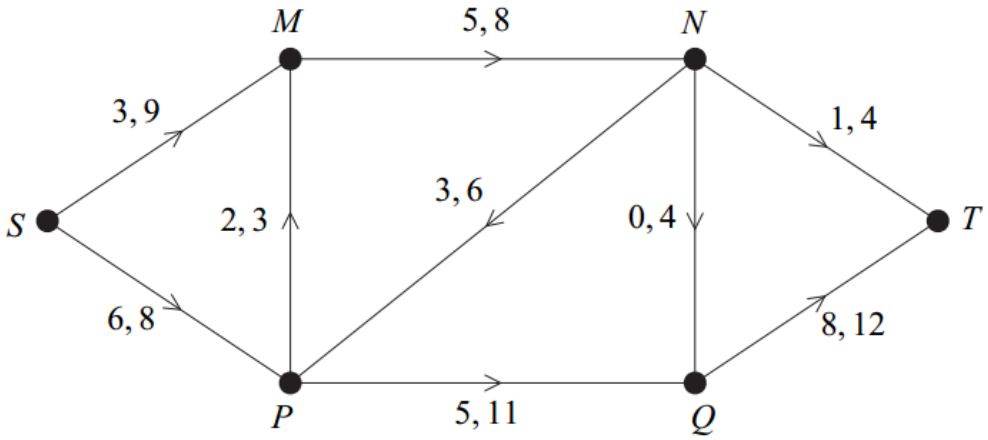
D2: Network Flows

Past Paper Questions
2006 - 2013

Name:

4 [**Figures 3**, **4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes, with the lower and upper capacities for each pipe in litres per second.



(a) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 10 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges MN, PQ, NP and NT. *(4 marks)*

(b) (i) Taking your answer from part (a) as an initial flow, use flow augmentation on **Figure 4** to find the maximum flow from S to T. *(6 marks)*

(ii) State the value of the maximum flow and illustrate this flow on **Figure 5**. *(2 marks)*

(c) Find a cut with capacity equal to that of the maximum flow. *(2 marks)*

Figure 3 (for use in part (a))

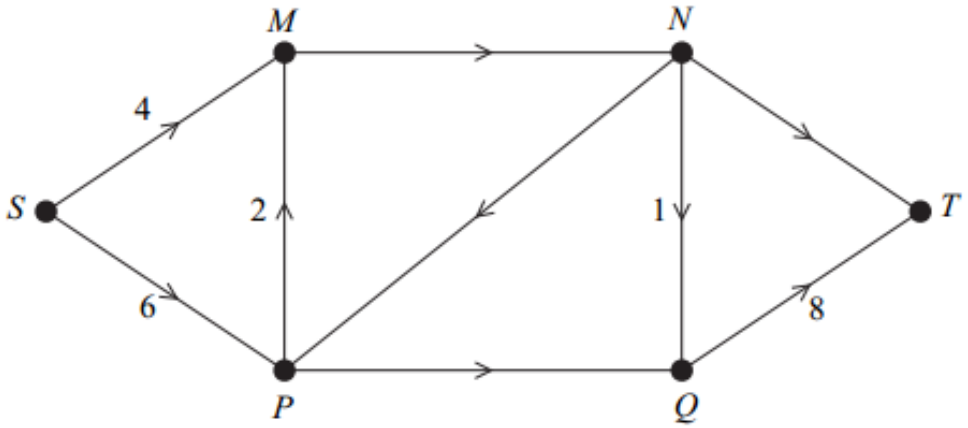
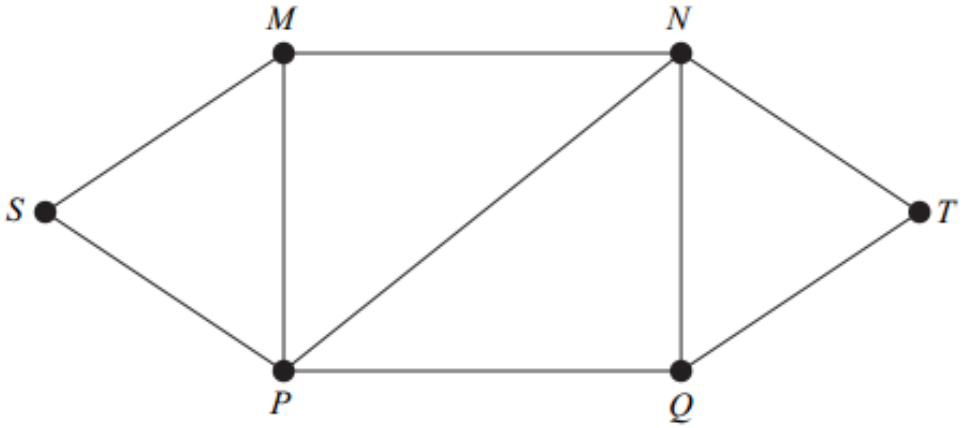
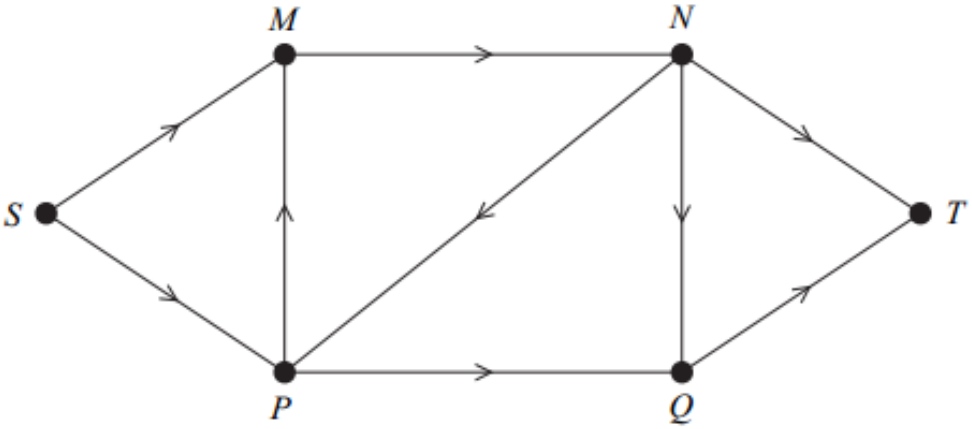


Figure 4 (for use in part (b)(i))



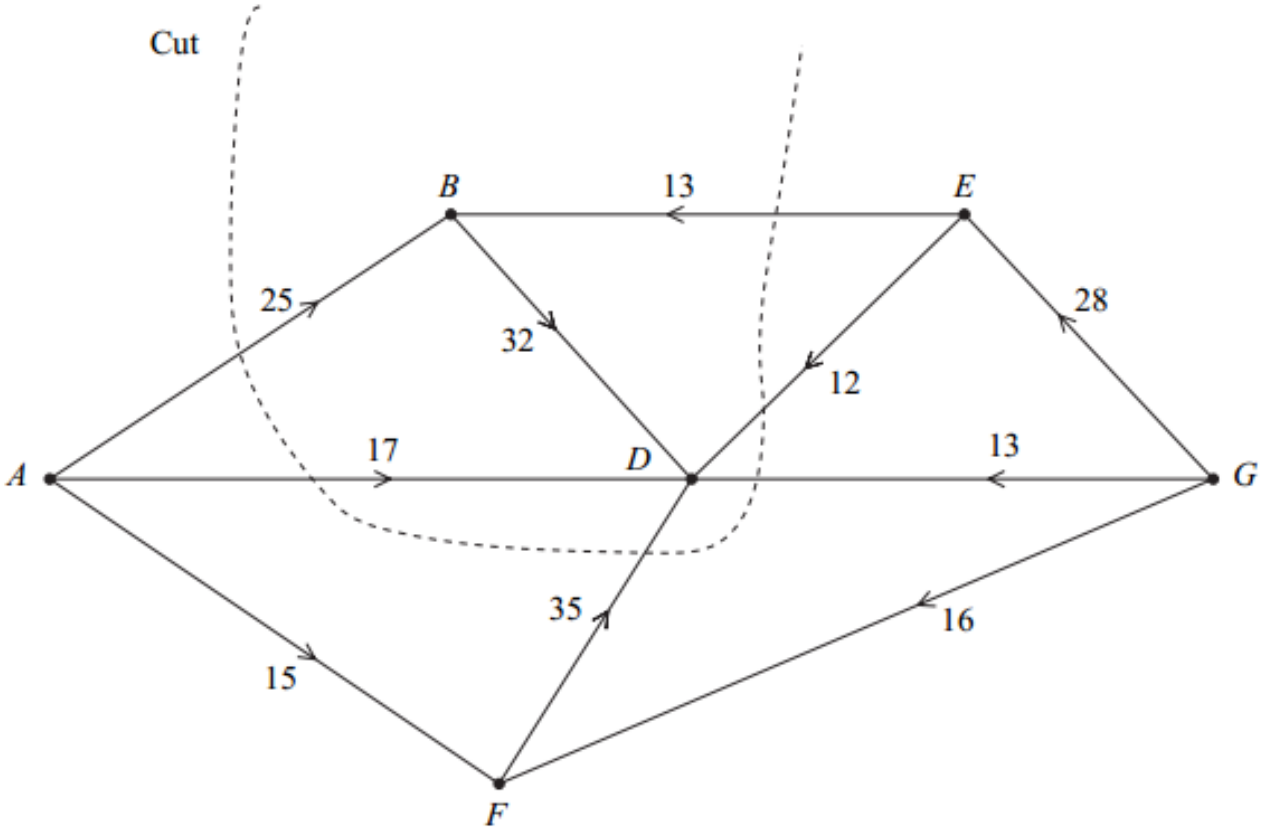
Route	Extra flow

Figure 5 (for use in part (b)(ii))



4 [**Figures 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows the routes along corridors from the playgrounds A and G to the assembly hall in a school. The number on each edge represents the maximum number of pupils that can travel along the corridor in one minute.



(a) State the vertex that represents the assembly hall. *(1 mark)*

(b) Find the value of the cut shown on the diagram. *(1 mark)*

(c) State the maximum flow along the routes ABD and GED. *(2 marks)*

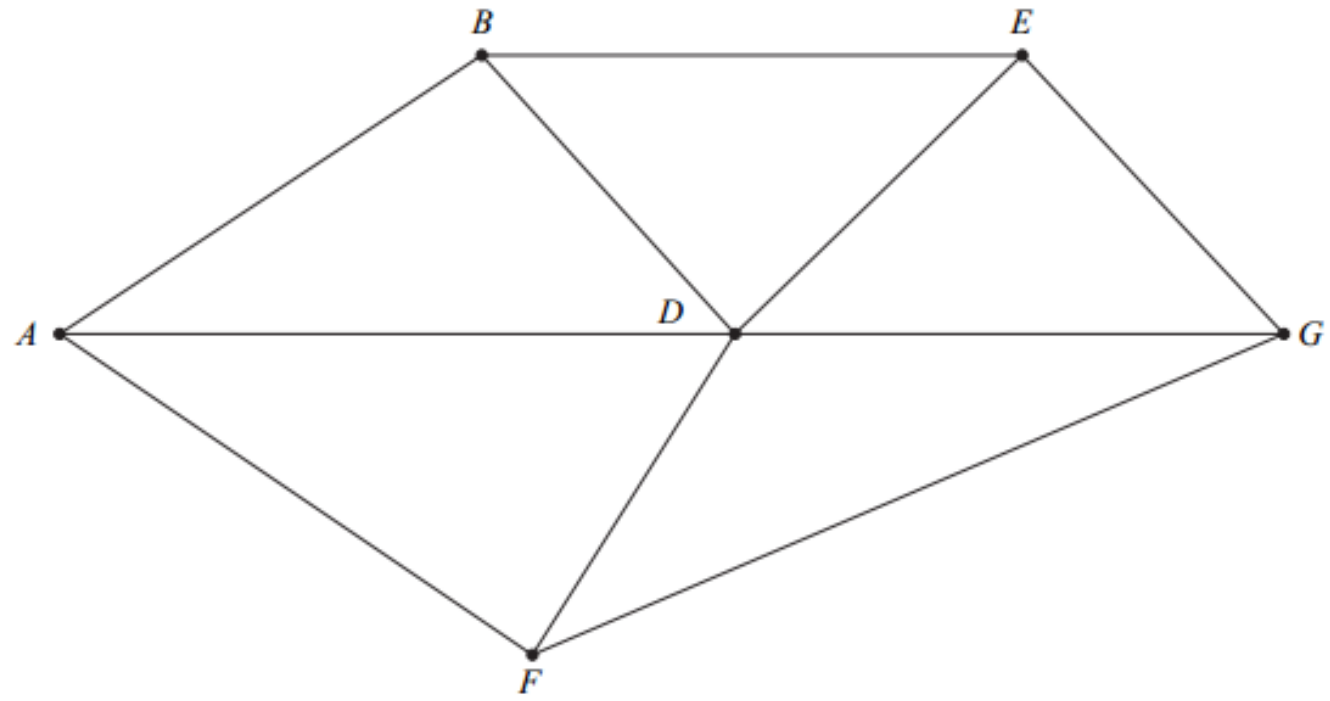
(d) (i) Taking your answers to part (c) as the initial flow, use a labelling procedure on **Figure 4** to find the maximum flow through the network. *(6 marks)*

(ii) State the value of the maximum flow and, on **Figure 5**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Verify that your flow is a maximum flow by finding a cut of the same value. *(2 marks)*

