Core 4: Binomial Theorem

Past Exam Questions
2006 - 2013

Name:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

January 2006

5 (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in x^2.

(2 marks)

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x.

(3 marks)

(b) Obtain the binomial expansion of $\frac{1}{(1-x)^2}$ up to and including the term in x^2.

(2 marks)

(c) Given that $\frac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\frac{A}{(3-2x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$, find the values of A, B and C.

(5 marks)

(d) Hence find the binomial expansion of $\frac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in x^2.

(3 marks)

June 2006

2 (a) Obtain the binomial expansion of $(1-x)^{-3}$ up to and including the term in x^2.

(2 marks)

(b) Hence obtain the binomial expansion of $\left(1-\frac{5}{2}x\right)^{-3}$ up to and including the term in x^2.

(2 marks)

(c) Find the range of values of x for which the binomial expansion of $\left(1-\frac{5}{2}x\right)^{-3}$ would be valid.

(2 marks)

(d) Given that x is small, show that $\left(\frac{4}{2-5x}\right)^3 \approx a + bx + cx^2$, where a, b and c are integers.

(2 marks)

January 2007

5 (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in x^2. *(2 marks)*

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2 + \frac{1}{4}x - \frac{1}{32}x^2$ for small values of x. *(3 marks)*

(ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. *(2 marks)*

June 2007

2 (a) (i) Find the binomial expansion of $(1+x)^{-1}$ up to the term in x^3. *(2 marks)*

(ii) Hence, or otherwise, obtain the binomial expansion of $\frac{1}{1+3x}$ up to the term in x^3. *(2 marks)*

(b) Express $\frac{1+4x}{(1+x)(1+3x)}$ in partial fractions. *(3 marks)*

(c) (i) Find the binomial expansion of $\frac{1+4x}{(1+x)(1+3x)}$ up to the term in x^3. *(3 marks)*

(ii) Find the range of values of x for which the binomial expansion of $\frac{1+4x}{(1+x)(1+3x)}$ is valid. *(2 marks)*

January 2008

3 (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ up to and including the term in x^2. *(2 marks)*

(b) Hence obtain the binomial expansion of $\sqrt{1+\frac{3}{2}x}$ up to and including the term in x^2. *(2 marks)*

(c) Hence show that $\sqrt{\frac{2+3x}{8}} \approx a + bx + cx^2$ for small values of x, where a, b and c are constants to be found. *(2 marks)*

June 2008

4 (a) (i) Obtain the binomial expansion of $(1-x)^{\frac{1}{4}}$ up to and including the term in x^2.
(2 marks)

(ii) Hence show that $(81-16x)^{\frac{1}{4}} \approx 3 - \frac{4}{27}x - \frac{8}{729}x^2$ for small values of x.
(3 marks)

(b) Use the result from part (a)(ii) to find an approximation for $\sqrt[4]{80}$, giving your answer to seven decimal places.
(2 marks)

January 2009

4 (a) (i) Find the binomial expansion of $(1-x)^{\frac{1}{2}}$ up to and including the term in x^2.
(2 marks)

(ii) Hence obtain the binomial expansion of $\sqrt{4-x}$ up to and including the term in x^2.
(3 marks)

(b) Use your answer to part (a)(ii) to find an approximate value for $\sqrt{3}$. Give your answer to three decimal places.
(2 marks)

June 2009

3 (a) Find the binomial expansion of $(1-x)^{-1}$ up to and including the term in x^2.
(2 marks)

(b) (i) Express $\frac{3x-1}{(1-x)(2-3x)}$ in the form $\frac{A}{1-x} + \frac{B}{2-3x}$, where A and B are integers.
(3 marks)

(ii) Find the binomial expansion of $\frac{3x-1}{(1-x)(2-3x)}$ up to and including the term in x^2.
(6 marks)

(c) Find the range of values of x for which the binomial expansion of $\frac{3x-1}{(1-x)(2-3x)}$ is valid.
(2 marks)

January 2010

3 (a) (i) Find the binomial expansion of $(1+x)^{-\frac{1}{3}}$ up to and including the term in x^2.
(2 marks)

