Core 1 - Calculus

Past Paper Questions
2006 - 2013

Name:

7 The volume, V m^3, of water in a tank at time t seconds is given by

$$V = \tfrac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{dV}{dt}$; *(3 marks)*

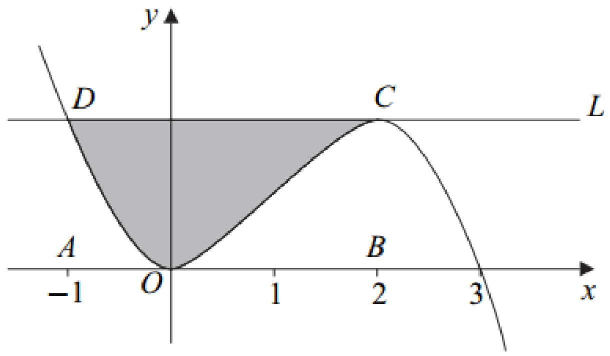
(ii) $\dfrac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in m^3 s^{-1}, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\dfrac{dy}{dx}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

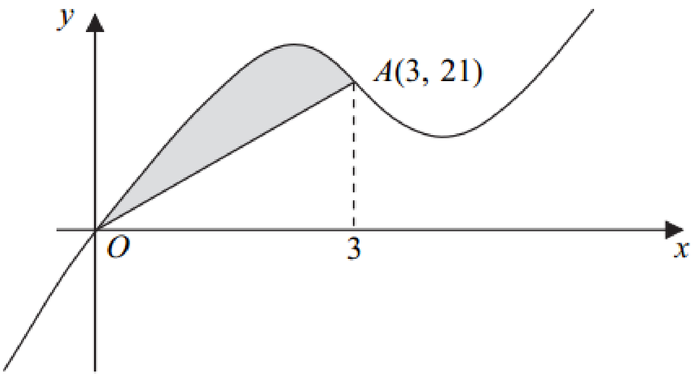
3 A curve has equation $y = 7 - 2x^5$.

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

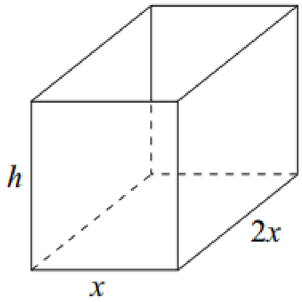
(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\, dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\, dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, $V\,\text{m}^3$, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$

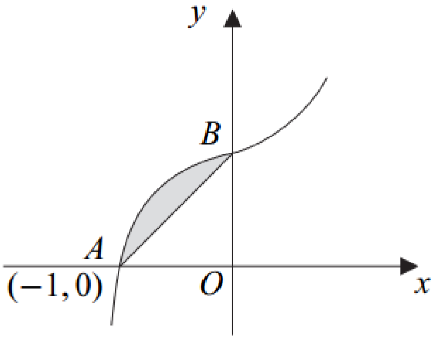
(1 mark)

(b) (i) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

(ii) Find $\int (3x^5 + 2x + 5)\, dx$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

June 2007

4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dy}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

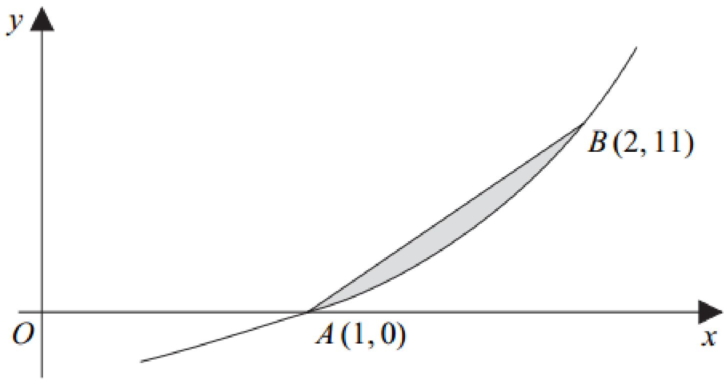
6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

(i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x-1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

January 2008

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

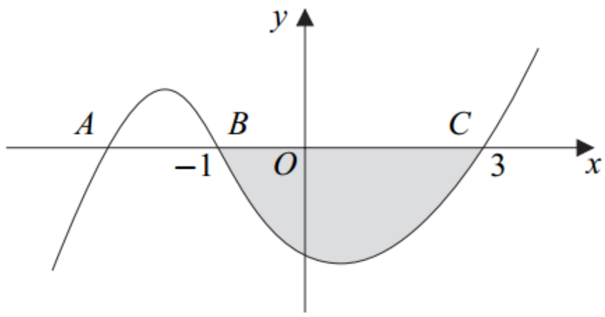
(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\int_{-1}^{3} (x^3 - 7x - 6)\, dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

3 Two numbers, x and y, are such that $3x + y = 9$, where $x \geqslant 0$ and $y \geqslant 0$.

It is given that $V = xy^2$.

