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# Core 1: Circle Geometry

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Past Paper Questions  
2006 - 2013

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Name:

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**5** A circle with centre  $C$  has equation  $x^2 + y^2 - 8x + 6y = 11$ .

- (a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

- (b) Write down:

(i) the coordinates of  $C$ ; (1 mark)

(ii) the radius of the circle. (1 mark)

- (c) The point  $O$  has coordinates  $(0, 0)$ .

(i) Find the length of  $CO$ . (2 marks)

(ii) Hence determine whether the point  $O$  lies inside or outside the circle, giving a reason for your answer. (2 marks)

**7** A circle has equation  $x^2 + y^2 - 4x - 14 = 0$ .

- (a) Find:

(i) the coordinates of the centre of the circle; (3 marks)

(ii) the radius of the circle in the form  $p\sqrt{2}$ , where  $p$  is an integer. (3 marks)

- (b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. (3 marks)

- (c) A line has equation  $y = 2k - x$ , where  $k$  is a constant.

(i) Show that the  $x$ -coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0 \quad (3 \text{ marks})$$

(ii) Find the values of  $k$  for which the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0$$

has equal roots. (4 marks)

(iii) Describe the geometrical relationship between the line and the circle when  $k$  takes either of the values found in part (c)(ii). (1 mark)

**4** A circle with centre  $C$  has equation  $x^2 + y^2 + 2x - 12y + 12 = 0$ .

- (a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

- (b) Write down:

(i) the coordinates of  $C$ ; (1 mark)

(ii) the radius of the circle. (1 mark)

- (c) Show that the circle does **not** intersect the  $x$ -axis. (2 marks)

- (d) The line with equation  $x + y = 4$  intersects the circle at the points  $P$  and  $Q$ .

- (i) Show that the  $x$ -coordinates of  $P$  and  $Q$  satisfy the equation

$$x^2 + 3x - 10 = 0 \quad (3 \text{ marks})$$

(ii) Given that  $P$  has coordinates  $(2, 2)$ , find the coordinates of  $Q$ . (2 marks)

(iii) Hence find the coordinates of the midpoint of  $PQ$ . (2 marks)

**5** A circle with centre  $C$  has equation  $(x + 3)^2 + (y - 2)^2 = 25$ .

- (a) Write down:

(i) the coordinates of  $C$ ; (2 marks)

(ii) the radius of the circle. (1 mark)

- (b) (i) Verify that the point  $N(0, -2)$  lies on the circle. (1 mark)

(ii) Sketch the circle. (2 marks)

(iii) Find an equation of the normal to the circle at the point  $N$ . (3 marks)

- (c) The point  $P$  has coordinates  $(2, 6)$ .

(i) Find the distance  $PC$ , leaving your answer in surd form. (2 marks)

(ii) Find the length of a tangent drawn from  $P$  to the circle. (3 marks)

January 2008

4 A circle with centre  $C$  has equation  $x^2 + y^2 - 10y + 20 = 0$ .

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k \quad (2 \text{ marks})$$

(b) Write down:

(i) the coordinates of  $C$ ; (1 mark)

(ii) the radius of the circle, leaving your answer in surd form. (1 mark)

(c) A line has equation  $y = 2x$ .

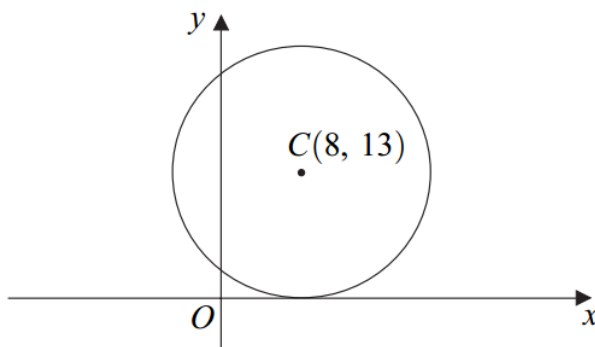
(i) Show that the  $x$ -coordinate of any point of intersection of the line and the circle satisfies the equation  $x^2 - 4x + 4 = 0$ . (2 marks)

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact,  $P$ . (3 marks)

(d) Prove that the point  $Q(-1, 4)$  lies inside the circle. (2 marks)

June 2008

7 The circle  $S$  has centre  $C(8, 13)$  and touches the  $x$ -axis, as shown in the diagram.



(a) Write down an equation for  $S$ , giving your answer in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (2 \text{ marks})$$

(b) The point  $P$  with coordinates  $(3, 1)$  lies on the circle.

