D2: Game Theory

Past Paper Questions
2006 - 2013

Name:

January 2006

6 Sam is playing a computer game in which he is trying to drive a car in different road conditions. He chooses a car and the computer decides the road conditions. The points scored by Sam are shown in the table.

Road Conditions

		C_1	C_2	C_3
	S_1	−2	2	4
Sam's Car	S_2	2	4	5
	S_3	5	1	2

Sam is trying to maximise his total points and the computer is trying to stop him.

(a) Explain why Sam should never choose S_1 and why the computer should not choose C_3. *(2 marks)*

(b) Find the play-safe strategies for the reduced 2 by 2 game for Sam and the computer, and hence show that this game does not have a stable solution. *(4 marks)*

(c) Sam uses random numbers to choose S_2 with probability p and S_3 with probability $1 - p$.

(i) Find expressions for the expected gain for Sam when the computer chooses each of its two remaining strategies. *(3 marks)*

(ii) Calculate the value of p for Sam to maximise his total points. *(2 marks)*

(iii) Hence find the expected points gain for Sam. *(1 mark)*

June 2006

6 Two people, Rowan and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rowan.

Colleen

	Strategy	C_1	C_2	C_3
	R_1	−3	−4	1
Rowan	R_2	1	5	−1
	R_3	−2	−3	4

(a) Explain the meaning of the term 'zero-sum game'. *(1 mark)*

(b) Show that this game has no stable solution. *(3 marks)*

(c) Explain why Rowan should never play strategy R_1. *(1 mark)*

(d) (i) Find the optimal mixed strategy for Rowan. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

4 (a) Two people, Ros and Col, play a zero-sum game. The game is represented by the following pay-off matrix for Ros.

		Col		
	Strategy	**X**	**Y**	**Z**
Ros	**I**	−4	−3	0
	II	5	−2	2
	III	1	−1	3

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player and state the value of the game. *(2 marks)*

(b) Ros and Col play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros.

		Col		
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Ros	$\mathbf{R_1}$	3	2	1
	$\mathbf{R_2}$	−2	−1	2

(i) Find the optimal mixed strategy for Ros. *(7 marks)*

(ii) Calculate the value of the game. *(1 mark)*

June 2007

3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

Callum

		C_1	C_2	C_3
	R_1	5	2	-1
Rose	R_2	-3	-1	5
	R_3	4	1	-2

(a) (i) State the play-safe strategy for Rose and give a reason for your answer. *(2 marks)*

(ii) Show that there is no stable solution for this game. *(2 marks)*

(b) Explain why Rose should never play strategy $\mathbf{R_3}$. *(1 mark)*

(c) Rose adopts a mixed strategy, choosing $\mathbf{R_1}$ with probability p and $\mathbf{R_2}$ with probability $1 - p$.

(i) Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions. *(3 marks)*

(ii) Illustrate graphically these expected gains for $0 \leqslant p \leqslant 1$. *(2 marks)*

(iii) Hence determine the optimal mixed strategy for Rose. *(3 marks)*

(iv) Find the value of the game. *(1 mark)*

January 2008

3 Two people, Rob and Con, play a zero-sum game.

The game is represented by the following pay-off matrix for Rob.

Con

	Strategy	C_1	C_2	C_3
	R_1	-2	5	3
Rob	R_2	3	-3	-1
	R_3	-3	3	2

(a) Explain what is meant by the term 'zero-sum game'. *(1 mark)*

(b) Show that this game has no stable solution. *(3 marks)*

(c) Explain why Rob should never play strategy R_3. *(1 mark)*

(d) (i) Find the optimal mixed strategy for Rob. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

June 2008

3 Two people, Roseanne and Collette, play a zero-sum game. The game is represented by the following pay-off matrix for Roseanne.

		Collette		
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Roseanne	$\mathbf{R_1}$	-3	2	3
	$\mathbf{R_2}$	2	-1	-4

(a) (i) Find the optimal mixed strategy for Roseanne. *(7 marks)*

(ii) Show that the value of the game is -0.5. *(1 mark)*

(b) (i) Collette plays strategy C_1 with probability p and strategy C_2 with probability q. Write down, in terms of p and q, the probability that she plays strategy C_3. *(1 mark)*

(ii) Hence, given that the value of the game is -0.5, find the optimal mixed strategy for Collette. *(4 marks)*

January 2009

4 (a) Two people, Raj and Cal, play a zero-sum game. The game is represented by the following pay-off matrix for Raj.

		Cal		
	Strategy	**X**	**Y**	**Z**
Raj	**I**	-7	8	-5
	II	6	2	-1
	III	-2	4	-3

Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) Ros and Carly play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros, where x is a constant.

		Carly	
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$
Ros	$\mathbf{R_1}$	5	x
	$\mathbf{R_2}$	-2	4

Ros chooses strategy R_1 with probability p.

(i) Find expressions for the expected gains for Ros when Carly chooses each of the strategies C_1 and C_2. *(2 marks)*

(ii) Given that the value of the game is $\frac{8}{3}$, find the value of p and the value of x. *(4 marks)*

June 2009

2 Two people, Rowena and Colin, play a zero-sum game.

The game is represented by the following pay-off matrix for Rowena.

Colin

	Strategy	C_1	C_2	C_3
	R_1	-4	5	4
Rowena	R_2	2	-3	-1
	R_3	-5	4	3

(a) Explain what is meant by the term 'zero-sum game'. *(1 mark)*

(b) Determine the play-safe strategy for Colin, giving a reason for your answer. *(2 marks)*

(c) Explain why Rowena should never play strategy R_3 . *(1 mark)*

(d) Find the optimal mixed strategy for Rowena. *(7 marks)*

January 2010

3 (a) Two people, Ann and Bill, play a zero-sum game. The game is represented by the following pay-off matrix for Ann.

