D2: Allocations

Past Paper Questions
2006 - 2013

Name:

January 2006

1 Five trainers, Ali, Bo, Chas, Dee and Eve, held an initial training session with each of four teams over an assault course. The completion times in minutes are recorded below.

	Ali	**Bo**	**Chas**	**Dee**	**Eve**
Team 1	16	19	18	25	24
Team 2	22	21	20	26	25
Team 3	21	22	23	21	24
Team 4	20	21	21	23	20

Each of the four teams is to be allocated a trainer and the overall time for the four teams is to be minimised. No trainer can train more than one team.

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four trainers should be allocated to which team. State the minimum total training time for the four teams using this matching. *(8 marks)*

June 2006

2 Four of the five students Phil, Quin, Ros, Sue and Tim are to be chosen to make up a team for a mathematical relay race. The team will be asked four questions, one each on the topics A, B, C and D. A different member of the team will answer each question. Each member has to give the correct answer to the question before the next question is asked. The team with the least overall time wins.
The average times, in seconds, for each student in some practice questions are given below.

	Phil	**Quin**	**Ros**	**Sue**	**Tim**
Topic A	18	15	19	20	17
Topic B	23	24	22	25	23
Topic C	20	16	18	22	19
Topic D	21	17	18	23	20

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first**, then rows, to decide which four students should be chosen for the team. State which student should be allocated to each topic and state the total time for the four students on the practice questions using this matching. *(8 marks)*

2 Five successful applicants received the following scores when matched against suitability criteria for five jobs in a company.

	Job 1	**Job 2**	**Job 3**	**Job 4**	**Job 5**
Alex	13	11	9	10	13
Bill	15	12	12	11	12
Cath	12	10	8	14	14
Don	11	12	13	14	10
Ed	12	14	14	13	14

It is intended to allocate each applicant to a different job so as to maximise the total score of the five applicants.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $15 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table from 15. Use the Hungarian algorithm to allocate the jobs to the applicants so that the total score is maximised. *(8 marks)*

(c) It is later discovered that Bill has already been allocated to Job 4. Decide how to alter the allocation of the other jobs so as to maximise the score now possible. *(3 marks)*

2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	A	**B**	**C**	**D**	**E**
Centre 1	10	11	8	12	5
Centre 2	11	5	11	6	7
Centre 3	12	8	7	11	4
Centre 4	10	9	14	10	6
Centre 5	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

(a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

(b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost. *(3 marks)*

(d) Find the value of this minimum daily total travel cost. *(1 mark)*

January 2008

2 The following table shows the times taken, in minutes, by five people, Ash, Bob, Col, Dan and Emma, to carry out the tasks 1, 2, 3 and 4. Dan cannot do task 3.

	Ash	**Bob**	**Col**	**Dan**	**Emma**
Task 1	14	10	12	12	14
Task 2	11	13	10	12	12
Task 3	13	11	12	**	12
Task 4	13	10	12	13	15

Each of the four tasks is to be given to a different one of the five people so that the overall time for the four tasks is minimised.

(a) Modify the table of values by adding an extra row of **non-zero** values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four people should be allocated to which task. State the minimum total time for the four tasks using this matching. *(8 marks)*

(c) After special training, Dan is able to complete task 3 in 12 minutes. Determine, giving a reason, whether the minimum total time found in part (b) could be improved. *(2 marks)*

June 2008

2 The following table shows the scores of five people, Alice, Baji, Cath, Dip and Ede, after playing five different computer games.

	Alice	**Baji**	**Cath**	**Dip**	**Ede**
Game 1	17	16	19	17	20
Game 2	20	13	15	16	18
Game 3	16	17	15	18	13
Game 4	13	14	18	15	17
Game 5	15	16	20	16	15

Each of the five games is to be assigned to one of the five people so that the total score is maximised. No person can be assigned to more than one game.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $20 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 20, and hence show that, by reducing **columns first** and then rows, the resulting table of values is as below.

3	1	1	1	0
0	4	5	2	2
4	0	5	0	7
5	1	0	1	1
5	1	0	2	5

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with one horizontal and three vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of games to the five people so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

1 The times taken in minutes for five people, P, Q, R, S and T, to complete each of five different tasks are recorded in the table.

	P	**Q**	**R**	**S**	**T**
Task 1	17	20	19	17	17
Task 2	19	18	18	18	15
Task 3	13	16	16	14	12
Task 4	13	13	15	13	13
Task 5	10	11	12	14	13

Using the Hungarian algorithm, each of the five people is to be allocated to a different task so that the total time for completing the five tasks is minimised.

(a) By reducing the **columns first** and then the rows, show that the new table of values is as follows.