(a) Show that $V = 81x - 54x^2 + 9x^3$. *(2 marks)*

(b) (i) Show that $\dfrac{\mathrm{d}V}{\mathrm{d}x} = k(x^2 - 4x + 3)$, and state the value of the integer k. *(4 marks)*

(ii) Hence find the two values of x for which $\dfrac{\mathrm{d}V}{\mathrm{d}x} = 0$. *(2 marks)*

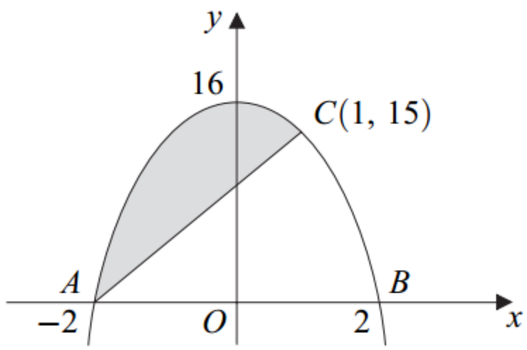
(c) Find $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}$. *(2 marks)*

(d) (i) Find the value of $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}$ for each of the two values of x found in part (b)(ii). *(1 mark)*

(ii) Hence determine the value of x for which V has a maximum value. *(1 mark)*

(iii) Find the maximum value of V. *(1 mark)*

5 The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC. *(3 marks)*

(b) (i) Find $\displaystyle\int_{-2}^{1} (16 - x^4)\,\mathrm{d}x$. *(5 marks)*

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC. *(3 marks)*

January 2009

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dx}{dt}$; *(3 marks)*

(ii) $\frac{d^2x}{dt^2}$. *(2 marks)*

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of x with respect to t when $t = 1$. *(2 marks)*

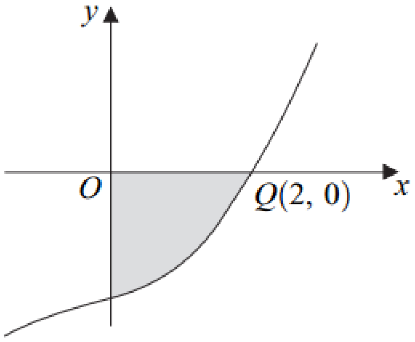
(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. *(2 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\, dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

June 2009

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Verify that the point P is a stationary point of the curve. *(2 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

(d) Find an equation of the tangent to the curve at the point where $x = 1$. *(4 marks)*

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

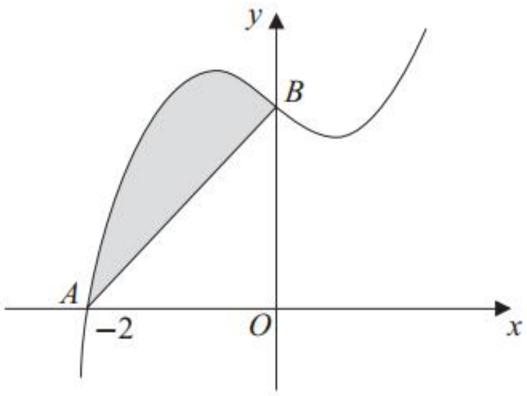
(i) Find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $p(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A(-2, 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\displaystyle\int_{-2}^{0} (x^3 - x + 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

3 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

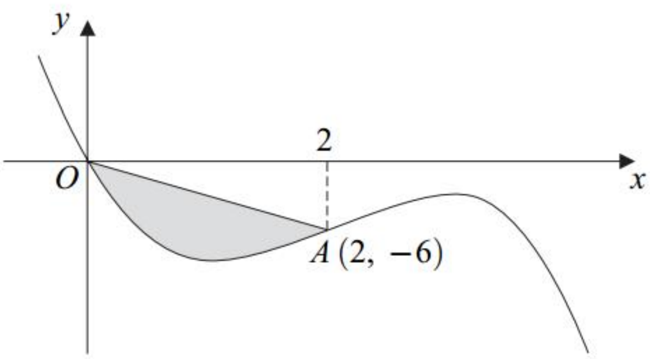
(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value. *(4 marks)*

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$. *(1 mark)*

6 The curve with equation $y = 12x^2 - 19x - 2x^3$ is sketched below.



The curve crosses the x-axis at the origin O, and the point $A\,(2, -6)$ lies on the curve.