Bill

	Strategy	B_1	B_2	B_3
	A_1	-1	0	-2
Ann	A_2	4	-2	-3
	A_3	-4	-5	-3

Show that this game has a stable solution and state the play-safe strategies for Ann and Bill. *(4 marks)*

(b) Russ and Carlos play a different zero-sum game, which does not have a stable solution. The game is represented by the following pay-off matrix for Russ.

Carlos

	Strategy	C_1	C_2	C_3
Russ	R_1	-4	7	-3
	R_2	2	-1	1

(i) Find the optimal mixed strategy for Russ. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

June 2010

4 Two people, Roger and Corrie, play a zero-sum game.

The game is represented by the following pay-off matrix for Roger.

Corrie

Roger

Strategy	**C_1**	**C_2**	**C_3**
R_1	7	3	-5
R_2	-2	-1	4

(a) (i) Find the optimal mixed strategy for Roger. *(7 marks)*

(ii) Show that the value of the game is $\frac{7}{13}$. *(1 mark)*

(b) Given that the value of the game is $\frac{7}{13}$, find the optimal mixed strategy for Corrie. *(5 marks)*

January 2011

3 Two people, Rhona and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rhona.

Colleen

Rhona

Strategy	**C_1**	**C_2**	**C_3**
R_1	2	6	4
R_2	3	-3	-1
R_3	x	$x + 3$	3

It is given that $x < 2$.

(a) (i) Write down the three row minima. *(1 mark)*

(ii) Show that there is no stable solution. *(3 marks)*

(b) Explain why Rhona should never play strategy R_3. *(1 mark)*

(c) (i) Find the optimal mixed strategy for Rhona. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

3 (a) Two people, Tom and Jerry, play a zero-sum game. The game is represented by the following pay-off matrix for Tom.

Jerry

	Strategy	**A**	**B**	**C**
	I	-4	5	-3
Tom	**II**	-3	-2	8
	III	-7	6	-2

Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) Rohan and Carla play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Rohan.

Carla

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Rohan	$\mathbf{R_1}$	3	5	-1
	$\mathbf{R_2}$	1	-2	4

(i) Find the optimal mixed strategy for Rohan and show that the value of the game is $\frac{3}{2}$. *(7 marks)*

(ii) Carla plays strategy C_1 with probability p, and strategy C_2 with probability q.

Find the values of p and q and hence find the optimal mixed strategy for Carla. *(4 marks)*

3 Two people, Roz and Colum, play a zero-sum game. The game is represented by the following pay-off matrix for Roz.

Colum

	Strategy	C_1	C_2	C_3
	R_1	-2	-6	-1
Roz	R_2	-5	2	-6
	R_3	-3	3	-4

(a) Explain what is meant by the term 'zero-sum game'. *(2 marks)*

(b) Determine the play-safe strategy for Colum, giving a reason for your answer. *(2 marks)*

(c) (i) Show that the matrix can be reduced to a 2 by 3 matrix, giving the reason for deleting one of the rows. *(2 marks)*

(ii) Hence find the optimal mixed strategy for Roz. *(7 marks)*

4 (a) Two people, Adam and Bill, play a zero-sum game. The game is represented by the following pay-off matrix for Adam.

Bill

Adam

Strategy	**B_1**	**B_2**	**B_3**
A_1	-6	-1	-5
A_2	5	2	-3
A_3	-5	4	-4
A_4	2	1	-4

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player. *(1 mark)*

(iii) State the value of the game for **Bill**. *(1 mark)*

4 (b) Roza plays a different zero-sum game against a computer. The game is represented by the following pay-off matrix for Roza.

Computer

Roza

Strategy	**C_1**	**C_2**	**C_3**
R_1	3	4	-3
R_2	-2	-1	5

(i) State which strategy the computer should never play, giving a reason for your answer. *(1 mark)*

(ii) Roza chooses strategy R_1 with probability p. Find expressions for the expected gains for Roza when the computer chooses each of its two remaining strategies. *(2 marks)*

(iii) Hence find the value of p for which Roza will maximise her expected gains. *(2 marks)*

January 2013

6 Kate and Pippa play a zero-sum game. The game is represented by the following pay-off matrix for Kate.

Pippa

Kate

Strategy	***D***	***E***	***F***
A	-2	0	3
B	3	-2	-2
C	4	1	-1

(a) Explain why Kate should not adopt strategy *B*. *(1 mark)*

(b) Find the optimal mixed strategy for Kate and find the value of the game. *(7 marks)*

(c) Find the optimal mixed strategy for Pippa. *(4 marks)*

June 2013

5 Romeo and Juliet play a zero-sum game. The game is represented by the following pay-off matrix for Romeo.

Juliet

Romeo

Strategy	**D**	**E**	**F**
A	4	-4	0
B	-2	-5	3
C	2	1	-2

(a) Find the play-safe strategy for each player. *(3 marks)*

(b) Show that there is no stable solution. *(1 mark)*

(c) Explain why Juliet should never play strategy D. *(1 mark)*

(d) (i) Explain why the following is a suitable pay-off matrix **for Juliet**.

4	5	-1
0	-3	2

(2 marks)

(ii) Hence find the optimal strategy for Juliet. *(7 marks)*

(iii) Find the value of the game for Juliet. *(1 mark)*