3	5	3	0	1
6	4	3	2	0
3	5	4	1	0
3	2	3	0	1
0	0	0	1	1

(3 marks)

(b) Show that the zeros in the table in part (a) can be covered with three lines, and use adjustments to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the two possible ways of allocating the five people to the five tasks so that the total completion time is minimised. *(3 marks)*

(d) Find the minimum total time for completing the five tasks. *(1 mark)*

3 Five lecturers were given the following scores when matched against criteria for teaching five courses in a college.

	Course 1	**Course 2**	**Course 3**	**Course 4**	**Course 5**
Ron	13	13	9	10	13
Sam	13	14	12	17	15
Tom	16	10	8	14	14
Una	11	14	12	16	10
Viv	12	14	14	13	15

Each lecturer is to be allocated to exactly one of the courses so as to maximise the total score of the five lecturers.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $17 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 17. Hence show that, by reducing **rows first** and then columns, the resulting table of values is as below.

0	0	3	3	0
4	3	4	0	2
0	6	7	2	2
5	2	3	0	6
3	1	0	2	0

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with two horizontal and two vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of courses to the five lecturers so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

2 The following table shows the times taken, in minutes, by five people, Ron, Sam, Tim, Vic and Zac, to carry out the tasks 1, 2, 3 and 4. Sam takes x minutes, where $8 \leqslant x \leqslant 12$, to do task 2.

	Ron	**Sam**	**Tim**	**Vic**	**Zac**
Task 1	8	7	9	10	8
Task 2	9	x	8	7	11
Task 3	12	10	9	9	10
Task 4	11	9	8	11	11

Each of the four tasks is to be given to a different one of the five people so that the total time for the four tasks is minimised.

(a) Modify the table of values by adding an extra row of **non-zero** values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) (i) Use the Hungarian algorithm, reducing **columns first** and then rows, to reduce the matrix to a form, in terms of x, from which the optimum matching can be made. *(5 marks)*

(ii) Hence find the possible way of allocating the four tasks so that the total time is minimised. *(2 marks)*

(iii) Find the minimum total time. *(1 mark)*

(c) After special training, Sam is able to complete task 2 in 7 minutes and is assigned to task 2.

Determine the possible ways of allocating the other three tasks so that the total time is minimised. *(2 marks)*

2 Five students attempted five different games, and penalty points were given for any mistakes that they made. The table shows the penalty points incurred by the students.

	Game 1	Game 2	Game 3	Game 4	Game 5
Ali	5	7	3	8	8
Beth	8	6	4	8	7
Cat	6	1	2	10	3
Di	4	4	3	10	7
Ell	4	6	4	7	9

Using the Hungarian algorithm, each of the five students is to be allocated to a different game so that the total number of penalty points is minimised.

(a) By reducing the **rows first** and then the columns, show that the new table of values is

2	4	0	2	3
4	2	0	1	1
5	0	1	k	0
1	1	0	4	2
0	2	0	0	3

and state the value of the constant k. *(3 marks)*

(b) Show that the zeros in the table in part **(a)** can be covered with three lines, and use augmentation to produce a table where five lines are required to cover the zeros. *(3 marks)*

(c) Hence find the possible ways of allocating the five students to the five games with the minimum total of penalty points. *(3 marks)*

(d) Find the minimum possible total of penalty points. *(1 mark)*

2 A farmer has five fields. He intends to grow a different crop in each of four fields and to leave one of the fields unused. The farmer tests the soil in each field and calculates a score for growing each of the four crops. The scores are given in the table below.

	Field A	**Field B**	**Field C**	**Field D**	**Field E**
Crop 1	16	12	8	18	14
Crop 2	20	15	8	16	12
Crop 3	9	10	12	17	12
Crop 4	18	11	17	15	19

The farmer's aim is to maximise the total score for the four crops.

(a) (i) Modify the table of values by first subtracting each value in the table above from 20 and then adding an extra row of equal values. *(1 mark)*

(ii) Explain why the Hungarian algorithm can now be applied to the new table of values to maximise the total score for the four crops. *(3 marks)*

(b) (i) By reducing **rows** first, show that the table from part **(a)(i)** becomes

2	6	10	0	p
0	5	12	4	8
8	7	5	0	q
1	8	2	4	0
0	0	0	0	0

State the values of the constants p and q. *(2 marks)*

(ii) Show that the zeros in the table from part **(b)(i)** can be covered by one horizontal and three vertical lines, and use the Hungarian algorithm to decide how the four crops should be allocated to the fields. *(6 marks)*

(iii) Hence find the maximum possible total score for the four crops. *(1 mark)*

2 The times taken, in minutes, for five people, A, B, C, D and E, to complete each of five different puzzles are recorded in the table below.

