FP1: Conics

Past Paper Questions
2006 - 2013

Name:

8 A curve has equation $y^2 = 12x$.

(a) Sketch the curve. *(2 marks)*

(b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*

(ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*

(c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x-coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0$$

(3 marks)

(ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*

(iii) Using this value of c, find the coordinates of the point where the line touches the curve. *(2 marks)*

(iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

June 2006

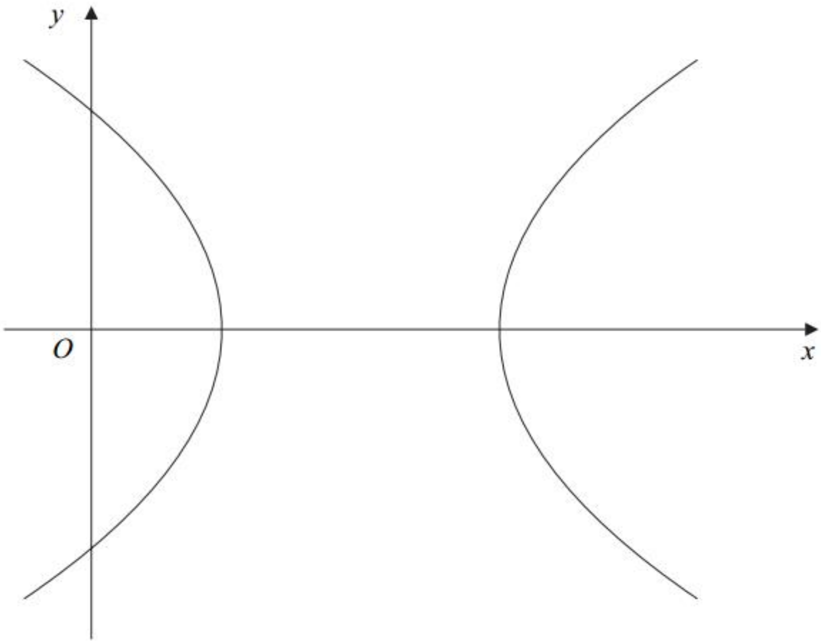
7 (a) Describe a geometrical transformation by which the hyperbola

$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$. *(2 marks)*

(b) The diagram shows the hyperbola H with equation

$$x^2 - y^2 - 4x + 3 = 0$$



By completing the square, describe a geometrical transformation by which the hyperbola H can be obtained from the hyperbola $x^2 - y^2 = 1$. *(4 marks)*

January 2007

8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

(a) Find the y-coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. *(3 marks)*

(c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. *(1 mark)*

(d) (i) Show that, if the line $y = x - 4$ intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0$$ *(3 marks)*

(ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C. *(2 marks)*

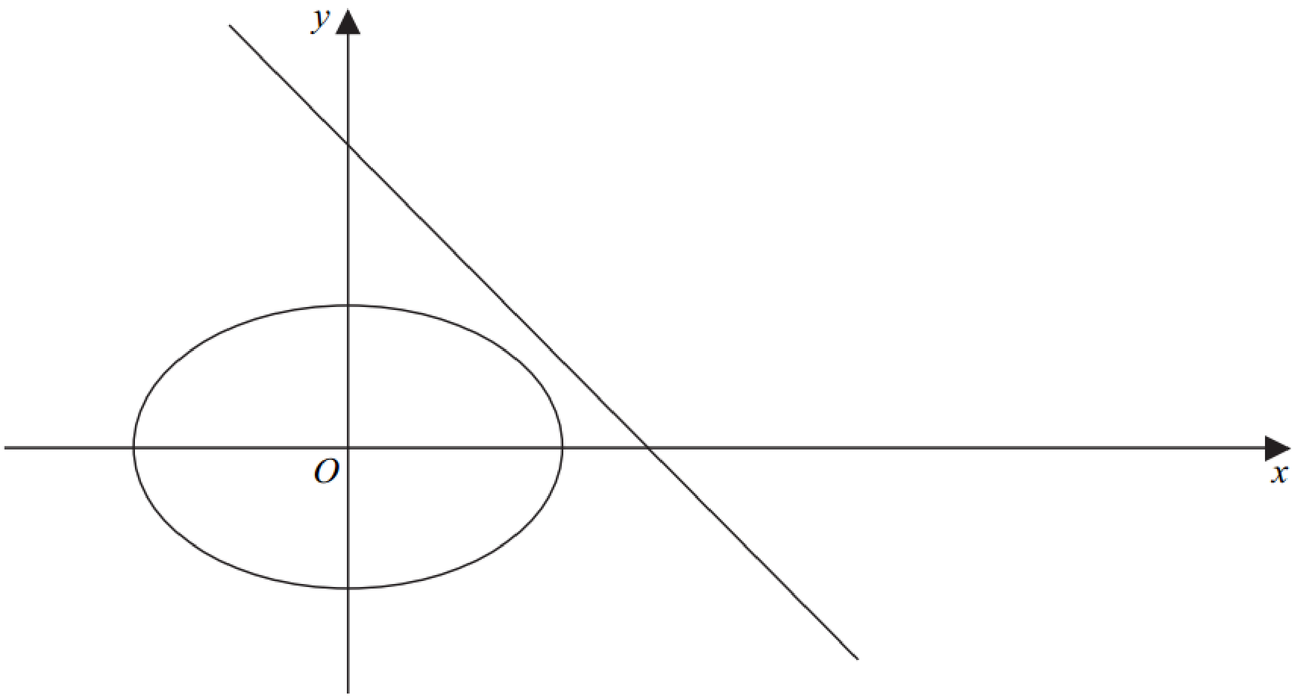
9 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



(a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. *(2 marks)*

(b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k, the equation of the curve after this translation. *(2 marks)*

(c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x-coordinates of the points of intersection must satisfy the equation

$$3x^2 - 2(k+4)x + (k^2 + 6) = 0$$ *(4 marks)*

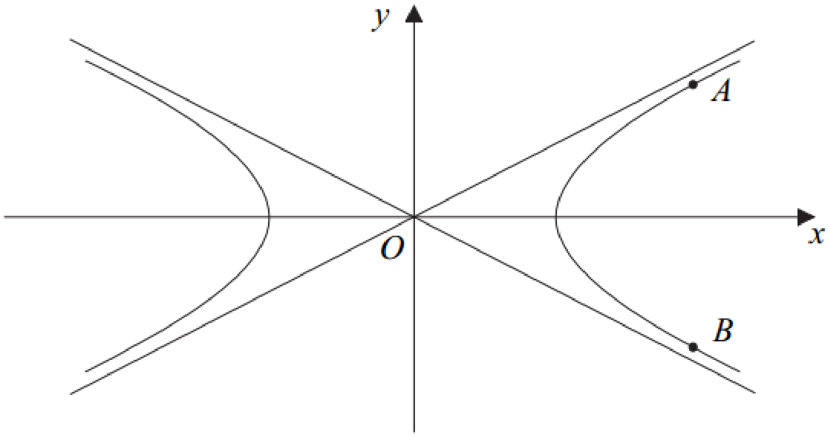
(d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. *(4 marks)*

(e) On **Figure 3**, show the translated curves corresponding to these two values of k. *(3 marks)*

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. *(2 marks)*

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B.

Find, in surd form, the y-coordinates of A and B. *(2 marks)*

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

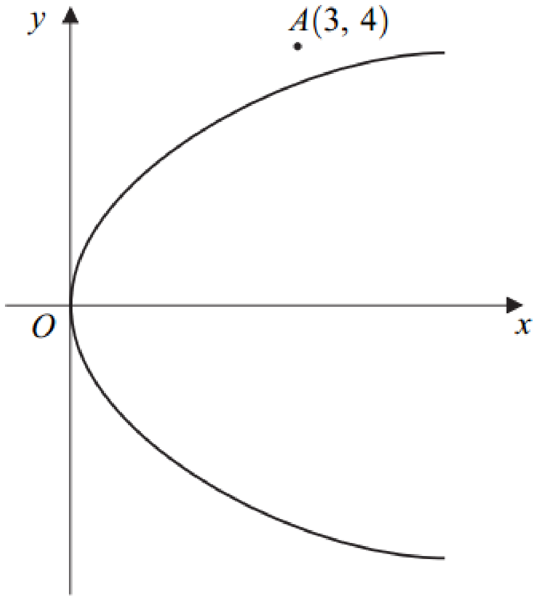
Write down:

(i) the y-coordinates of the image points of A and B under this translation; *(1 mark)*

(ii) the equations of the hyperbola and the asymptotes after the translation. *(3 marks)*

June 2008

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*

(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0$$ *(3 marks)*

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

(d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

January 2009

9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

(a) Find the equations of the two asymptotes of H, giving each answer in the form $y = mx$. *(2 marks)*

(b) Draw a sketch of the two asymptotes of H, using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H. *(3 marks)*

(c) (i) Show that, if the line $y = x + c$ intersects H, the x-coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$ *(4 marks)*

(ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c. *(2 marks)*

(iii) Find, in terms of c, the y-coordinates of these two points. *(3 marks)*

June 2009

6 An ellipse E has equation

$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

(a) Sketch the ellipse E, showing the coordinates of the points of intersection of the ellipse with the coordinate axes. *(3 marks)*

(b) The ellipse E is stretched with scale factor 2 parallel to the y-axis.

Find and simplify the equation of the curve after the stretch. *(3 marks)*

(c) The **original** ellipse, E, is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$. The equation of the translated ellipse is

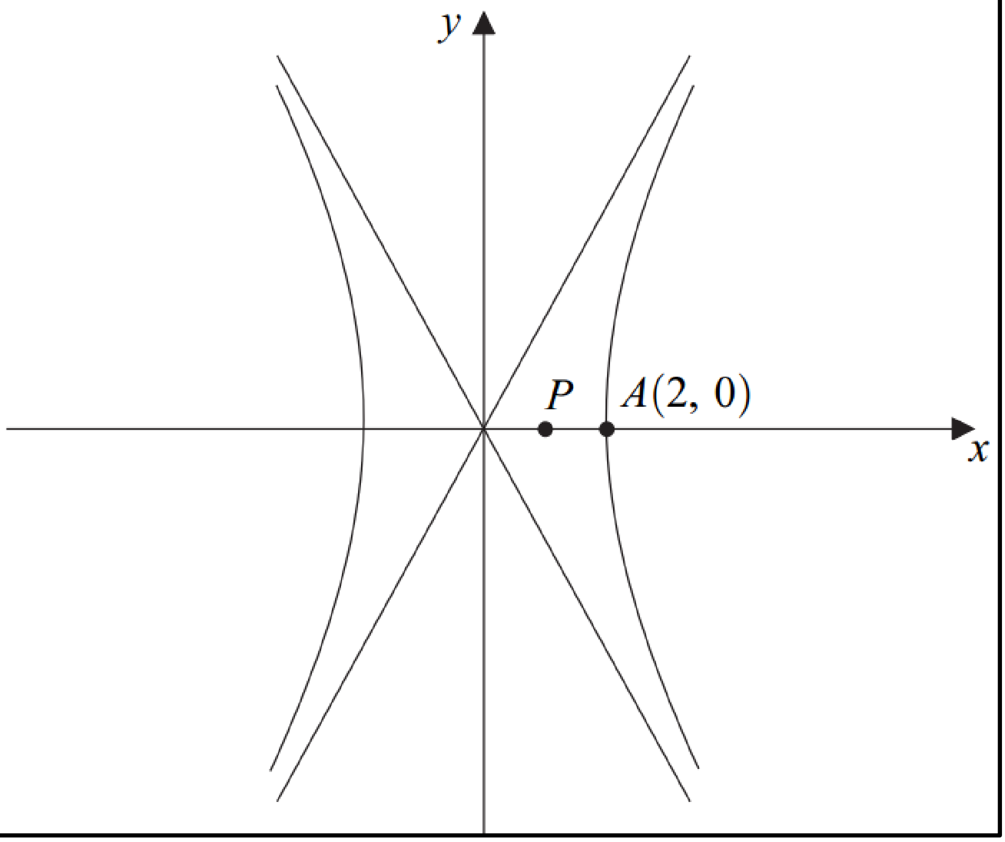
$$4x^2 + 3y^2 - 8x + 6y = 5$$

Find the values of a and b. *(5 marks)*

9 The diagram shows the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and its asymptotes.



The constants a and b are positive integers.

