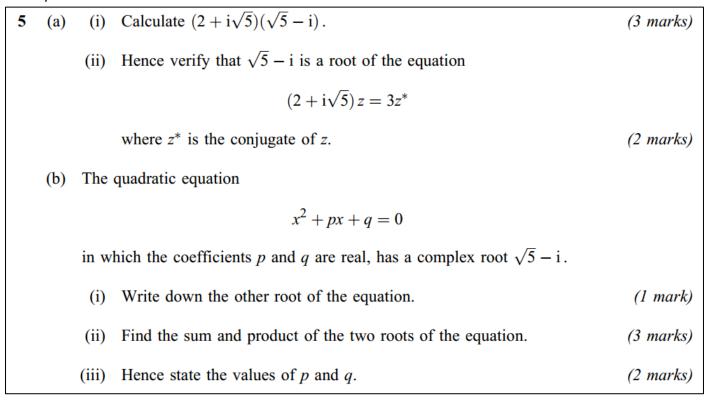
## FP1: Complex Numbers

Past Exam Questions 2006 - 2013

Name:



June 2006

6 It is given that z = x + iy, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for

 $(z + i)^*$ 

where  $(z + i)^*$  denotes the complex conjugate of (z + i).

(b) Solve the equation

 $(z+i)^* = 2iz + 1$ 

giving your answer in the form a + bi.

(5 marks)

(2 marks)

1(a) Solve the following equations, giving each root in the form a + bi:(i)  $x^2 + 16 = 0$ ;(2 marks)(ii)  $x^2 - 2x + 17 = 0$ .(2 marks)(b) (i) Expand  $(1 + x)^3$ .(2 marks)(ii) Express  $(1 + i)^3$  in the form a + bi.(2 marks)(iii) Hence, or otherwise, verify that x = 1 + i satisfies the equation(2 marks) $x^3 + 2x - 4i = 0$ (2 marks)

## June 2007

3 It is given that z = x + iy, where x and y are real numbers.
(a) Find, in terms of x and y, the real and imaginary parts of z - 3iz\*
where z\* is the complex conjugate of z. (3 marks)
(b) Find the complex number z such that z - 3iz\* = 16 (3 marks)

January 2008

1 It is given that 
$$z_1 = 2 + i$$
 and that  $z_1^*$  is the complex conjugate of  $z_1$ .

Find the real numbers x and y such that

$$x + 3iy = z_1 + 4iz_1^* \tag{4 marks}$$

June 2008

2	The complex number $2 + 3i$ is a root of the quadratic equation	
	$x^2 + bx + c = 0$	
	where $b$ and $c$ are real numbers.	
	(a) Write down the other root of this equation.	(1 mark)

June 2009

3	The complex number $z$ is defined by	
	z = x + 2i	
	where x is real.	
(a)	Find, in terms of x, the real and imaginary parts of:	
(i)	$z^2;$	(3 marks)
(ii)	$z^2 + 2z^*$ .	(2 marks)
(b)	Show that there is exactly one value of x for which $z^2 + 2z^*$ is real.	(2 marks)

January 2010

2	The complex number $z$ is defined by	
	z = 1 + i	
	(a) Find the value of $z^2$ , giving your answer in its simplest form.	(2 marks)
	(b) Hence show that $z^8 = 16$ .	(2 marks)
	(c) Show that $(z^*)^2 = -z^2$ .	(2 marks)

June 2010

2	It is given that $z = x + iy$ , where x and y are real numbers.	
(a)	Find, in terms of $x$ and $y$ , the real and imaginary parts of	
	$(1 - 2i)z - z^*$	(4 marks)
(b)	Hence find the complex number $z$ such that	
	$(1-2i)z - z^* = 10(2+i)$	(2 marks)

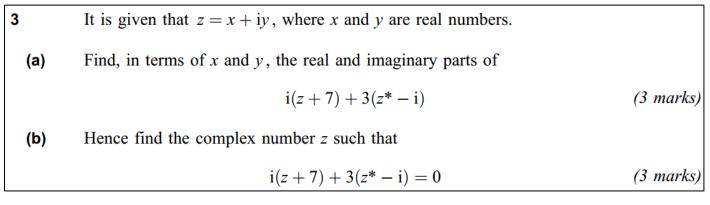
5 (a) It is given that z<sub>1</sub> = <sup>1</sup>/<sub>2</sub> - i.
(i) Calculate the value of z<sub>1</sub><sup>2</sup>, giving your answer in the form a + bi. (2 marks)
(ii) Hence verify that z<sub>1</sub> is a root of the equation
z<sup>2</sup> + z\* + <sup>1</sup>/<sub>4</sub> = 0 (2 marks)
(b) Show that z<sub>2</sub> = <sup>1</sup>/<sub>2</sub> + i also satisfies the equation in part (a)(ii). (2 marks)
(c) Show that the equation in part (a)(ii) has two equal real roots. (2 marks)

## June 2011

3	It is given that $z = x + iy$ , where x and y are real.	
(a)	Find, in terms of $x$ and $y$ , the real and imaginary parts of	
	$(z - i)(z^* - i)$	(3 marks)
(b)	Given that	
	$(z - i)(z^* - i) = 24 - 8i$	
	find the two possible values of z.	(4 marks)

January 2012

3 (a)	Solve the following equations, giving each root in the form $a + bi$ :	
(i)	$x^2 + 9 = 0;$	(1 mark)
(ii)	$(x+2)^2 + 9 = 0.$	(1 mark)
(b) (i)	Expand $(1+x)^3$ .	(1 mark)
(ii)	Express $(1+2i)^3$ in the form $a+bi$ .	(3 marks)
(iii)	Given that $z = 1 + 2i$ , find the value of	
	$z^* - z^3$	(2 marks)



January 2013

2 (a)	Solve the equation $w^2 + 6w + 34 = 0$ , giving your answers in the form p where p and q are integers.	+ qi, (3 marks)
(b)	It is given that $z = i(1+i)(2+i)$ .	
(i)	Express z in the form $a + bi$ , where a and b are integers.	(3 marks)
(ii)	Find integers m and n such that $z + mz^* = ni$ .	(3 marks)

June 2013

4 (a) It is given that z = x + yi, where x and y are real numbers.
(i) Write down, in terms of x and y, an expression for (z - 2i)\*. (1 mark)
(ii) Solve the equation
 (z - 2i)\* = 4iz + 3
 giving your answer in the form a + bi. (5 marks)
(b) It is given that p + qi, where p and q are real numbers, is a root of the equation
 z<sup>2</sup> + 10iz - 29 = 0.
 Without finding the values of p and q, state why p - qi is not a root of the

(1 mark)

equation  $z^2 + 10iz - 29 = 0$ .