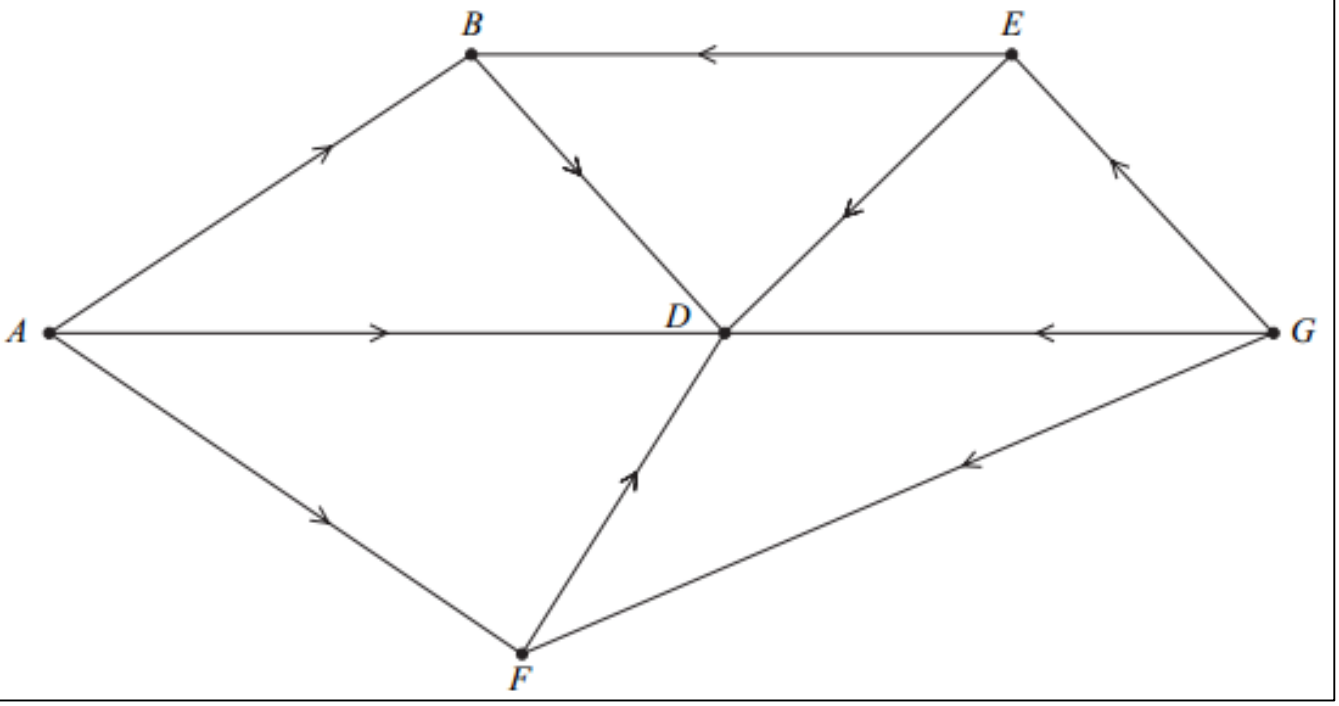
(e) On a particular day, there is an obstruction allowing no more than 15 pupils per minute to pass through vertex E. State the maximum number of pupils that can move through the network per minute on this particular day. *(2 marks)*

Figure 4 (for use in Question 4 part (d)(i))



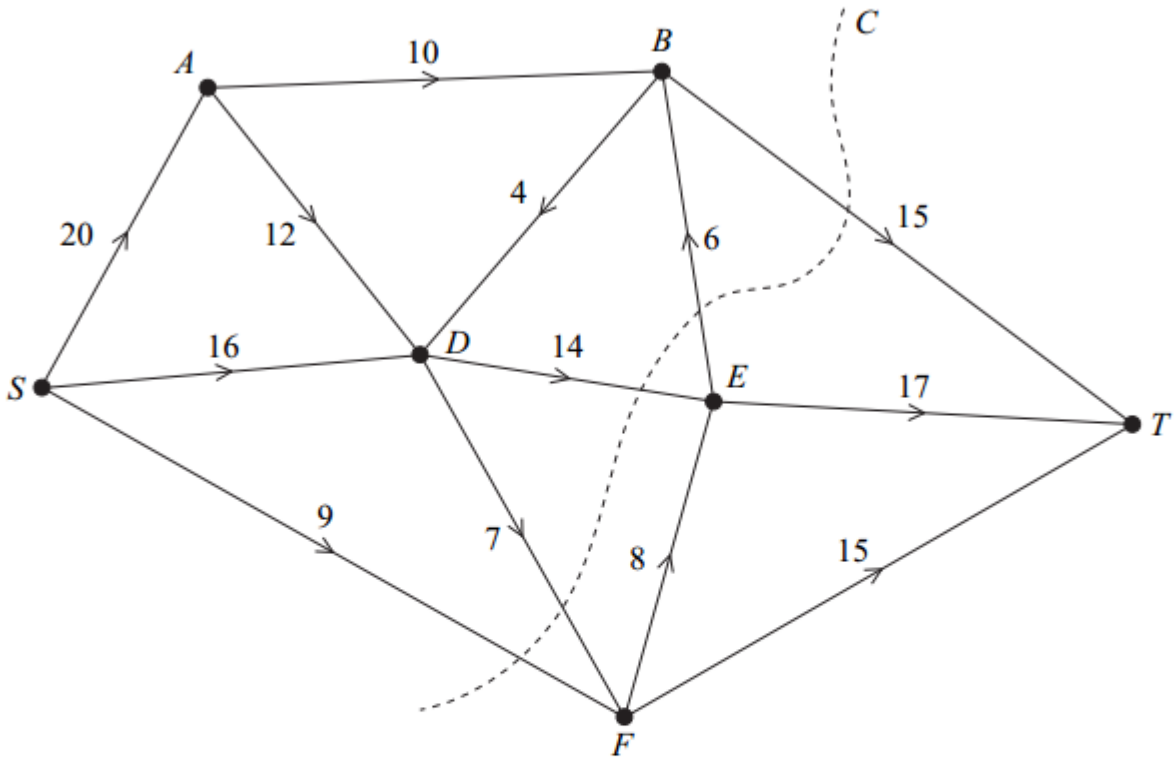
Route	Flow
ABD *GED*	

Figure 5 (for use in Question 4 part (d)(ii))



6 [**Figures 2** and **3**, printed on the insert, are provided for use in this question.]

The diagram shows a network of pipelines through which oil can travel. The oil field is at S, the refinery is at T and the other vertices are intermediate stations. The weights on the edges show the capacities in millions of barrels per hour that can flow through each pipeline.



(a) (i) Find the value of the cut marked C on the diagram. *(1 mark)*

(ii) Hence make a deduction about the maximum flow of oil through the network. *(2 marks)*

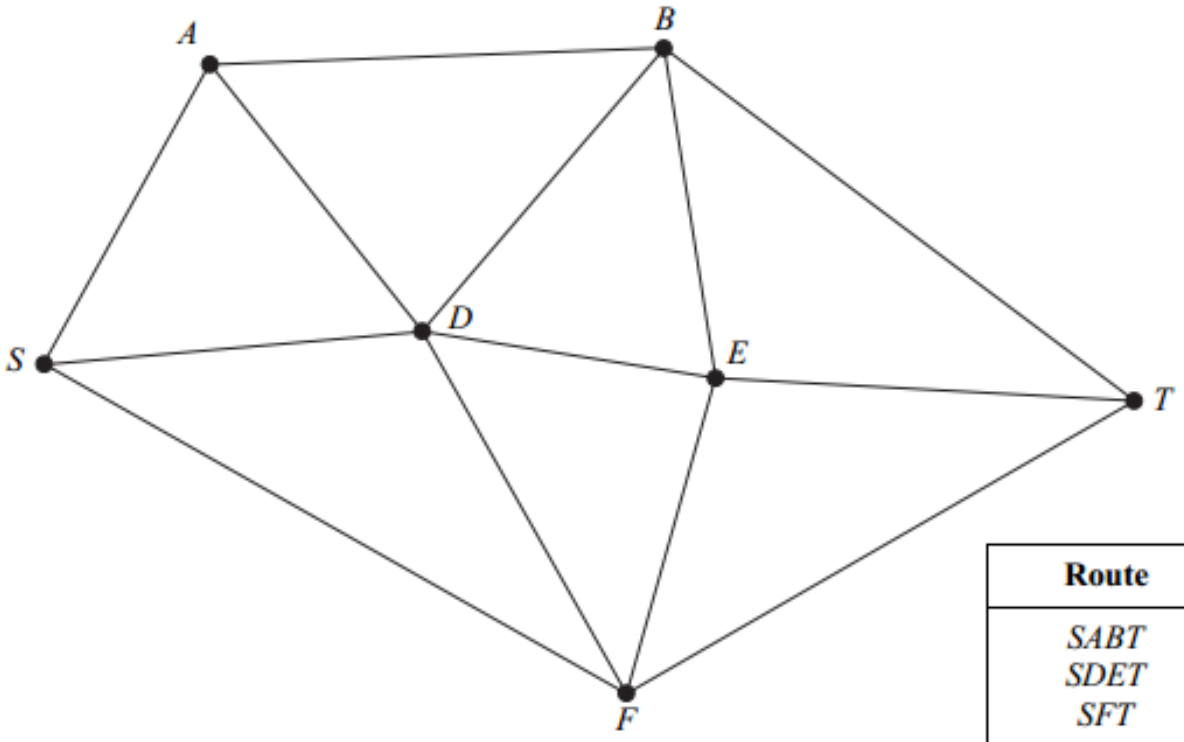
(b) State the maximum possible flows along the routes $SABT$, $SDET$ and SFT. *(2 marks)*

(c) (i) Taking your answer to part (b) as the initial flow, use a labelling procedure on **Figure 2** to find the maximum flow from S to T. Record your routes and flows in the table provided and show the augmented flows on the network diagram. *(6 marks)*

(ii) State the value of the maximum flow, and, on **Figure 3**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

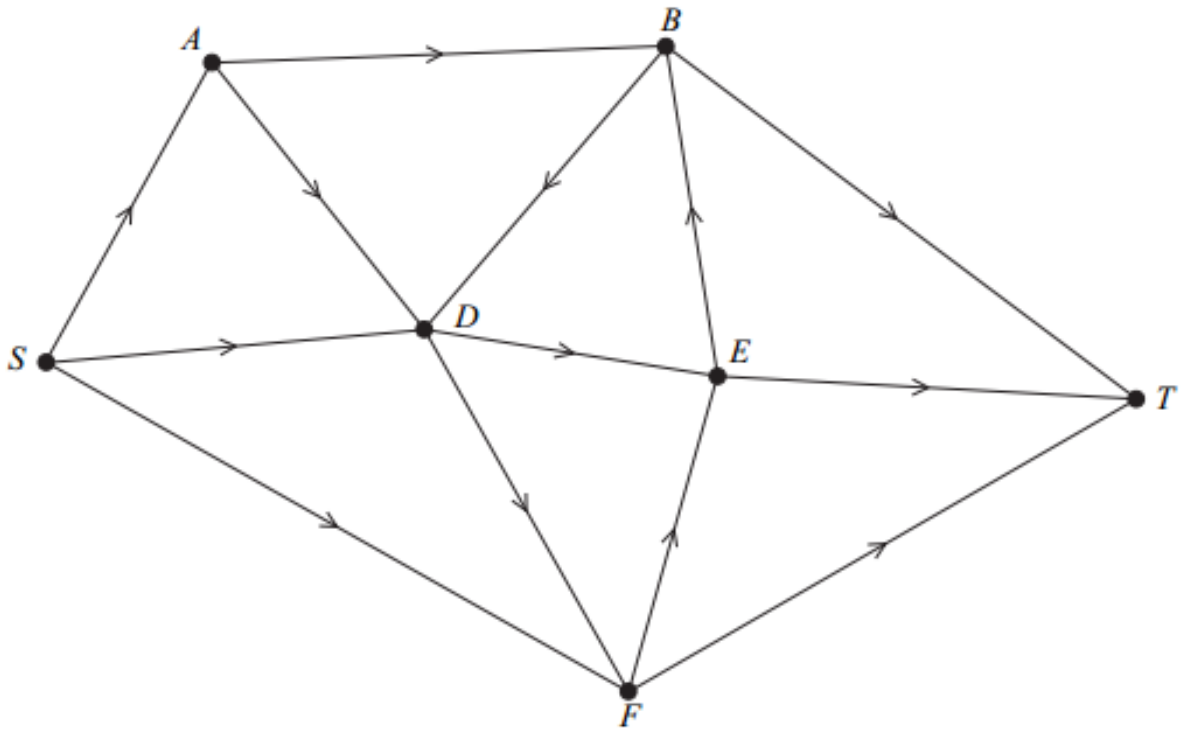
(iii) Prove that your flow in part (c)(ii) is a maximum. *(2 marks)*

Figure 2 (for use in Question 6(c)(i))



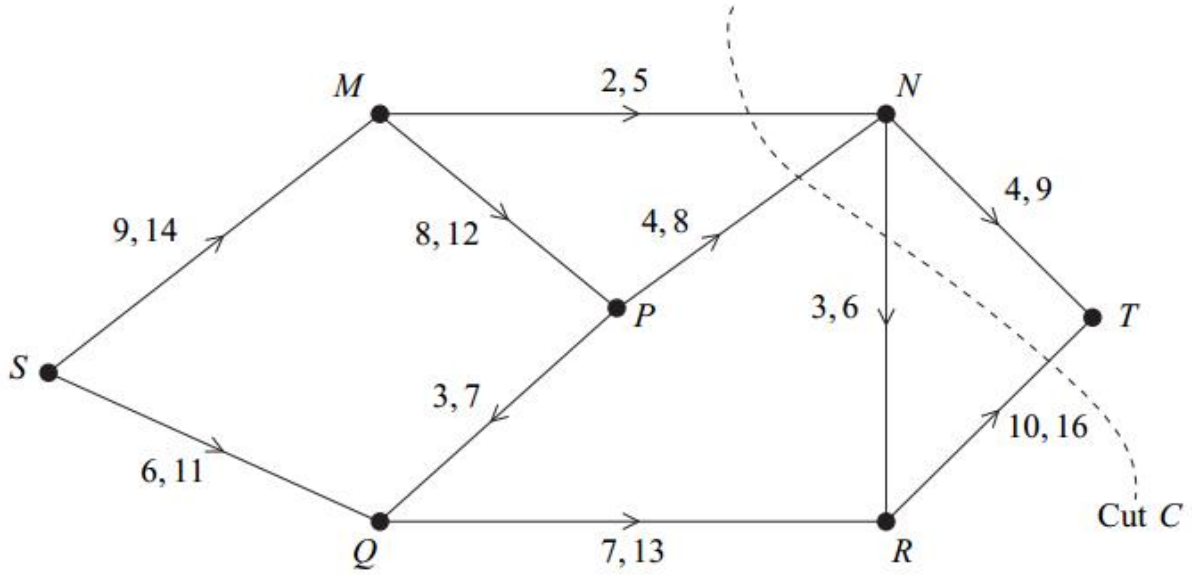
Route	Flow
SABT *SDET* *SFT*	

Figure 3 (for use in Question 6(c)(ii))



6 [**Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges MP, PN, QR and NR. *(4 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(2 marks)*