(ii) Hence find the binomial expansion of $\left(1+\frac{3}{4}x\right)^{-\frac{1}{3}}$ up to and including the term in x^2.
(2 marks)

(b) Hence show that $\sqrt[3]{\frac{256}{4+3x}} \approx a+bx+cx^2$ for small values of x, stating the values of the constants a, b and c.
(3 marks)

June 2010

4 (a) (i) Find the binomial expansion of $(1+x)^{\frac{3}{2}}$ up to and including the term in x^2.
(2 marks)

(ii) Find the binomial expansion of $(16+9x)^{\frac{3}{2}}$ up to and including the term in x^2.
(3 marks)

(b) Use your answer to part **(a)(ii)** to show that $13^{\frac{3}{2}} \approx 46+\frac{a}{b}$, stating the values of the integers a and b.
(2 marks)

January 2011

3 (a) Express $\frac{3+9x}{(1+x)(3+5x)}$ in the form $\frac{A}{1+x}+\frac{B}{3+5x}$, where A and B are integers.
(3 marks)

(b) Hence, or otherwise, find the binomial expansion of $\frac{3+9x}{(1+x)(3+5x)}$ up to and including the term in x^2.
(7 marks)

(c) Find the range of values of x for which the binomial expansion of $\frac{3+9x}{(1+x)(3+5x)}$ is valid.
(2 marks)

June 2011

3 (a) (i) Find the binomial expansion of $(1-x)^{\frac{1}{3}}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence, or otherwise, show that

$$(125-27x)^{\frac{1}{3}} \approx 5+\frac{m}{25}x+\frac{n}{3125}x^2$$

for small values of x, stating the values of the integers m and n. *(3 marks)*

(b) Use your result from part **(a)(ii)** to find an approximate value of $\sqrt[3]{119}$, giving your answer to five decimal places. *(2 marks)*

January 2012

3 (a) Find the binomial expansion of $(1+6x)^{\frac{2}{3}}$ up to and including the term in x^2. *(2 marks)*

(b) Find the binomial expansion of $(8+6x)^{\frac{2}{3}}$ up to and including the term in x^2. *(3 marks)*

(c) Use your answer from part **(b)** to find an estimate for $\sqrt[3]{100}$ in the form $\frac{a}{b}$, where a and b are integers. *(2 marks)*

June 2012

3 (a) Find the binomial expansion of $(1+4x)^{\frac{1}{2}}$ up to and including the term in x^2. *(2 marks)*

(b) (i) Find the binomial expansion of $(4-x)^{-\frac{1}{2}}$ up to and including the term in x^2. *(3 marks)*

(ii) State the range of values of x for which the expansion in part **(b)(i)** is valid. *(1 mark)*

(c) Find the binomial expansion of $\sqrt{\frac{1+4x}{4-x}}$ up to and including the term in x^2. *(2 marks)*

January 2013

2 It is given that $f(x) = \dfrac{7x-1}{(1+3x)(3-x)}$.

(a) Express $f(x)$ in the form $\dfrac{A}{3-x} + \dfrac{B}{1+3x}$, where A and B are integers. *(3 marks)*

(b) (i) Find the first three terms of the binomial expansion of $f(x)$ in the form $a + bx + cx^2$, where a, b and c are rational numbers. *(7 marks)*

(ii) State why the binomial expansion cannot be expected to give a good approximation to $f(x)$ at $x = 0.4$. *(1 mark)*

June 2013

3 (a) Find the binomial expansion of $(1+6x)^{-\frac{1}{3}}$ up to and including the term in x^2. *(2 marks)*

(b) (i) Find the binomial expansion of $(27+6x)^{-\frac{1}{3}}$ up to and including the term in x^2, simplifying the coefficients. *(3 marks)*

(ii) Given that $\sqrt[3]{\dfrac{2}{7}} = \dfrac{2}{\sqrt[3]{28}}$, use your binomial expansion from part **(b)(i)** to obtain an approximation to $\sqrt[3]{\dfrac{2}{7}}$, giving your answer to six decimal places. *(2 marks)*