(i) Find the gradient of the straight line passing through  $P$  and  $C$ . (1 mark)

(ii) Hence find an equation of the tangent to the circle  $S$  at the point  $P$ , giving your answer in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers. (4 marks)

(iii) The point  $Q$  also lies on the circle  $S$ , and the length of  $PQ$  is 10. Calculate the shortest distance from  $C$  to the chord  $PQ$ . (3 marks)

**7** A circle with centre  $C$  has equation  $x^2 + y^2 - 6x + 10y + 9 = 0$ .

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of  $C$ ;

(ii) the radius of the circle. (2 marks)

(c) The point  $D$  has coordinates  $(7, -2)$ .

(i) Verify that the point  $D$  lies on the circle. (1 mark)

(ii) Find an equation of the normal to the circle at the point  $D$ , giving your answer in the form  $mx + ny = p$ , where  $m$ ,  $n$  and  $p$  are integers. (3 marks)

(d) (i) A line has equation  $y = kx$ . Show that the  $x$ -coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0 \quad (2 \text{ marks})$$

(ii) Find the values of  $k$  for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots. (5 marks)

(iii) Describe the geometrical relationship between the line and the circle when  $k$  takes either of the values found in part (d)(ii). (1 mark)

**5** A circle with centre  $C$  has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

(a) Write down:

(i) the coordinates of  $C$ ; (1 mark)

(ii) the radius of the circle. (1 mark)

(b) (i) Verify that the circle passes through the origin  $O$ . (1 mark)

(ii) Given that the circle also passes through the points  $(10, 0)$  and  $(0, p)$ , sketch the circle and find the value of  $p$ . (3 marks)

(c) The point  $A(-7, -7)$  lies on the circle.

(i) Find the gradient of  $AC$ . (2 marks)

(ii) Hence find an equation of the tangent to the circle at the point  $A$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (3 marks)

**7** A circle with centre  $C$  has equation  $x^2 + y^2 - 4x + 12y + 15 = 0$ .

(a) Find:

(i) the coordinates of  $C$ ; (2 marks)

(ii) the radius of the circle. (2 marks)

(b) Explain why the circle lies entirely below the  $x$ -axis. (2 marks)

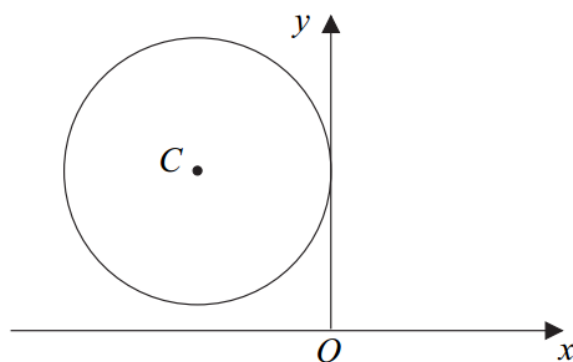
(c) The point  $P$  with coordinates  $(5, k)$  lies outside the circle.

(i) Show that  $PC^2 = k^2 + 12k + 45$ . (2 marks)

(ii) Hence show that  $k^2 + 12k + 20 > 0$ . (1 mark)

(iii) Find the possible values of  $k$ . (4 marks)

- 5** A circle with centre  $C(-5, 6)$  touches the  $y$ -axis, as shown in the diagram.



- (a)** Find the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

- (b) (i)** Verify that the point  $P(-2, 2)$  lies on the circle. (1 mark)
- (ii)** Find an equation of the normal to the circle at the point  $P$ . (3 marks)
- (iii)** The mid-point of  $PC$  is  $M$ . Determine whether the point  $P$  is closer to the point  $M$  or to the origin  $O$ . (4 marks)

- 6** A circle has centre  $C(-3, 1)$  and radius  $\sqrt{13}$ .

- (a) (i)** Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k \quad (2 \text{ marks})$$

- (ii)** Hence find the equation of the circle in the form

$$x^2 + y^2 + mx + ny + p = 0$$

where  $m$ ,  $n$  and  $p$  are integers. (3 marks)

- (b)** The circle cuts the  $y$ -axis at the points  $A$  and  $B$ . Find the distance  $AB$ . (3 marks)
- (c) (i)** Verify that the point  $D(-5, -2)$  lies on the circle. (1 mark)
- (ii)** Find the gradient of  $CD$ . (2 marks)
- (iii)** Hence find an equation of the tangent to the circle at the point  $D$ . (2 marks)