Figure 4 (for use in Question 6)

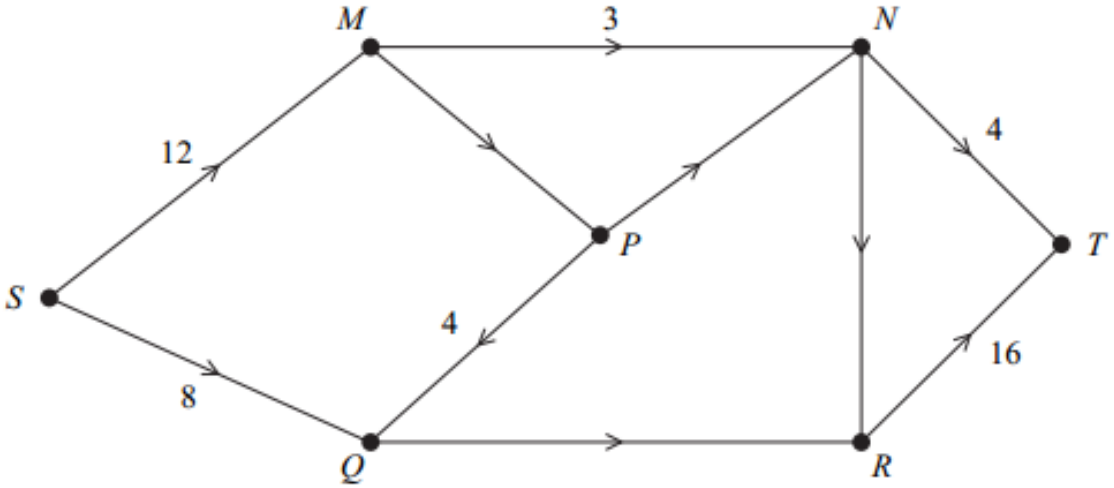
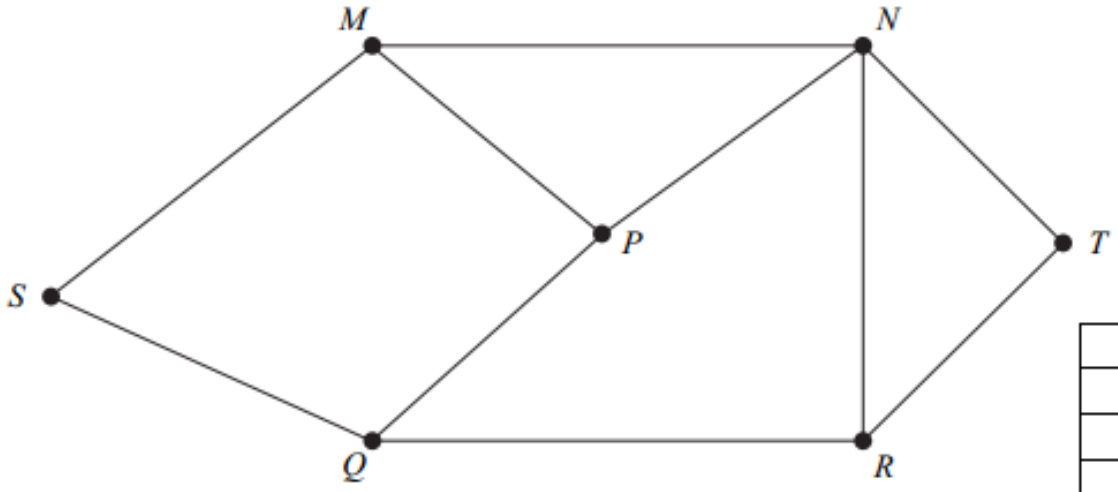
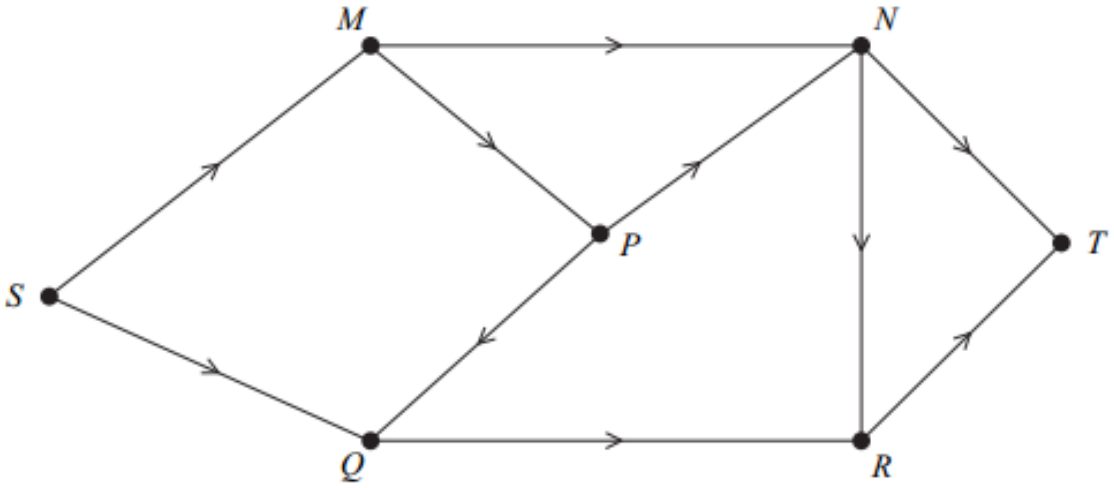


Figure 5 (for use in Question 6)



Path	Flow

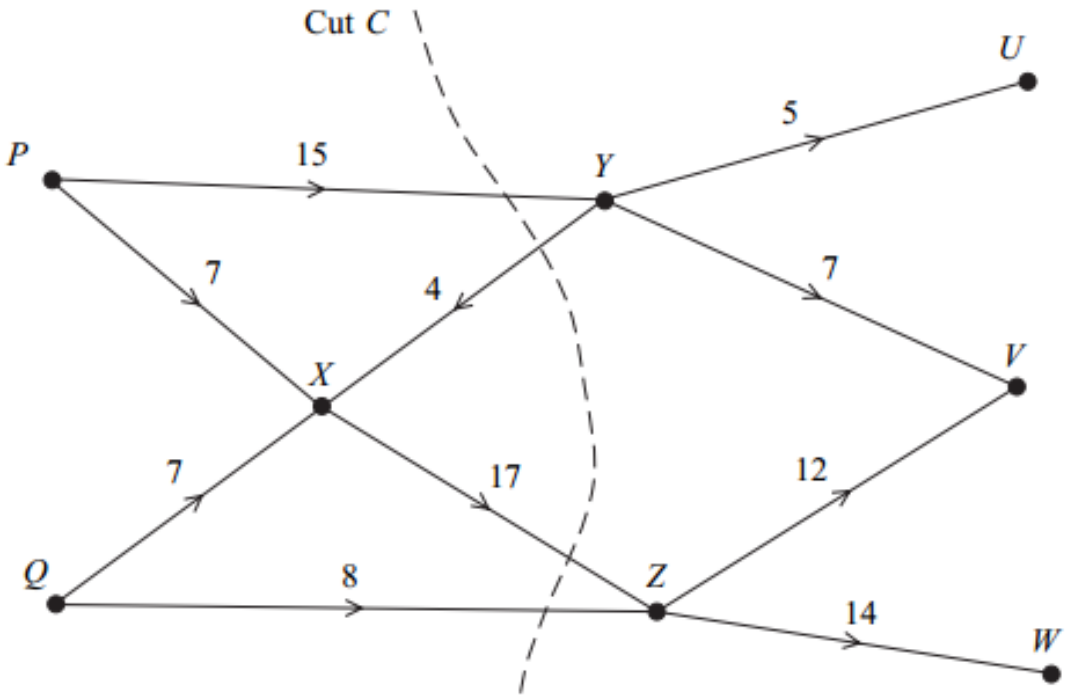
Figure 6 (for use in Question 6)



6 **[Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

Water has to be transferred from two mountain lakes P and Q to three urban reservoirs U, V and W. There are pumping stations at X, Y and Z.

The possible routes with the capacities along each edge, in millions of litres per hour, are shown in the following diagram.



(a) On **Figure 4**, add a super-source, S, and a super-sink, T, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each of the edges you have added. *(2 marks)*

(b) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(c) On **Figure 5**, write down the maximum flows along the routes $SQZWT$ and $SPYXZVT$. *(2 marks)*

(d) (i) On **Figure 6**, add the vertices S and T and the edges connecting S and T to the network. Using the maximum flows along the routes $SQZWT$ and $SPYXZVT$ found in part (c) as the initial flow, indicate the potential increases and decreases of flow on each edge. *(2 marks)*

(ii) Use flow augmentation to find the maximum flow from S to T. You should indicate any flow augmenting paths on **Figure 5** and modify the potential increases and decreases of the flow on **Figure 6**. *(4 marks)*

(e) State the value of the flow from Y to X in millions of litres per hour when the maximum flow is achieved. *(1 mark)*

Figure 4 (for use in Question 6)

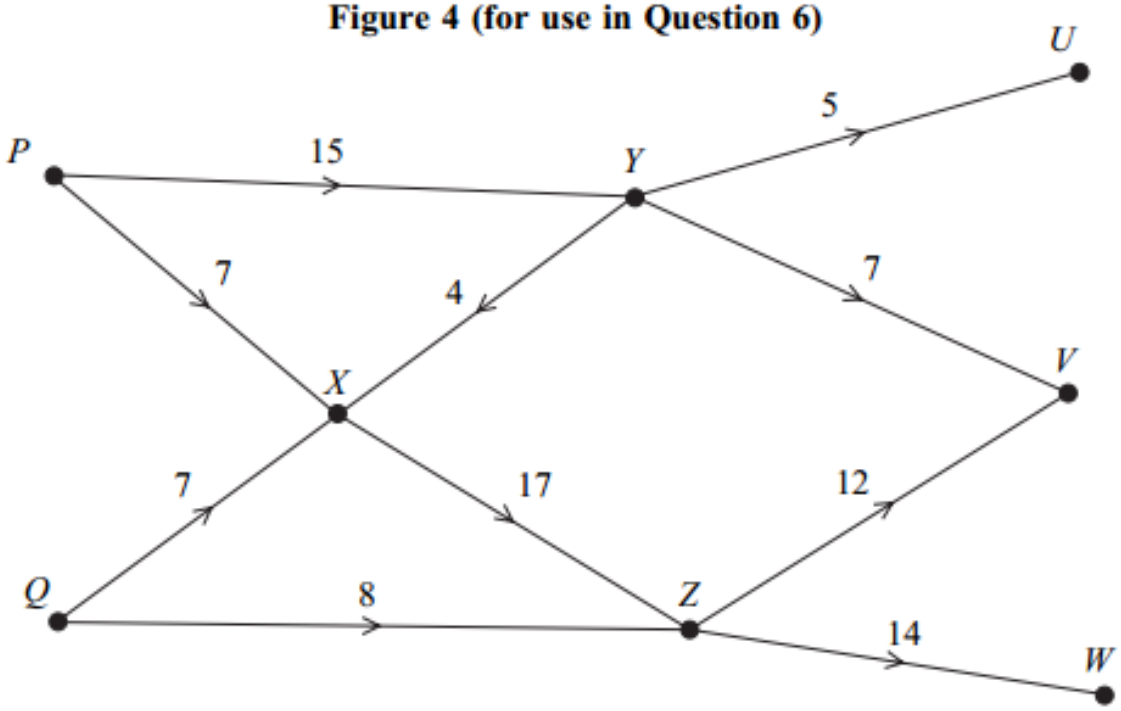
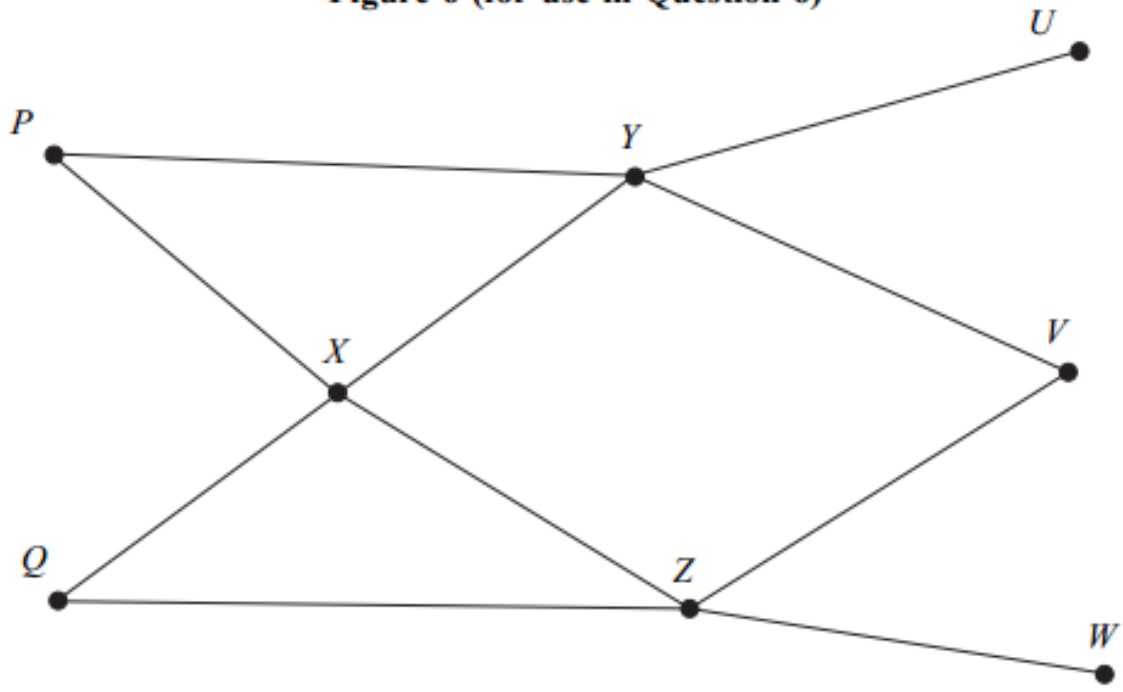


Figure 5 (for use in Question 6)

