marks)*

(ii) Hence, given that $x = 6$ when $t = 0$, show that $x = -2\ln\left(\frac{kt^2}{4} + e^{-3}\right)$. *(3 marks)*

June 2009

8 (a) Solve the differential equation

$$\frac{dx}{dt} = \frac{150 \cos 2t}{x}$$

given that $x = 20$ when $t = \frac{\pi}{4}$, giving your solution in the form $x^2 = f(t)$. *(6 marks)*

(b) The oscillations of a 'baby bouncy cradle' are modelled by the differential equation

$$\frac{dx}{dt} = \frac{150 \cos 2t}{x}$$

where x cm is the height of the cradle above its base t seconds after the cradle begins to oscillate.

Given that the cradle is 20 cm above its base at time $t = \frac{\pi}{4}$ seconds, find:

(i) the height of the cradle above its base 13 seconds after it starts oscillating, giving your answer to the nearest millimetre; *(2 marks)*

(ii) the time at which the cradle will first be 11 cm above its base, giving your answer to the nearest tenth of a second. *(2 marks)*

January 2010

7 Solve the differential equation $\frac{dy}{dx} = \frac{1}{y}\cos\left(\frac{x}{3}\right)$, given that $y = 1$ when $x = \frac{\pi}{2}$.

Write your answer in the form $y^2 = f(x)$. *(6 marks)*

January 2011

7 (a) (i) Solve the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{x}\sin\left(\frac{t}{2}\right)$ to find x in terms of t. *(3 marks)*

(ii) Given that $x = 1$ when $t = 0$, show that the solution can be written as

$$x = (a - \cos bt)^2$$

where a and b are constants to be found. *(3 marks)*

(b) The height, x metres, above the ground of a car in a fairground ride at time t seconds is modelled by the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{x}\sin\left(\frac{t}{2}\right)$.

The car is 1 metre above the ground when $t = 0$.

(i) Find the greatest height above the ground reached by the car during the ride. *(2 marks)*

(ii) Find the value of t when the car is first 5 metres above the ground, giving your answer to one decimal place. *(2 marks)*

June 2011

7 A giant snowball is melting. The snowball can be modelled as a sphere whose surface area is decreasing at a constant rate with respect to time. The surface area of the sphere is $A\,\text{cm}^2$ at time t days after it begins to melt.

(a) Write down a differential equation in terms of the variables A and t and a constant k, where $k > 0$, to model the melting snowball. *(2 marks)*

(b) (i) Initially, the radius of the snowball is 60 cm, and 9 days later, the radius has halved.

Show that $A = 1200\pi(12 - t)$.

(You may assume that the surface area of a sphere is given by $A = 4\pi r^2$, where r is the radius.) *(4 marks)*

(ii) Use this model to find the number of days that it takes the snowball to melt completely. *(1 mark)*

January 2012

7 Solve the differential equation

$$\frac{dy}{dx} = y^2 x \sin 3x$$

given that $y = 1$ when $x = \frac{\pi}{6}$. Give your answer in the form $y = \frac{9}{f(x)}$. *(9 marks)*

June 2012

8 (a) A water tank has a height of 2 metres. The depth of the water in the tank is h metres at time t minutes after water begins to enter the tank. The rate at which the depth of the water in the tank increases is proportional to the difference between the height of the tank and the depth of the water.

Write down a differential equation in the variables h and t and a positive constant k.

(You are not required to solve your differential equation.) *(3 marks)*

(b) (i) Another water tank is filling in such a way that t minutes after the water is turned on, the depth of the water, x metres, increases according to the differential equation

$$\frac{dx}{dt} = \frac{1}{15x\sqrt{2x-1}}$$

The depth of the water is 1 metre when the water is first turned on.

Solve this differential equation to find t as a function of x. *(8 marks)*

(ii) Calculate the time taken for the depth of the water in the tank to reach 2 metres, giving your answer to the nearest 0.1 of a minute. *(1 mark)*

January 2013

5 (a) Find $\int x\sqrt{x^2+3}\,dx$. *(2 marks)*

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+3}}{e^{2y}}$$

given that $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$. *(7 marks)*

7 The height of the tide in a certain harbour is h metres at time t hours. Successive high tides occur every 12 hours.

The **rate of change** of the height of the tide can be modelled by a function of the form $a\cos(kt)$, where a and k are constants. The largest value of this rate of change is 1.3 metres per hour.

Write down a differential equation in the variables h and t. State the values of the constants a and k. *(3 marks)*

8 (a) Find $\int t\cos\left(\frac{\pi}{4}t\right)\,\mathrm{d}t$. *(4 marks)*

(b) The platform of a theme park ride oscillates vertically. For the first 75 seconds of the ride,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{t\cos\left(\frac{\pi}{4}t\right)}{32x}$$

where x metres is the height of the platform above the ground after time t seconds.

At $t = 0$, the height of the platform above the ground is 4 metres.

Find the height of the platform after 45 seconds, giving your answer to the nearest centimetre. *(6 marks)*