	A	B	C	D	E
Puzzle 1	16	13	15	16	15
Puzzle 2	14	16	16	14	18
Puzzle 3	14	12	18	13	16
Puzzle 4	15	15	17	12	14
Puzzle 5	13	17	16	14	15

Using the Hungarian algorithm, each of the five people is to be allocated to a different puzzle so that the total time for completing the five puzzles is minimised.

(a) By reducing the **columns first** and then the rows, show that the new table of values is

3	1	0	4	1
0	k	0	1	3
1	0	3	1	2
2	3	2	0	0
0	5	1	2	1

State the value of the constant k. *(2 marks)*

(b) (i) Show that the zeros in the table in part **(a)** can be covered with one horizontal and three vertical lines. *(1 mark)*

(ii) Use augmentation to produce a table where five lines are required to cover the zeros. *(2 marks)*

(c) Hence find all the possible ways of allocating the five people to the five puzzles so that the total completion time is minimised. *(3 marks)*

(d) Find the minimum total time for completing the five puzzles. *(1 mark)*

(e) Explain how you would modify the original table if the Hungarian algorithm were to be used to find the **maximum** total time for completing the five puzzles using five different people. *(1 mark)*

2 A team with five members is training to take part in a quiz. The team members, Abby, Bob, Cait, Drew and Ellie, attempted sample questions on each of the five topics and their scores are given in the table.

	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5
Abby	27	29	25	35	31
Bob	33	22	17	29	29
Cait	23	29	25	33	21
Drew	22	29	29	27	31
Ellie	27	27	19	21	27

For the actual quiz, each topic must be allocated to exactly one of the team members. The maximum total score for the sample questions is to be used to allocate the different topics to the team members.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $35 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 35. Hence show that, by reducing **rows first** then columns, the resulting table of values is as below, stating the values of the constants p and q.

8	6	8	0	4
0	11	p	4	4
10	4	6	0	12
q	2	0	4	0
0	0	6	6	0

(3 marks)

(c) Show that the zeros in the table in part **(b)** can be covered with two horizontal and two vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) (i) Hence find the possible allocations of topics to the five team members so that the total score for the sample questions is maximised. *(3 marks)*

(ii) State the value of this maximum total score. *(1 mark)*

June 2012

2 The times taken in minutes for five people, Ann, Baz, Cal, Di and Ez, to complete each of five different tasks are recorded in the table below. Neither Ann nor Di can do task 2, as indicated by the asterisks in the table.

	Ann	Baz	Cal	Di	Ez
Task 1	13	14	15	17	16
Task 2	***	21	21	***	18
Task 3	16	19	19	17	15
Task 4	16	16	18	16	16
Task 5	20	23	22	20	20

Using the Hungarian algorithm, each of the five people is to be allocated to a different task so that the total time for completing the five tasks is minimised.

(a) By reducing the **rows first** and then the columns, show that the zeros in the new table of values can be covered with four lines. *(3 marks)*

(b) Use adjustments to produce a table where five lines are required to cover the zeros. *(3 marks)*

(c) Hence find the possible ways of allocating the five people to the five tasks in the minimum total time. *(3 marks)*

(d) State the minimum total time for completing the five tasks. *(1 mark)*

January 2013

3 Four pupils, Wendy, Xiong, Yasmin and Zaira, are each to be allocated a different memory coach from five available coaches: Asif, Bill, Connie, Deidre and Eric. Each pupil has an initial training session with each coach, and a test which scores their improvement in memory-recall produces the following results.

	Asif	Bill	Connie	Deidre	Eric
Wendy	35	38	43	34	37
Xiong	38	37	38	34	36
Yasmin	32	33	31	31	32
Zaira	34	38	35	31	34

(a) Modify the table of results by subtracting each value from 43. *(1 mark)*

(b) Use the Hungarian algorithm, reducing the **rows first**, to assign one coach to one pupil so that the total improvement of the four pupils is maximised.

State the total improvement of the four pupils. *(8 marks)*

3 The table shows the times taken, in minutes, by five people, A, B, C, D and E, to carry out the tasks V, W, X, Y and Z.

	A	***B***	***C***	***D***	***E***
Task *V*	100	110	112	102	95
Task *W*	125	130	110	120	115
Task *X*	105	110	101	108	120
Task *Y*	115	115	120	135	110
Task *Z*	100	98	99	100	102

Each of the five tasks is to be given to a different one of the five people so that the total time for the five tasks is minimised. The Hungarian algorithm is to be used.

(a) By reducing the **columns first**, and then the rows, show that the new table of values is

0	12	13	2	0
14	21	0	k	9
3	10	0	6	23
0	2	6	20	0
0	0	0	0	7

and state the value of the constant k. *(3 marks)*

(b) Show that the zeros in the table in part **(a)** can be covered with four lines. Use augmentation **twice** to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the possible ways of allocating the five tasks to the five people to achieve the minimum total time. *(3 marks)*

(d) Find the minimum total time. *(1 mark)*