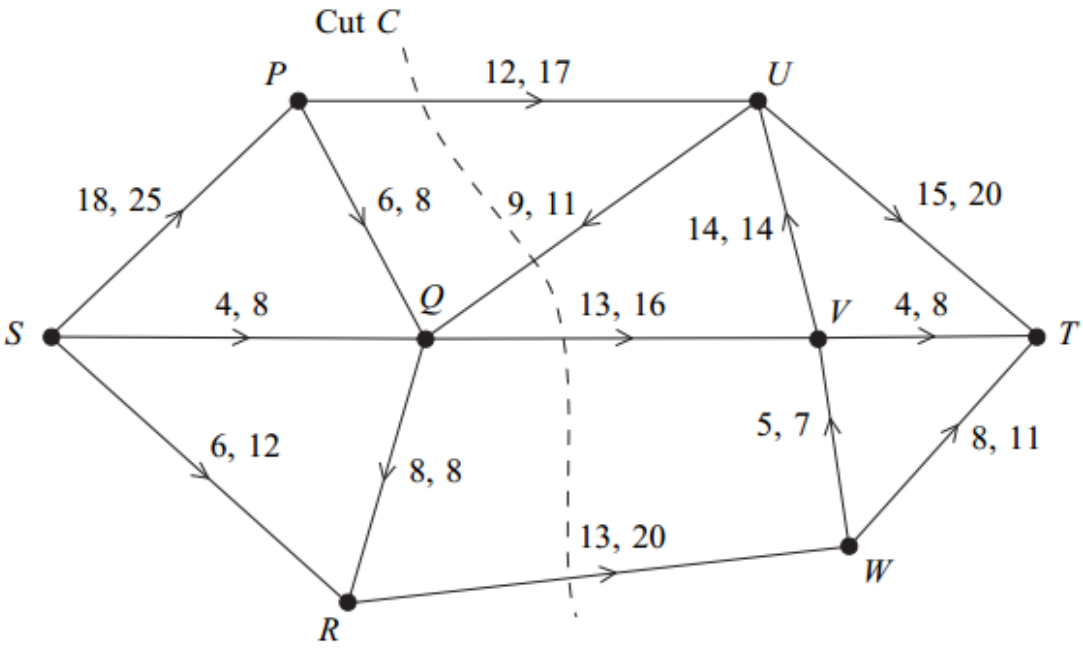
Route	Flow
SQZWT	
SPYXZVT	

Figure 6 (for use in Question 6)



6 [**Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) Hence state what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 32 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges PQ, UQ and UT. *(3 marks)*

(c) (i) Taking your feasible flow from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(1 mark)*

Figure 4 (for use in Question 6)

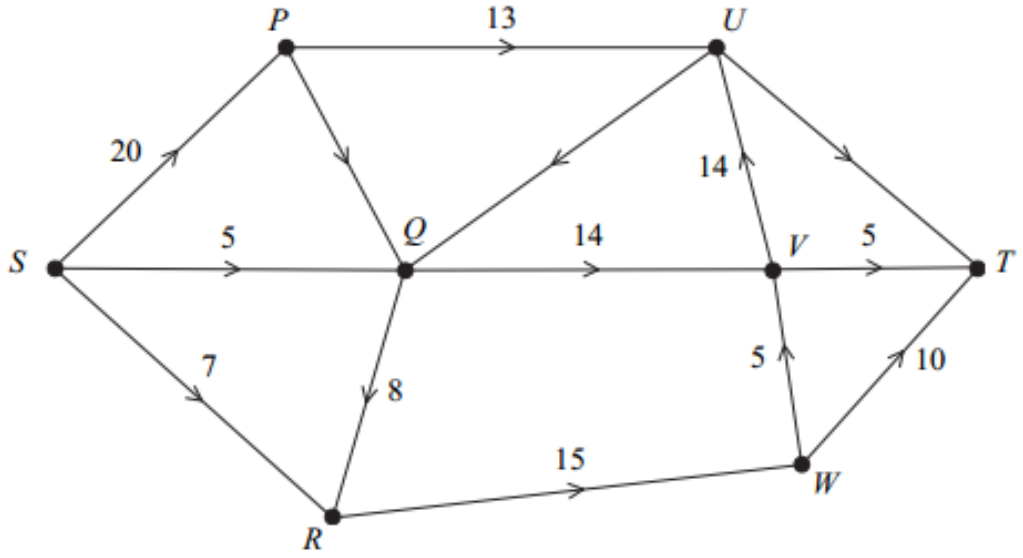
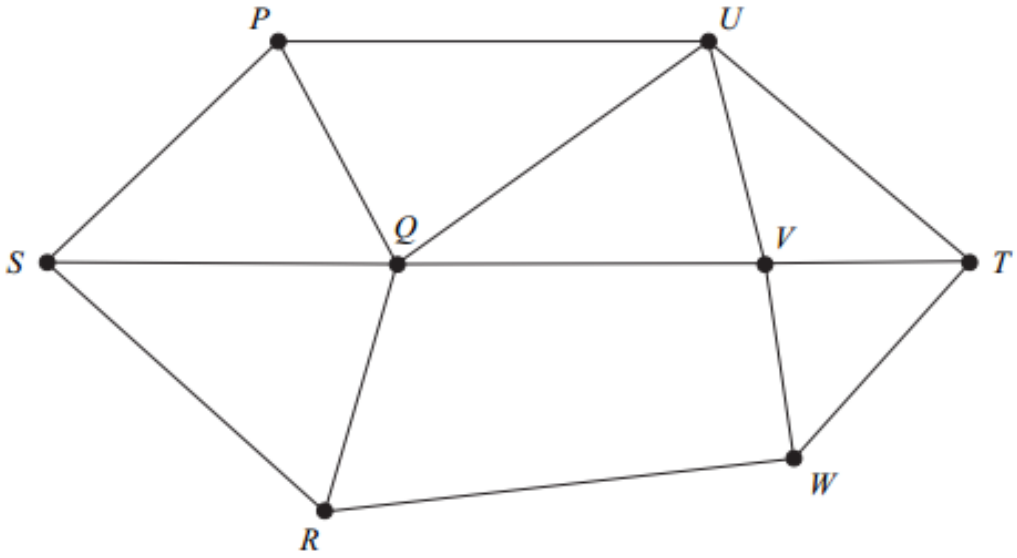
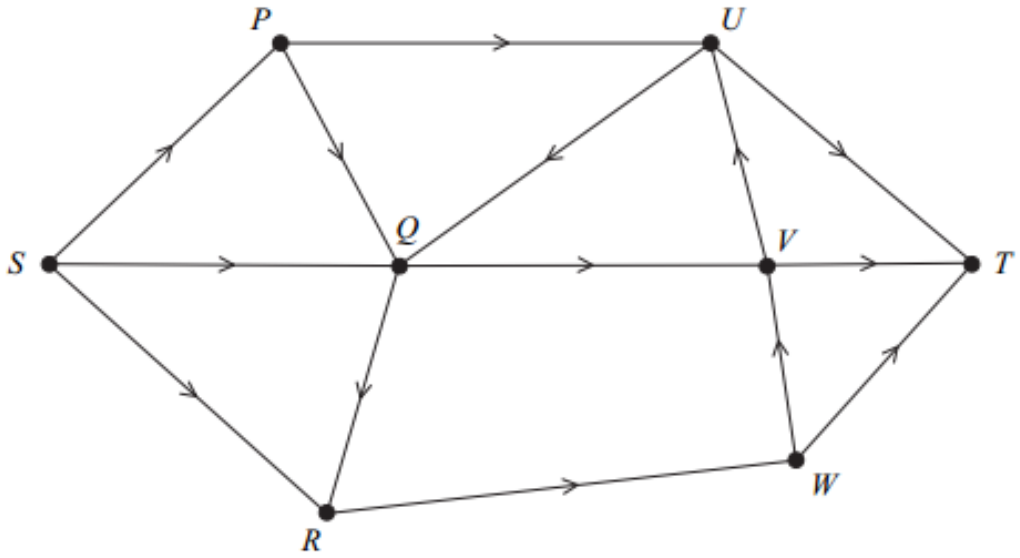


Figure 5 (for use in Question 6)



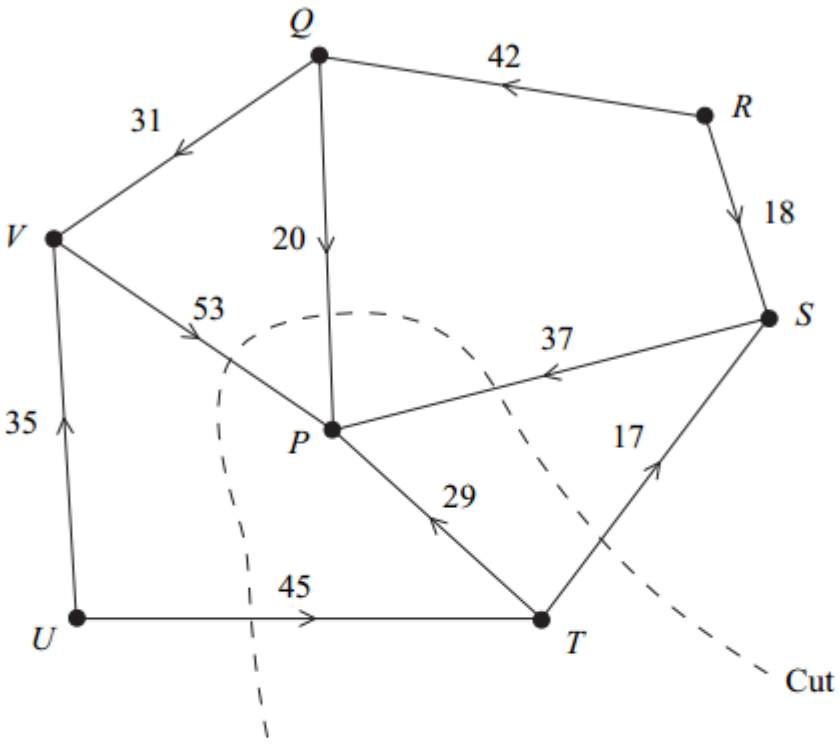
Path	Additional Flow

Figure 6 (for use in Question 6)



6 [**Figures 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows the routes along corridors from two arrival gates to the passport control area, P, in a small airport. The number on each edge represents the maximum number of passengers that can travel along a particular corridor in one minute.



(a) State the vertices that represent the arrival gates. *(1 mark)*

(b) Find the value of the cut shown on the network. *(1 mark)*

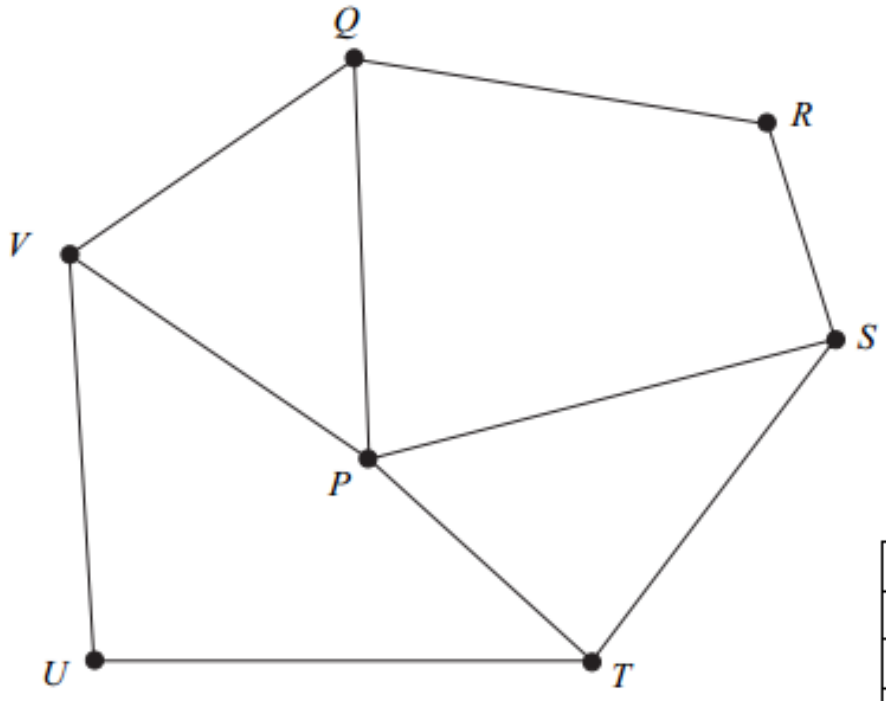
(c) State the maximum flow along each of the routes $UTSP$ and $RQVP$. *(2 marks)*

(d) (i) Taking your answers to part (c) as the initial flow, use the labelling procedure on **Figure 4** to find the maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(6 marks)*

(ii) State the value of the maximum flow, and, on **Figure 5**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(e) On a particular day, there is an obstruction allowing no more than 50 passengers per minute to pass through vertex V. State the maximum number of passengers that can move through the network per minute on this particular day. *(2 marks)*

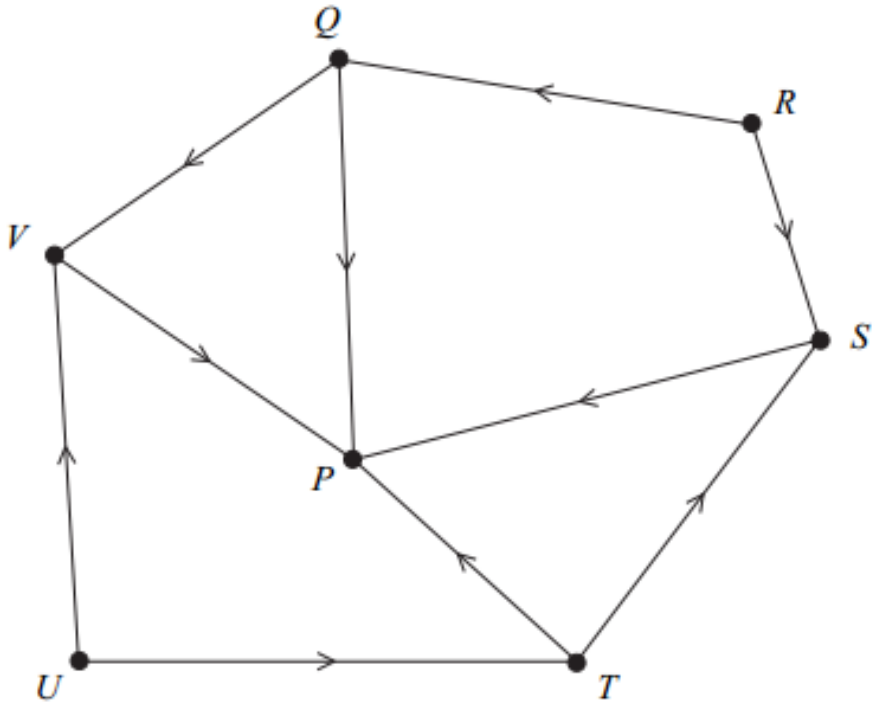
Figure 4 (for use in Question 6)