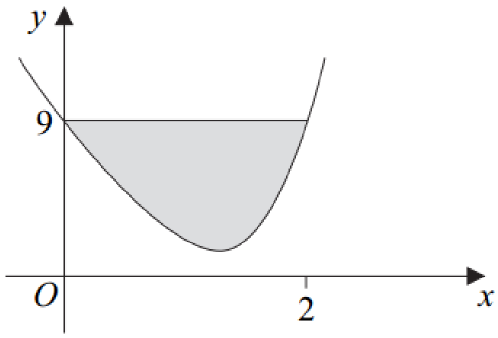
(a) (i) Find the gradient of the curve with equation $y = 12x^2 - 19x - 2x^3$ at the point A. *(4 marks)*

(ii) Hence find the equation of the normal to the curve at the point A, giving your answer in the form $x + py + q = 0$, where p and q are integers. *(3 marks)*

(b) (i) Find the value of $\int_0^2 (12x^2 - 19x - 2x^3)\, dx$. *(5 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

4 The curve with equation $y = x^4 - 8x + 9$ is sketched below.



The point $(2, 9)$ lies on the curve.

(a) (i) Find $\int_0^2 (x^4 - 8x + 9)\,dx$. *(5 marks)*

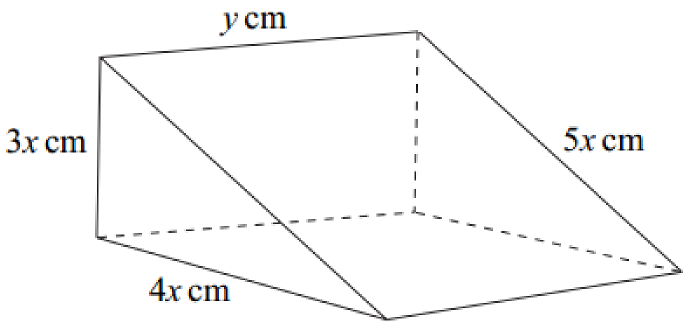
(ii) Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

(b) The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i) Find the gradient of the curve at the point A. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

6 The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is 144 cm^2.

(a) (i) Show that $xy + x^2 = 12$. *(3 marks)*

(ii) Hence show that the volume of the block, $V\text{ cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Show that V has a stationary value when $x = 2$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

January 2011

1 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

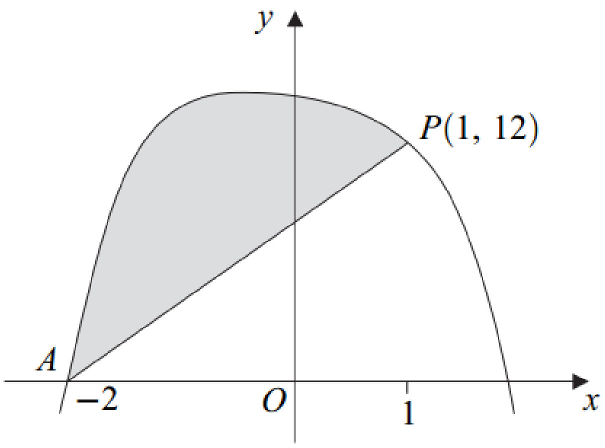
(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point. *(3 marks)*

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

4 The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

(a) (i) Find the gradient of the curve at the point P. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(2 marks)*

(b) (i) Find $\int_{-2}^{1} (14 - x - x^4)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. *(2 marks)*

June 2011

3 The volume, $V\text{ m}^3$, of water in a tank after time t seconds is given by

$$V = \frac{t^3}{4} - 3t + 5$$

(a) Find $\frac{dV}{dt}$. *(2 marks)*

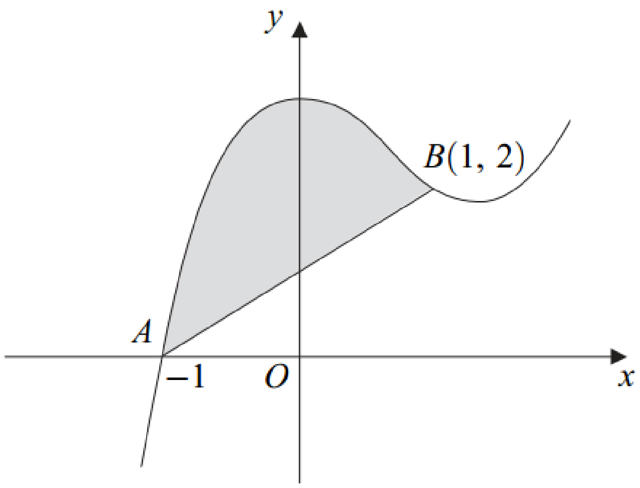
(b) (i) Find the rate of change of volume, in $\text{m}^3\,\text{s}^{-1}$, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the volume is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the positive value of t for which V has a stationary value. *(3 marks)*