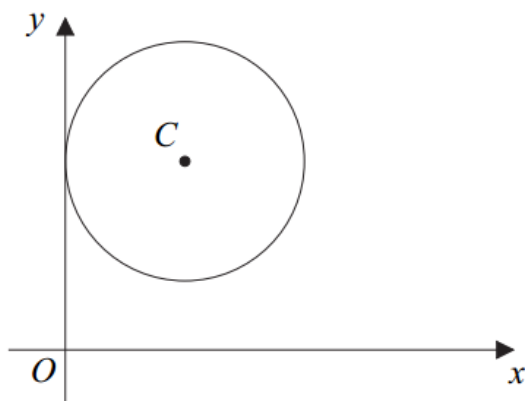


- 8** A circle has centre  $C(3, -8)$  and radius 10.
- (a)** Express the equation of the circle in the form
- $$(x - a)^2 + (y - b)^2 = k \quad (2 \text{ marks})$$
- (b)** Find the  $x$ -coordinates of the points where the circle crosses the  $x$ -axis. (3 marks)
- (c)** The tangent to the circle at the point  $A$  has gradient  $\frac{5}{2}$ . Find an equation of the line  $CA$ , giving your answer in the form  $rx + sy + t = 0$ , where  $r$ ,  $s$  and  $t$  are integers. (3 marks)
- (d)** The line with equation  $y = 2x + 1$  intersects the circle.
- (i)** Show that the  $x$ -coordinates of the points of intersection satisfy the equation
- $$x^2 + 6x - 2 = 0 \quad (3 \text{ marks})$$
- (ii)** Hence show that the  $x$ -coordinates of the points of intersection are of the form  $m \pm \sqrt{n}$ , where  $m$  and  $n$  are integers. (2 marks)

- 7** A circle with centre  $C$  has equation  $x^2 + y^2 + 14x - 10y + 49 = 0$ .
- (a)** Express this equation in the form
- $$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$
- (b)** Write down:
- (i)** the coordinates of  $C$ ;
- (ii)** the radius of the circle. (2 marks)
- (c)** Sketch the circle. (2 marks)
- (d)** A line has equation  $y = kx + 6$ , where  $k$  is a constant.
- (i)** Show that the  $x$ -coordinates of any points of intersection of the line and the circle satisfy the equation  $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ . (2 marks)
- (ii)** The equation  $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$  has equal roots. Show that
- $$12k^2 - 7k - 12 = 0 \quad (3 \text{ marks})$$
- (iii)** Hence find the values of  $k$  for which the line is a tangent to the circle. (2 marks)



- 6** The circle with centre  $C(5, 8)$  touches the  $y$ -axis, as shown in the diagram.



- (a)** Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k \quad (2 \text{ marks})$$

- (b) (i)** Verify that the point  $A(2, 12)$  lies on the circle. (1 mark)

- (ii)** Find an equation of the tangent to the circle at the point  $A$ , giving your answer in the form  $sx + ty + u = 0$ , where  $s$ ,  $t$  and  $u$  are integers. (5 marks)

- (c)** The points  $P$  and  $Q$  lie on the circle, and the mid-point of  $PQ$  is  $M(7, 12)$ .

- (i)** Show that the length of  $CM$  is  $n\sqrt{5}$ , where  $n$  is an integer. (2 marks)

- (ii)** Hence find the area of triangle  $PCQ$ . (3 marks)

- 7** A circle with centre  $C(-3, 2)$  has equation

$$x^2 + y^2 + 6x - 4y = 12$$

- (a)** Find the  $y$ -coordinates of the points where the circle crosses the  $y$ -axis. (3 marks)

- (b)** Find the radius of the circle. (3 marks)

- (c)** The point  $P(2, 5)$  lies outside the circle.

- (i)** Find the length of  $CP$ , giving your answer in the form  $\sqrt{n}$ , where  $n$  is an integer. (2 marks)

- (ii)** The point  $Q$  lies on the circle so that  $PQ$  is a tangent to the circle. Find the length of  $PQ$ . (2 marks)

**3** A circle  $C$  has equation

$$x^2 + y^2 - 10x + 14y + 25 = 0$$

**(a)** Write the equation of  $C$  in the form

$$(x - a)^2 + (y - b)^2 = k$$

where  $a$ ,  $b$  and  $k$  are integers.

*(3 marks)*

**(b)** Hence, for the circle  $C$ , write down:

**(i)** the coordinates of its centre;

*(1 mark)*

**(ii)** its radius.

*(1 mark)*

**(c) (i)** Sketch the circle  $C$ .

*(2 marks)*

**(ii)** Write down the coordinates of the point on  $C$  that is furthest away from the  $x$ -axis.

*(2 marks)*

**(d)** Given that  $k$  has the same value as in part **(a)**, describe geometrically the transformation which maps the circle with equation  $(x + 1)^2 + y^2 = k$  onto the circle  $C$ .

*(3 marks)*