Route	Value of Flow
UTSP	
RQVP	

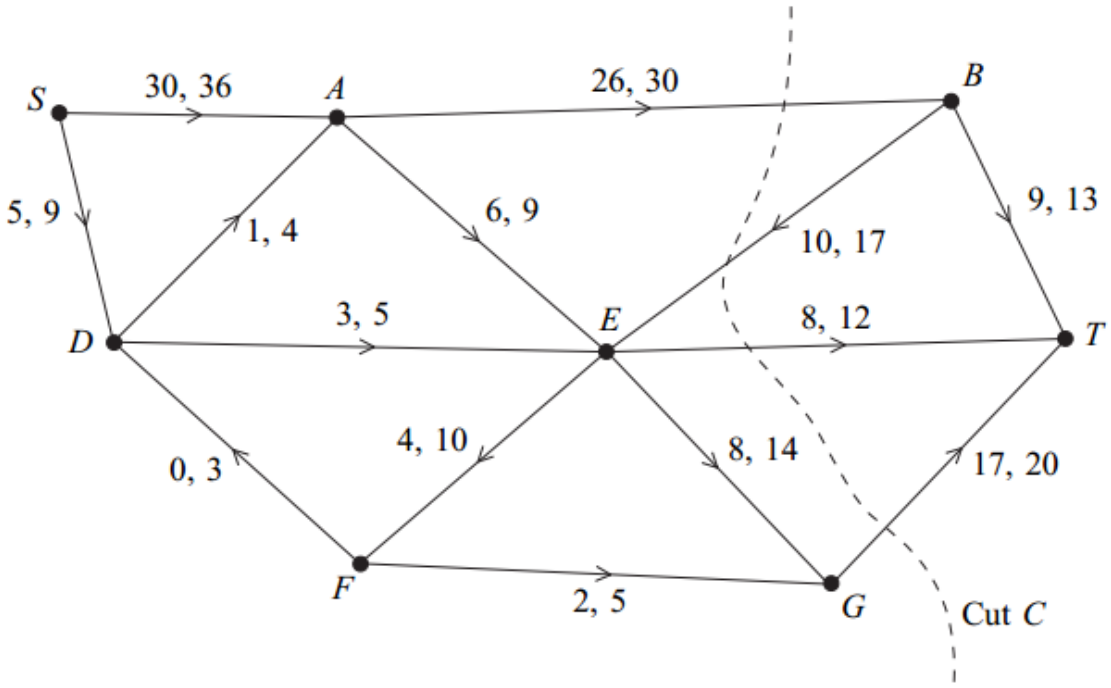
Figure 5 (for use in Question 6)

Maximum flow is _____ passengers per minute.



6 [**Figures 3, 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut C. *(2 marks)*

(b) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 40 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges AE, EF and FG. *(3 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4**. *(3 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

(d) Illustrate the maximum flow on **Figure 5**. *(2 marks)*

(e) Find a cut with value equal to that of the maximum flow. *(2 marks)*

Figure 3 (for use in Question 6)

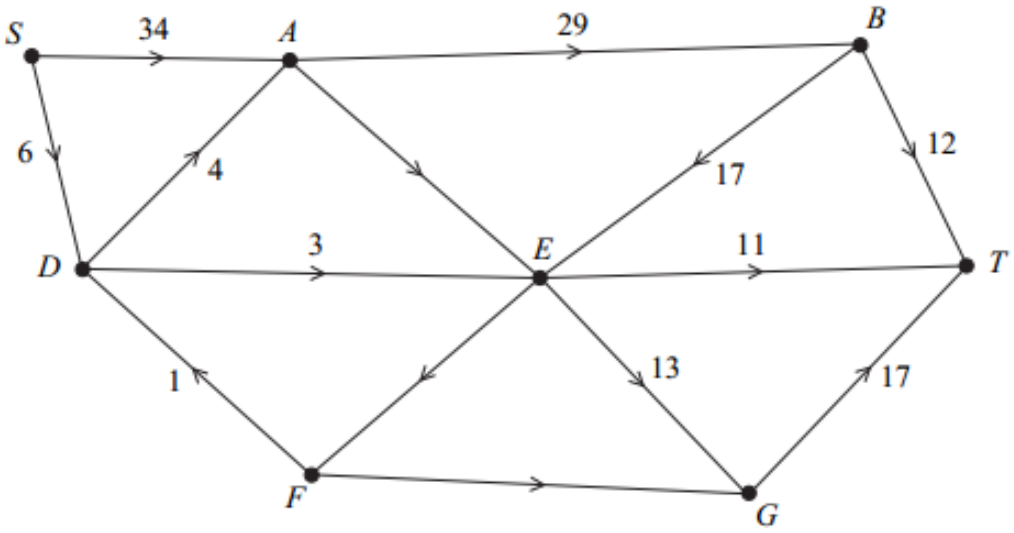
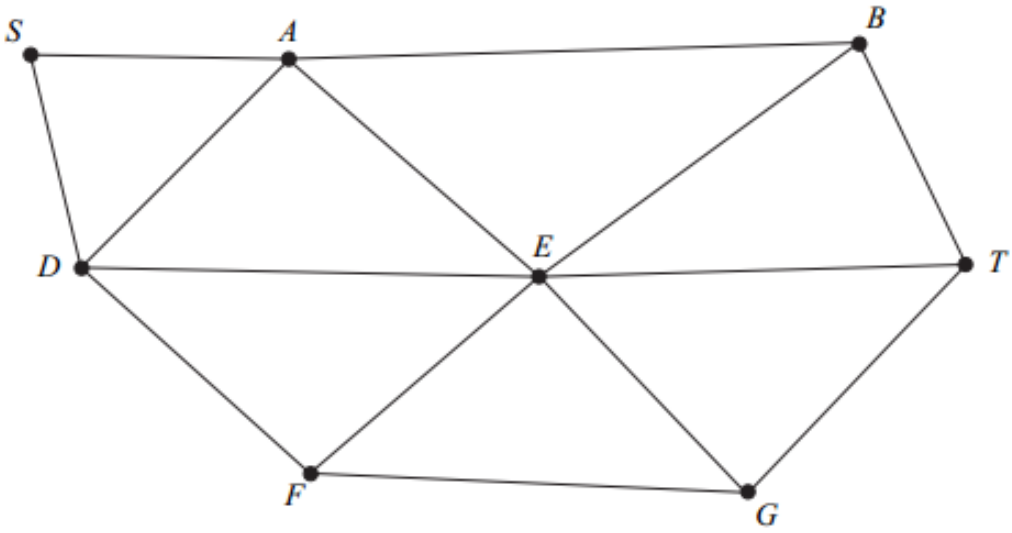
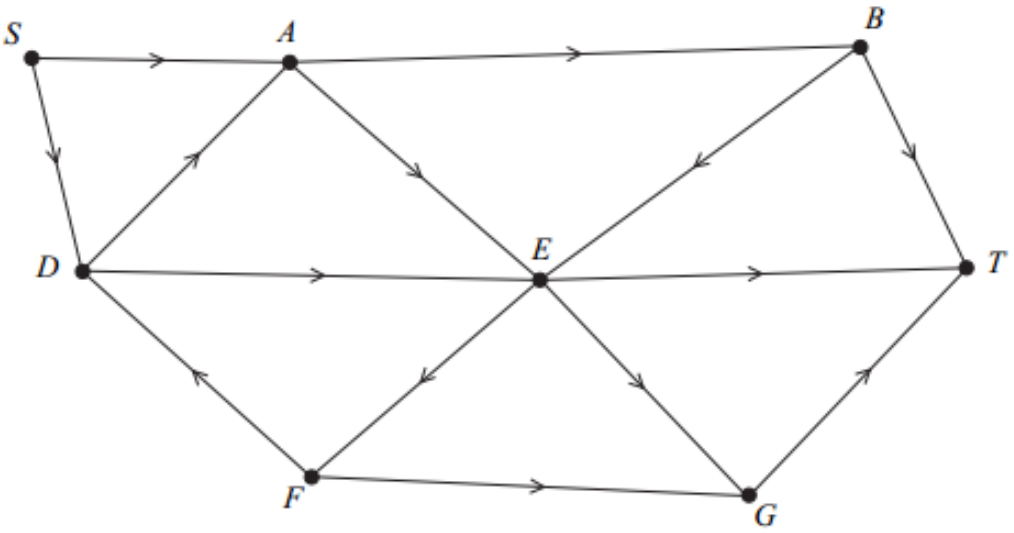


Figure 4 (for use in Question 6)



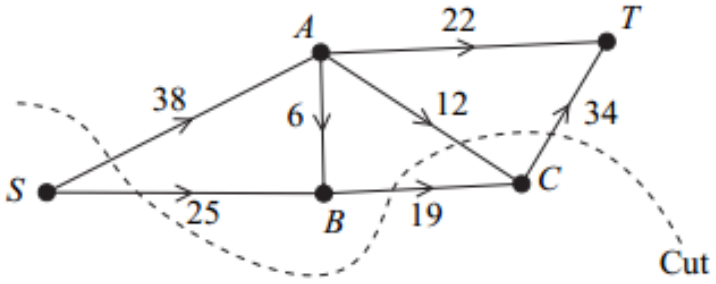
Path	Extra Flow

Figure 5 (for use in Question 6)



6 [**Figures 4, 5, 6 and 7**, printed on the insert, are provided for use in this question.]

(a) The network shows a flow from S to T along a system of pipes, with the capacity, in litres per minute, indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 97. *(1 mark)*

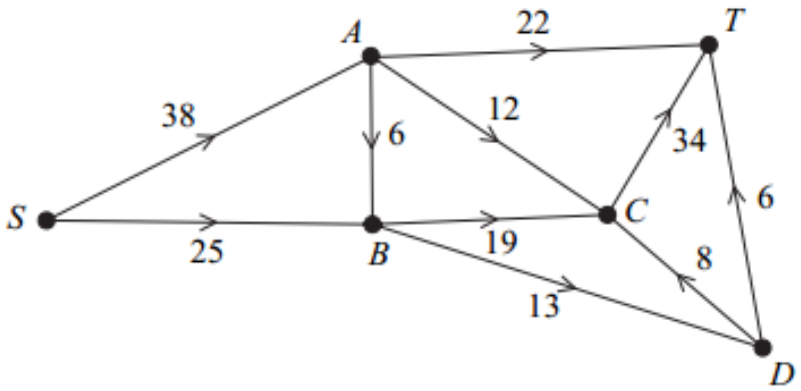
(ii) The cut shown on the diagram can be represented as $\{S, C\}$, $\{A, B, T\}$.

Complete the table on **Figure 4**, giving the value of each of the 8 possible cuts. *(4 marks)*

(iii) State the value of the maximum flow through the network, giving a reason for your answer. *(2 marks)*

(iv) Indicate on **Figure 5** a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(b) Extra pipes, BD, CD and DT, are added to form a new system, with the capacity, in litres per minute, indicated on each edge of the network below.



(i) Taking your values from **Figure 5** as the initial flow, use the labelling procedure on **Figure 6** to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

(ii) State the value of the new maximum flow, and, on **Figure 7**, indicate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

Figure 4

Cut		Value
$\{S\}$	$\{A, B, C, T\}$	63
$\{S, A\}$	$\{B, C, T\}$	
$\{S, B\}$	$\{A, C, T\}$	
$\{S, C\}$	$\{A, B, T\}$	97
$\{S, A, B\}$	$\{C, T\}$	53
$\{S, A, C\}$	$\{B, T\}$	87
$\{S, B, C\}$	$\{A, T\}$	
$\{S, A, B, C\}$	$\{T\}$	

Figure 5

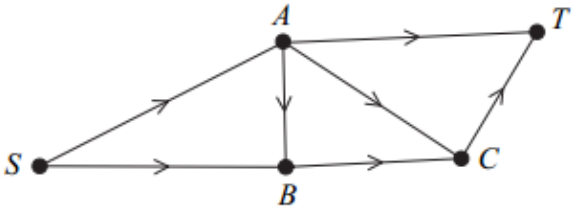
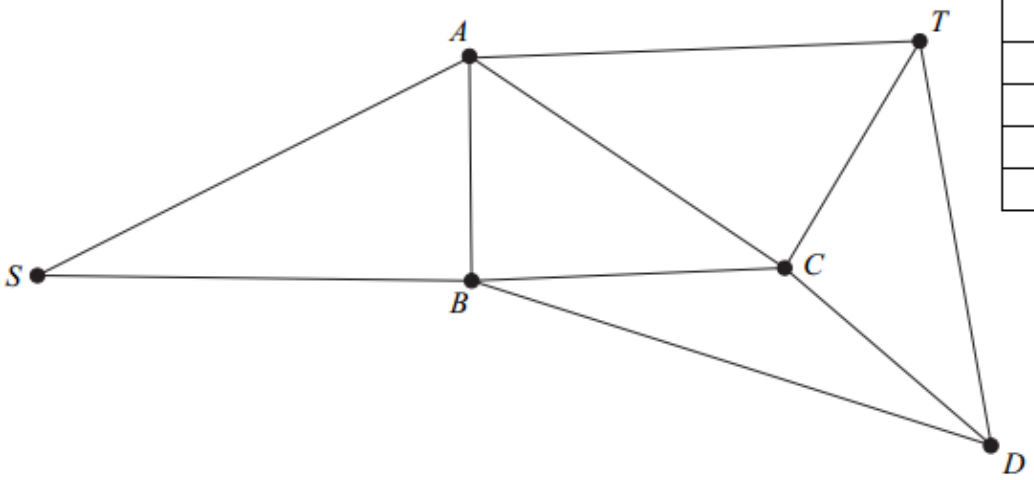
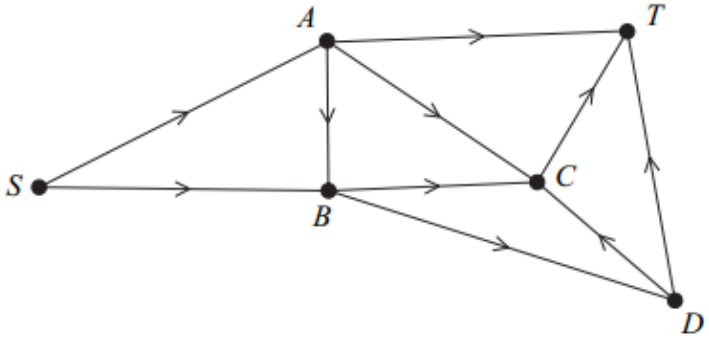