(ii) Find $\frac{d^2V}{dt^2}$, and hence determine whether this stationary value is a maximum value or a minimum value. *(3 marks)*

6 The curve with equation $y = x^3 - 2x^2 + 3$ is sketched below.

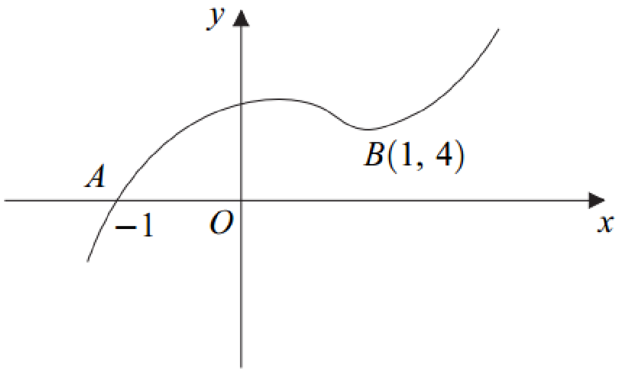


The curve cuts the x-axis at the point $A(-1, 0)$ and passes through the point $B(1, 2)$.

(a) Find $\int_{-1}^{1} (x^3 - 2x^2 + 3)\,dx$. *(5 marks)*

(b) Hence find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. *(3 marks)*

4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

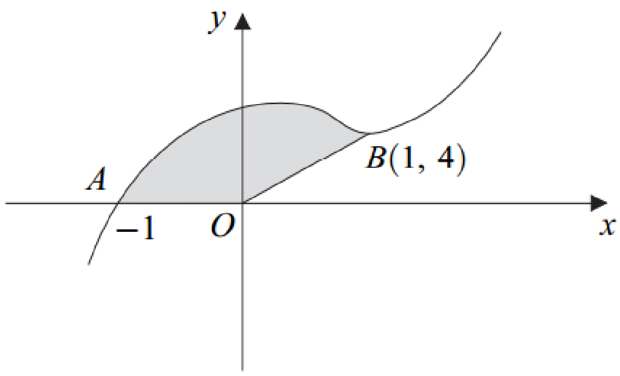
(i) $\dfrac{dy}{dx}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. *(2 marks)*

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve. *(3 marks)*

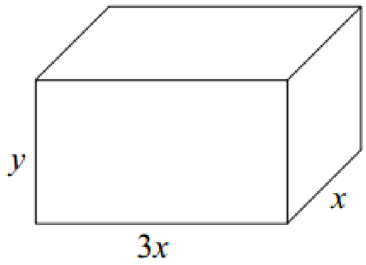
4 (d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(i) Find $\displaystyle\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB. *(2 marks)*

4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is 32 cm^2.

(a) (i) Show that $3x^2 + 4xy = 16$. *(2 marks)*

(ii) Hence show that the volume, $V\text{ cm}^3$, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$ *(2 marks)*

(b) Find $\frac{dV}{dx}$. *(2 marks)*

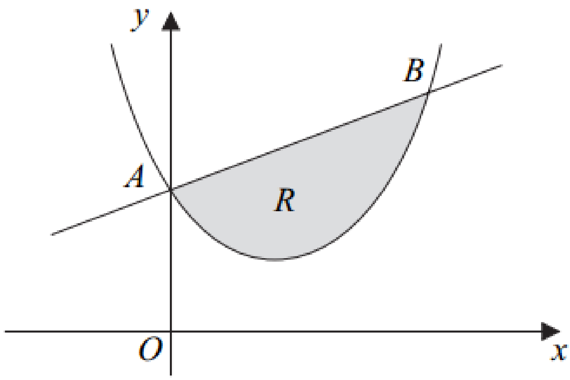
(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$. *(2 marks)*

(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$. *(2 marks)*

5 (a) (i) Express $x^2 - 3x + 5$ in the form $(x-p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

(b) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.



(i) Find the coordinates of the point B. *(3 marks)*

(ii) Find $\int (x^2 - 3x + 5)\,dx$. *(3 marks)*

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. *(4 marks)*

January 2013

2 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \leqslant t \leqslant 4$$

(a) Find $\frac{dy}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$. *(2 marks)*

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the value of $\frac{d^2y}{dt^2}$ when $t = 2$. *(2 marks)*

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value. *(1 mark)*

6 The gradient, $\frac{dy}{dx}$, of a curve at the point (x, y) is given by

$$\frac{dy}{dx} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(3 marks)*

(b) Find the equation of the curve. *(5 marks)*