Figure 6



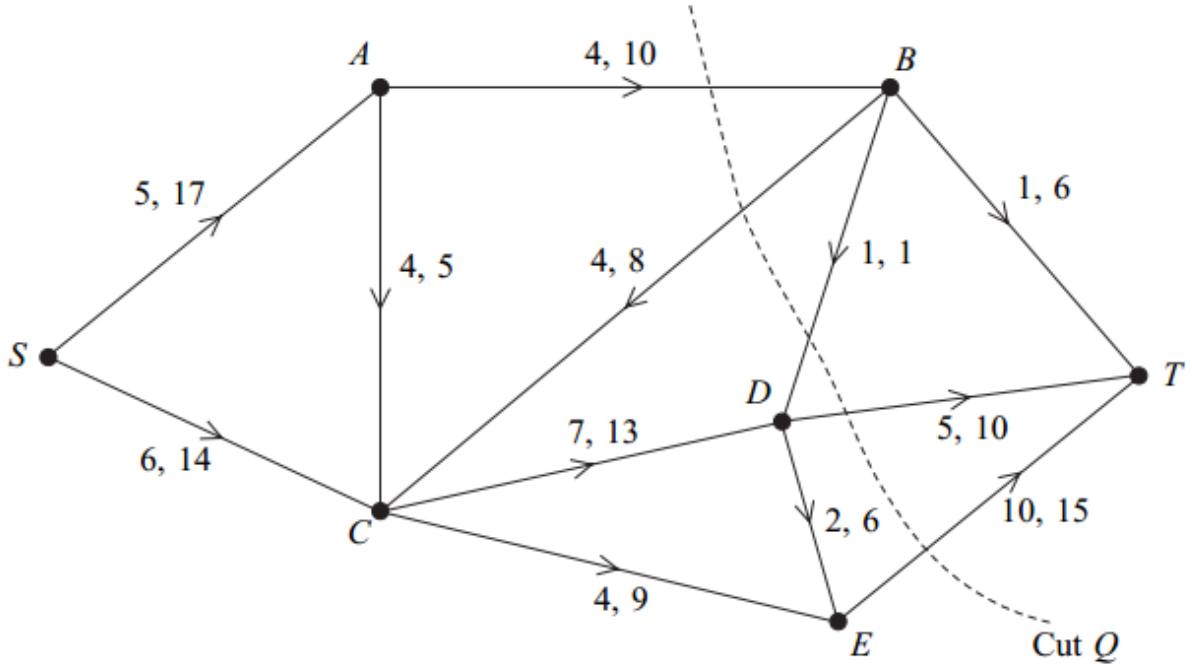
Path	Additional flow

Figure 7



New maximum flow is _____

6 The network shows a system of pipes with the lower and upper capacities for each pipe in litres per minute.



(a) Find the value of the cut Q. *(2 marks)*

(b) **Figure 3** opposite shows a partially completed diagram for a feasible flow of 24 litres per minute from S to T. Indicate, on **Figure 3**, the flows along the edges BT, DE and ET. *(2 marks)*

(c) (i) Taking your answer from part **(b)** as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4** opposite. *(2 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T.

You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 5** opposite. *(2 marks)*

(d) Find a cut with value equal to that of the maximum flow.

You may wish to show the cut on the network above. *(1 mark)*

Figure 3

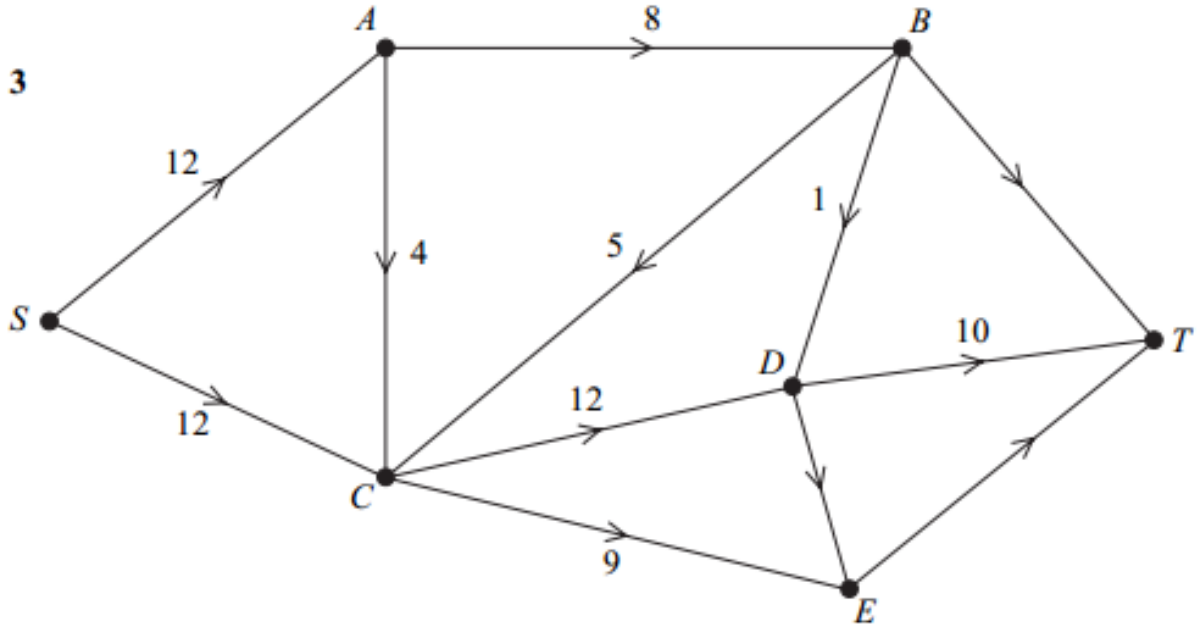
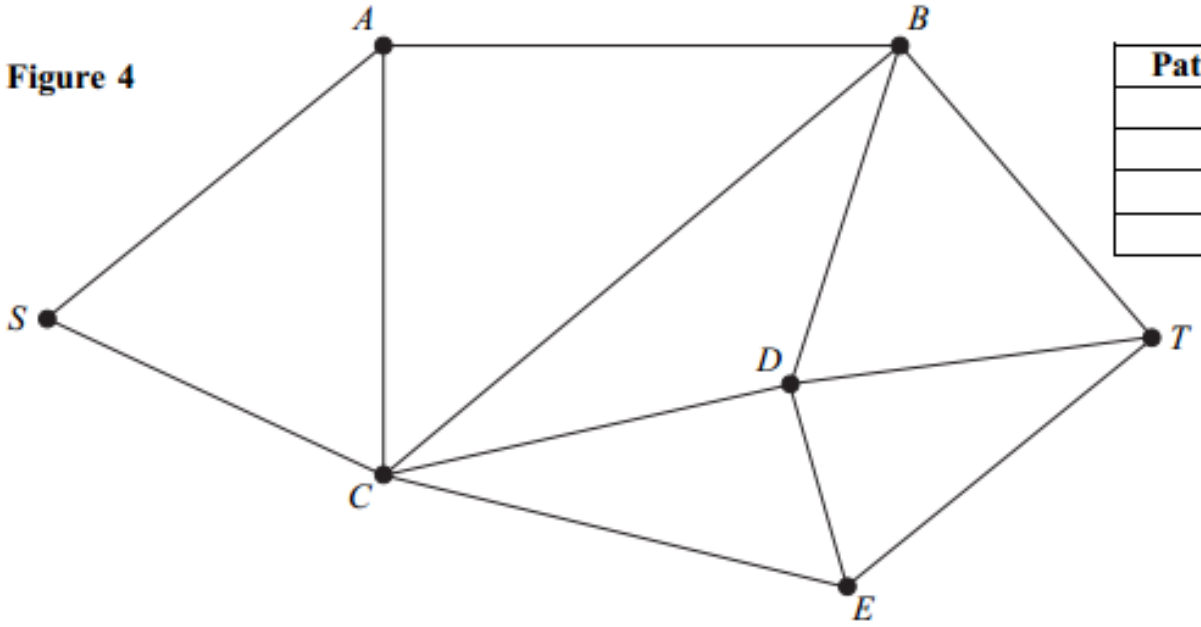
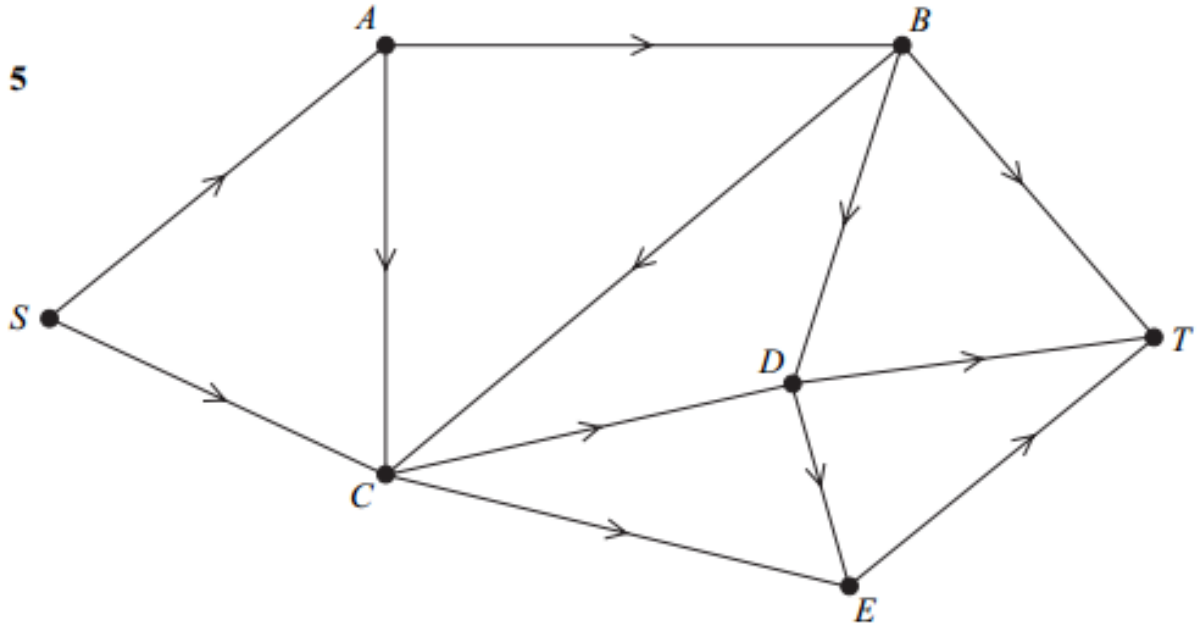


Figure 4

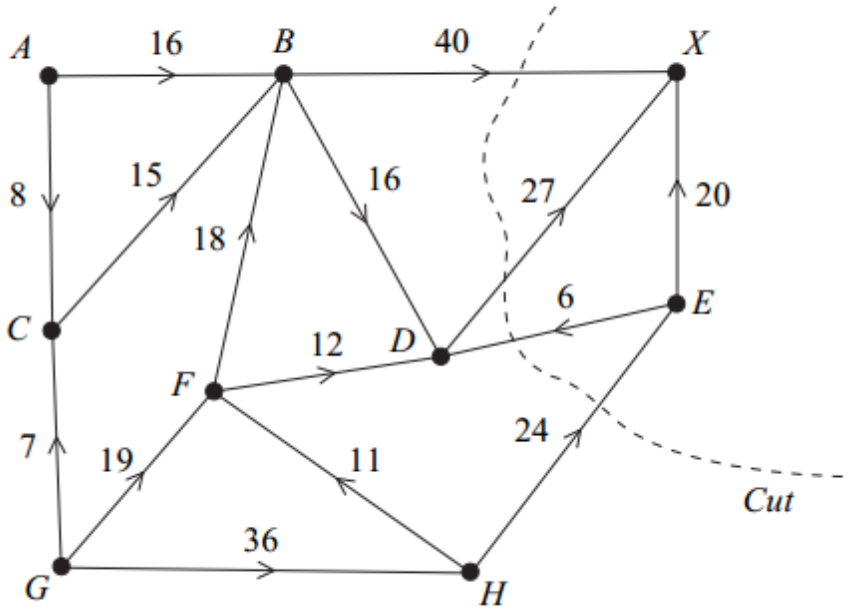


Path	Flow

Figure 5



5 The network shows the evacuation routes along corridors in a college, from two teaching areas to the exit, in case of a fire alarm sounding.



The two teaching areas are at A and G and the exit is at X.

The number on each edge represents the maximum number of people that can travel along a particular corridor in one minute.

(a) Find the value of the cut shown on the diagram. *(1 mark)*

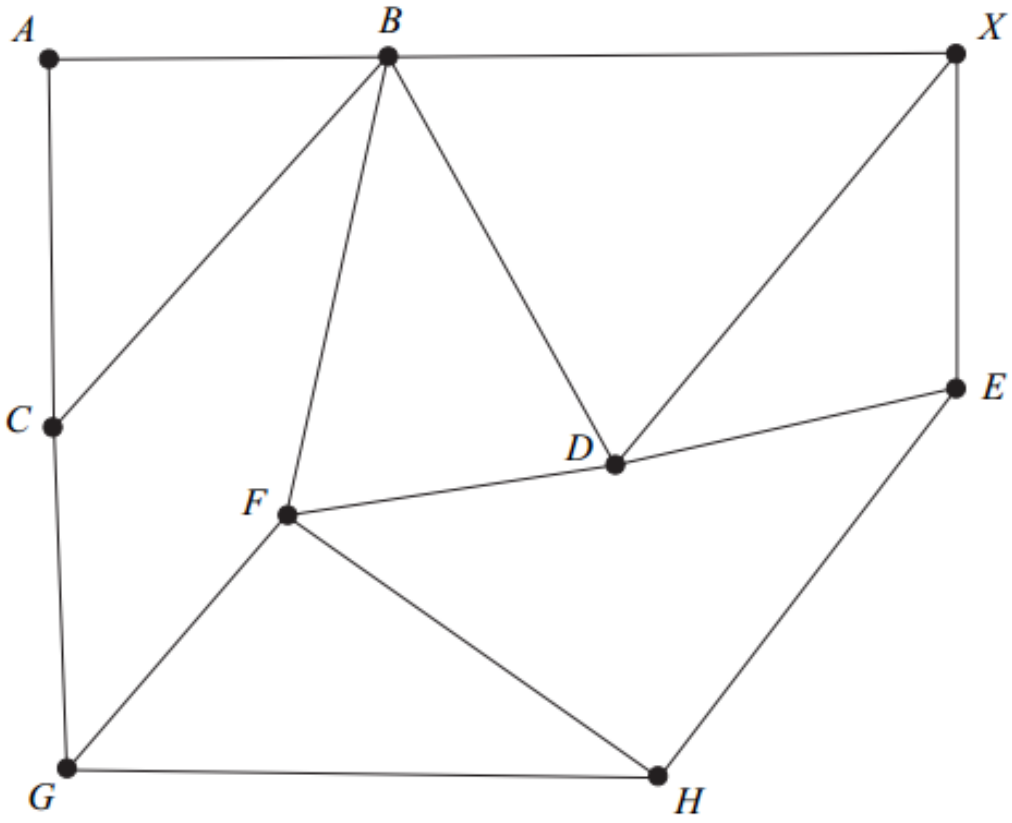
(b) Find the maximum flow along each of the routes $ABDX$, $GFBX$ and $GHEX$ and enter their values in the table on **Figure 3** opposite. *(3 marks)*

(c) (i) Taking your answers to part **(b)** as the initial flow, use the labelling procedure on **Figure 3** to find the maximum flow through the network. You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow, and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

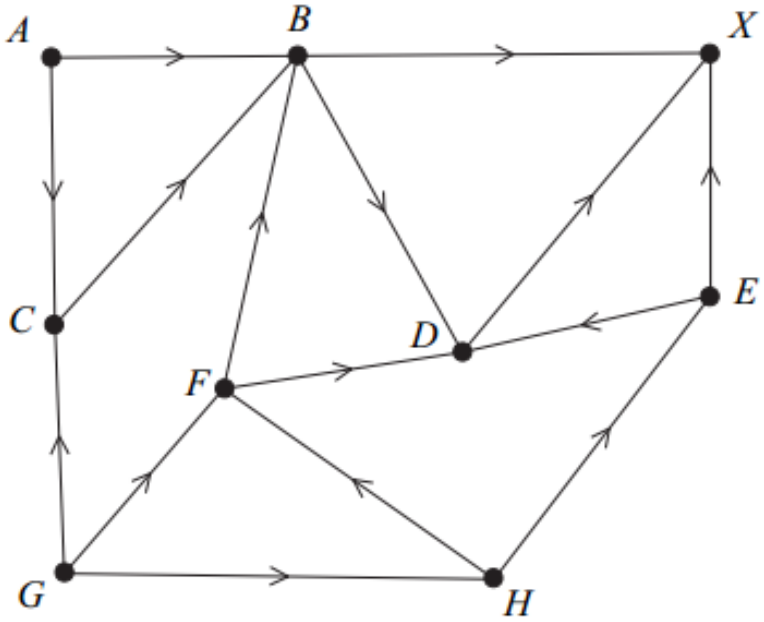
(d) During one particular fire drill, there is an obstruction allowing no more than 45 people per minute to pass through vertex B. State the maximum number of people that can move through the network per minute during this fire drill. *(2 marks)*

Figure 3



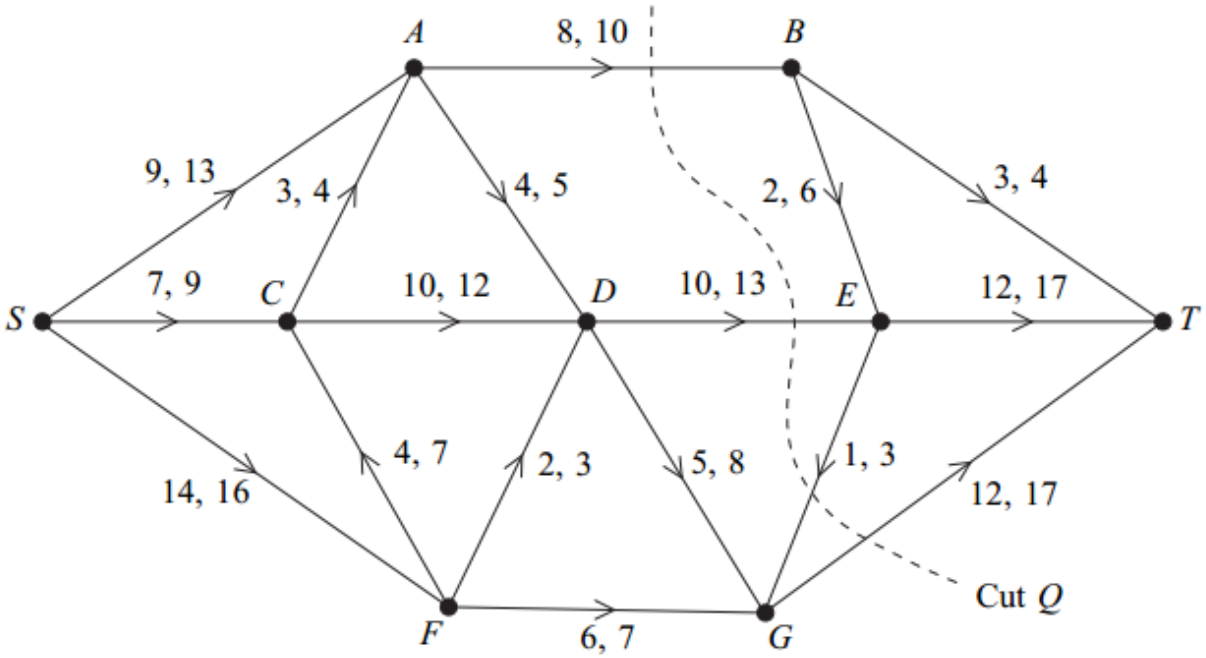
Route	Flow
ABDX	
GFBX	
GHEX	

Figure 4



Maximum flow is ..... people per minute.

6 The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut Q. *(2 marks)*

(b) **Figure 2** shows most of the values of a feasible flow of 34 litres per second from S to T.

(i) Insert the missing values of the flows along DE and FG on **Figure 2**. *(2 marks)*

(ii) Using this feasible flow as the initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 3**. *(2 marks)*

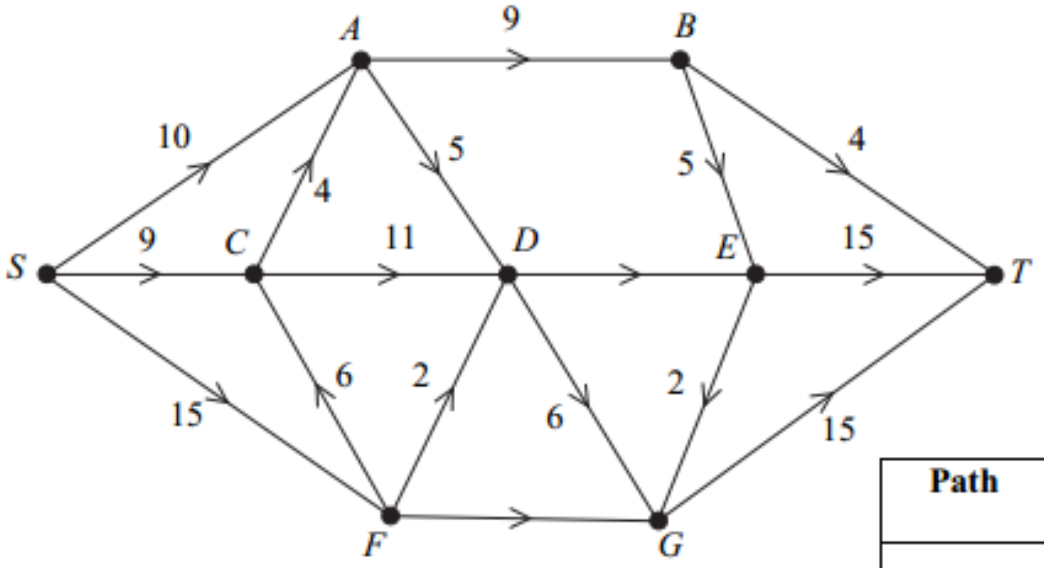
(iii) Use flow augmentation on **Figure 3** to find the maximum flow from S to T. You should indicate any flow-augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

(c) (i) State the value of the maximum flow. *(1 mark)*

(ii) Illustrate your maximum flow on **Figure 4**. *(2 marks)*

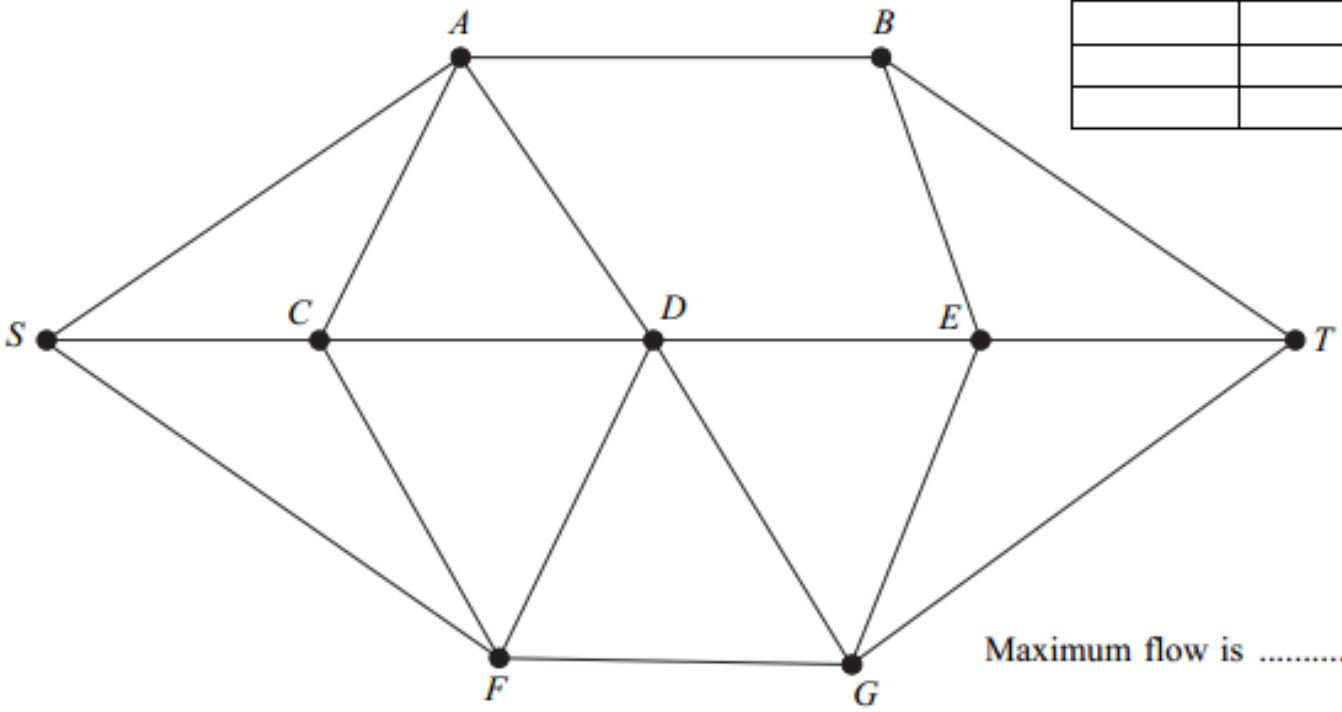
(d) Find a cut with capacity equal to that of the maximum flow. *(1 mark)*

Figure 2



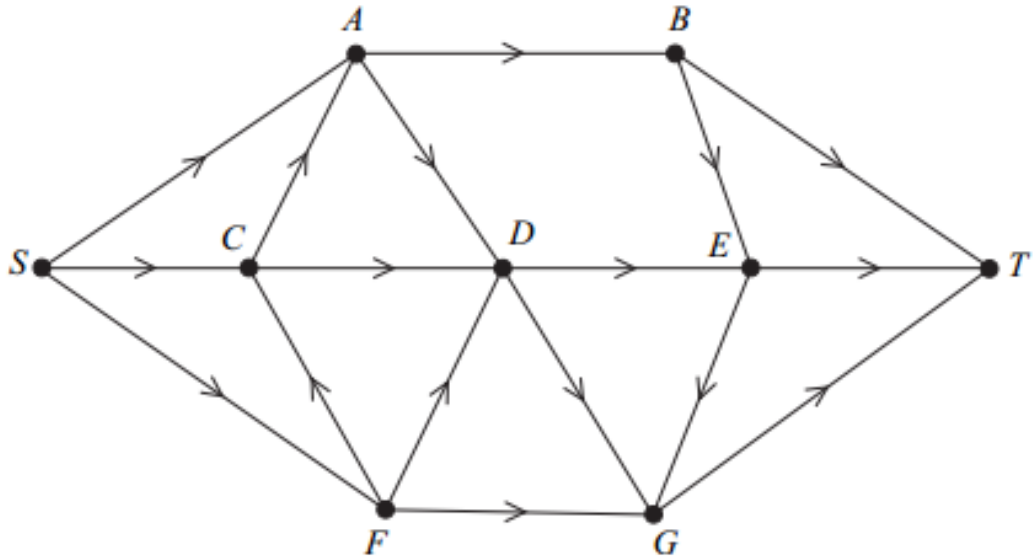
Path	Extra Flow

Figure 3

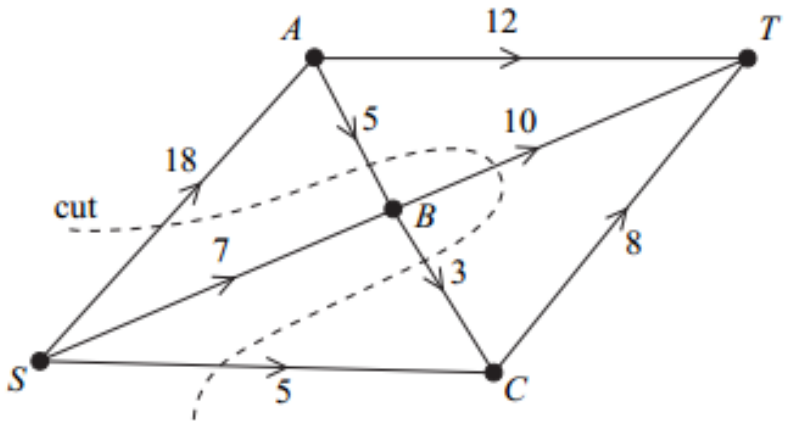


Maximum flow is .....

Figure 4



6 (a) The network shows a flow from S to T along a system of pipes, with the capacity in litres per second indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 36. *(1 mark)*

.....

.....

(ii) The cut shown on the diagram can be represented as $\{S, B\}$, $\{A, C, T\}$.

Complete the table below to give the value of each of the 8 possible cuts. *(3 marks)*

Cut		Value
$\{S\}$	$\{A, B, C, T\}$	30
$\{S, A\}$	$\{B, C, T\}$	29
$\{S, B\}$	$\{A, C, T\}$	36
$\{S, C\}$	$\{A, B, T\}$	33
$\{S, A, B\}$	$\{C, T\}$	
$\{S, A, C\}$	$\{B, T\}$	
$\{S, B, C\}$	$\{A, T\}$	
$\{S, A, B, C\}$	$\{T\}$	30

(iii) State the value of the maximum flow through the network, giving a reason for your answer. *(2 marks)*

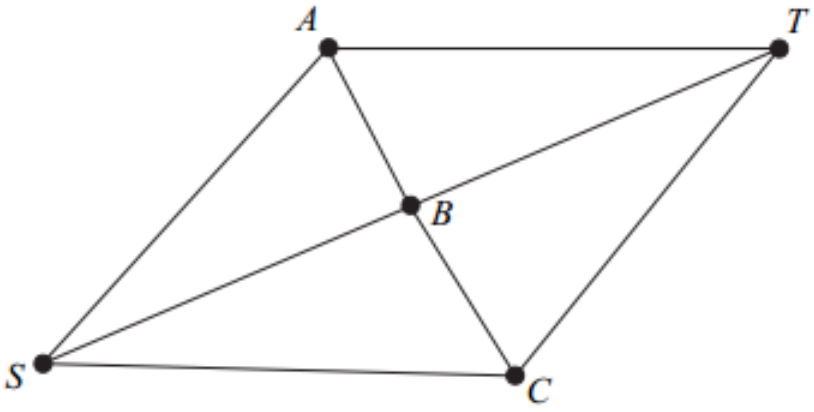
Maximum flow is .....

because .....

.....

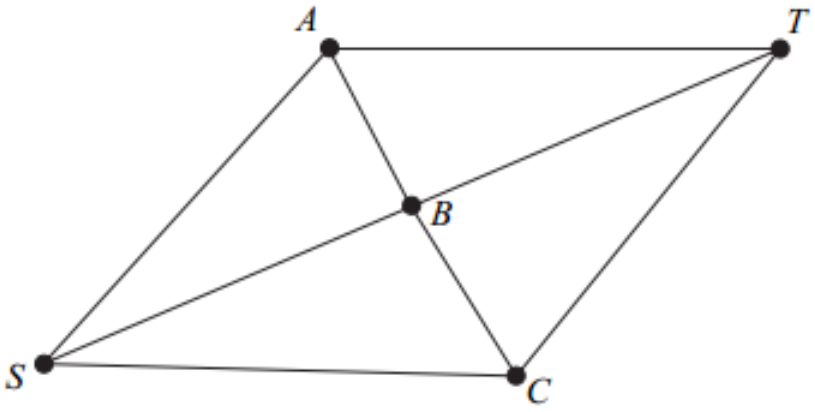
.....

(iv) Indicate on the diagram below a possible flow along each edge corresponding to this maximum flow. *(1 mark)*



(b) The capacities along SC and along AT are each increased by 4 litres per second.

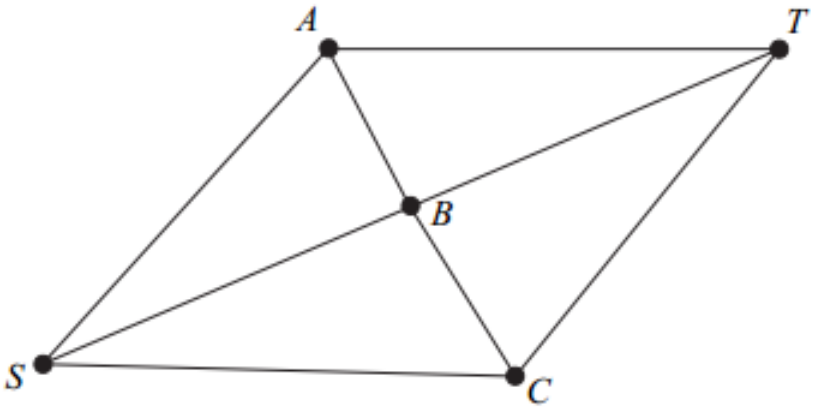
(i) Using your **values from part (a)(iv) as the initial flow**, indicate potential increases and decreases on the diagram below and use the labelling procedure to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the diagram. *(6 marks)*



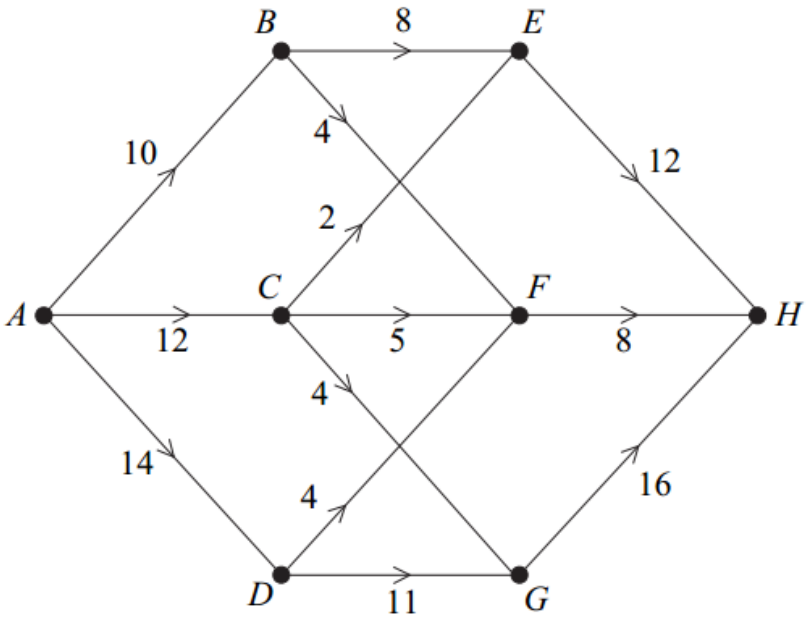
Path	Additional Flow

(ii) Use your results from part **(b)(i)** to illustrate the flow along each edge that gives this new maximum flow, and state the value of the new maximum flow. *(3 marks)*

New maximum flow is .....



8 The network below represents a system of pipes. The capacity of each pipe, in litres per second, is indicated on the corresponding edge.



(a) Find the maximum flow along each of the routes *ABEH*, *ACFH* and *ADGH* and enter their values in the table on **Figure 4** opposite. *(1 mark)*

(b) (i) Taking your answers to part **(a)** as the initial flow, use the labelling procedure on **Figure 4** to find the maximum flow through the network. You should indicate any flow-augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow and, on **Figure 5** opposite, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut. *(1 mark)*

Figure 4

Route	Flow
ABEH	
ACFH	
ADGH	

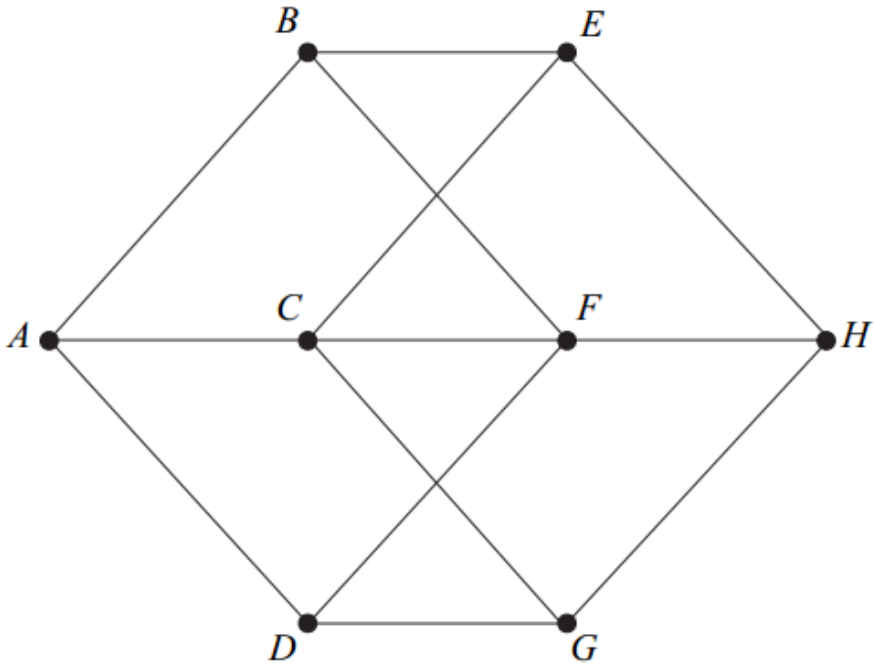
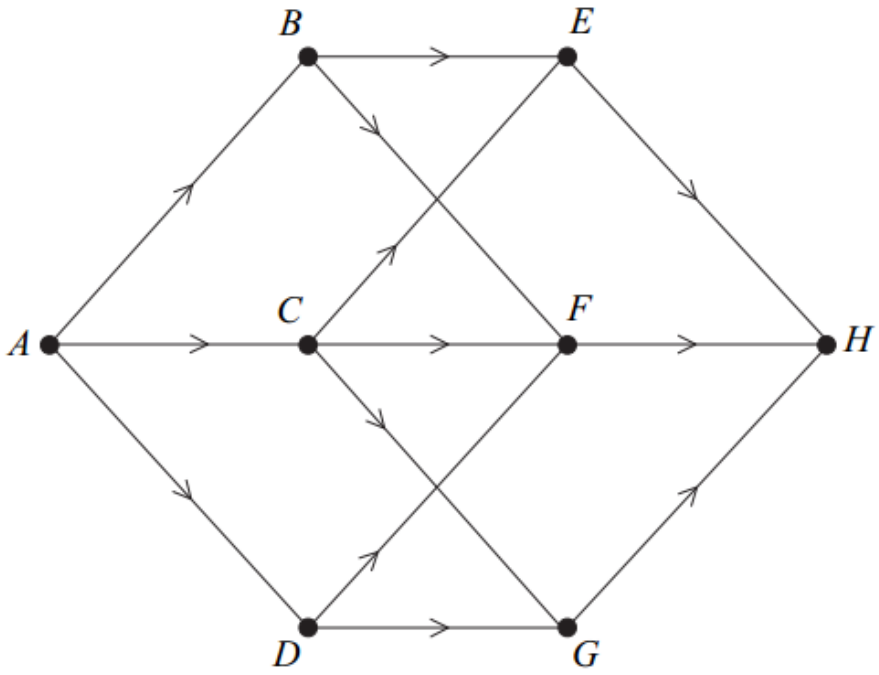
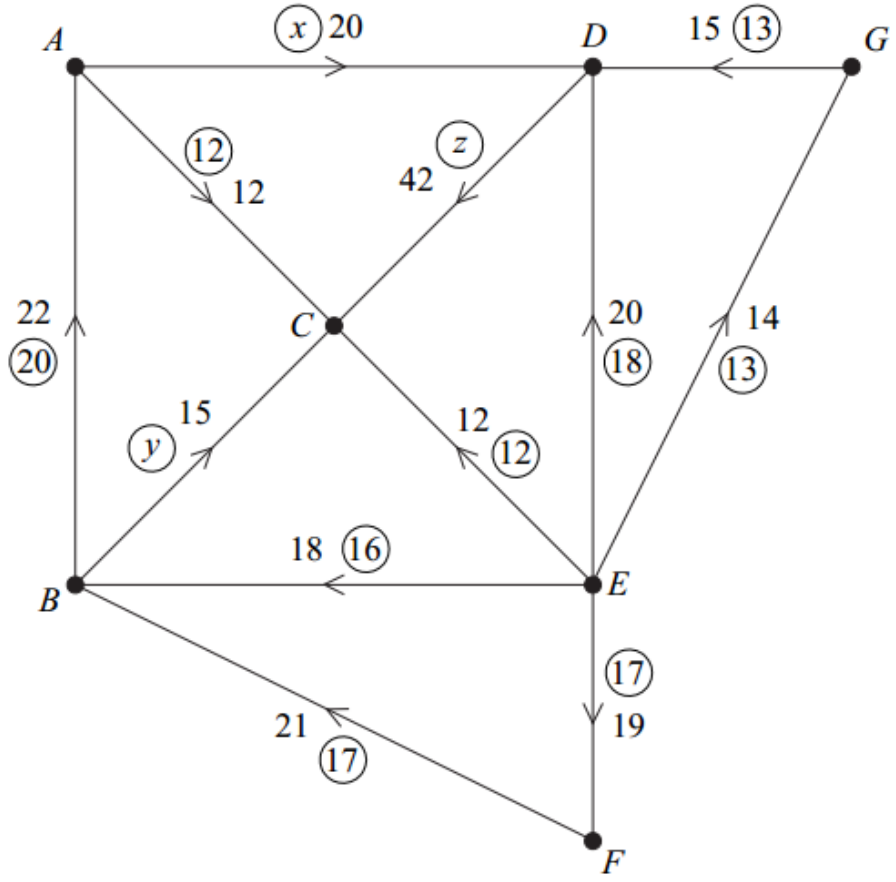


Figure 5



2 The network below represents a system of pipes. The number **not** circled on each edge represents the capacity of each pipe in litres per second. The number or letter in each circle represents an initial flow in litres per second.



(a) Write down the capacity of edge EF. *(1 mark)*

(b) State the source vertex. *(1 mark)*

(c) State the sink vertex. *(1 mark)*

(d) Find the values of x, y and z. *(3 marks)*

(e) Find the value of the initial flow. *(1 mark)*

(f) Find the value of a cut through the edges EB, EC, ED, EF and EG. *(1 mark)*