The point A on the hyperbola has coordinates $(2, 0)$.

The equations of the asymptotes are $y = 2x$ and $y = -2x$.

(a) Show that $a = 2$ and $b = 4$. *(4 marks)*

(b) The point P has coordinates $(1, 0)$. A straight line passes through P and has gradient m. Show that, if this line intersects the hyperbola, the x-coordinates of the points of intersection satisfy the equation

$$(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$$ *(4 marks)*

(c) Show that this equation has equal roots if $3m^2 = 16$. *(3 marks)*

(d) There are two tangents to the hyperbola which pass through P. Find the coordinates of the points at which these tangents touch the hyperbola.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

9 A parabola P has equation $y^2 = x - 2$.

(a) (i) Sketch the parabola P. *(2 marks)*

(ii) On your sketch, draw the two tangents to P which pass through the point $(-2, 0)$. *(2 marks)*

(b) (i) Show that, if the line $y = m(x + 2)$ intersects P, then the x-coordinates of the points of intersection must satisfy the equation

$$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$ *(3 marks)*

(ii) Show that, if this equation has equal roots, then

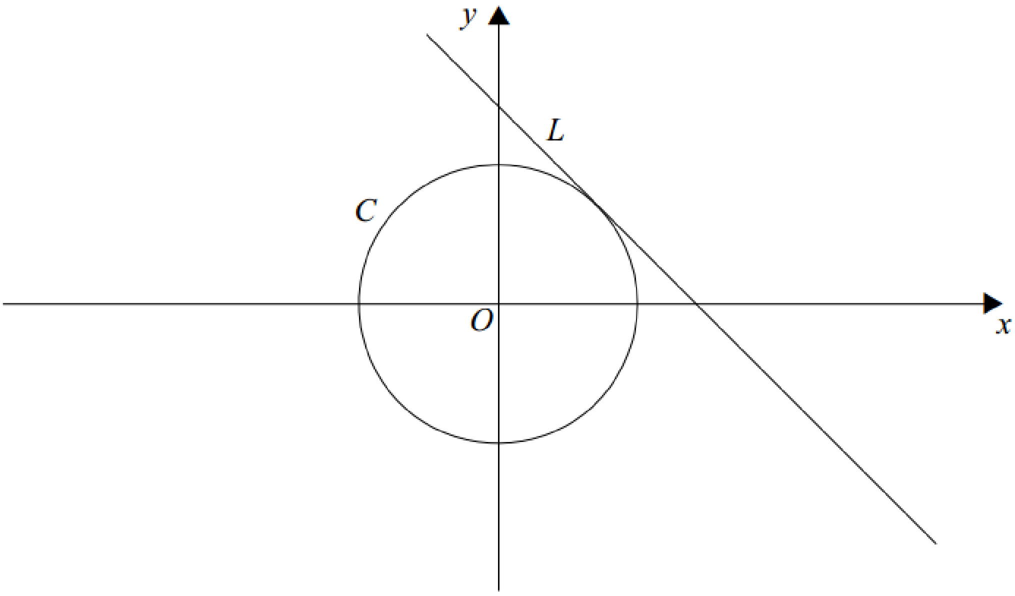
$$16m^2 = 1$$ *(3 marks)*

(iii) Hence find the coordinates of the points at which the tangents to P from the point $(-2, 0)$ touch the parabola P. *(3 marks)*

6 The diagram shows a circle C and a line L, which is the tangent to C at the point $(1, 1)$. The equations of C and L are

$$x^2 + y^2 = 2 \quad \text{and} \quad x + y = 2$$

respectively.



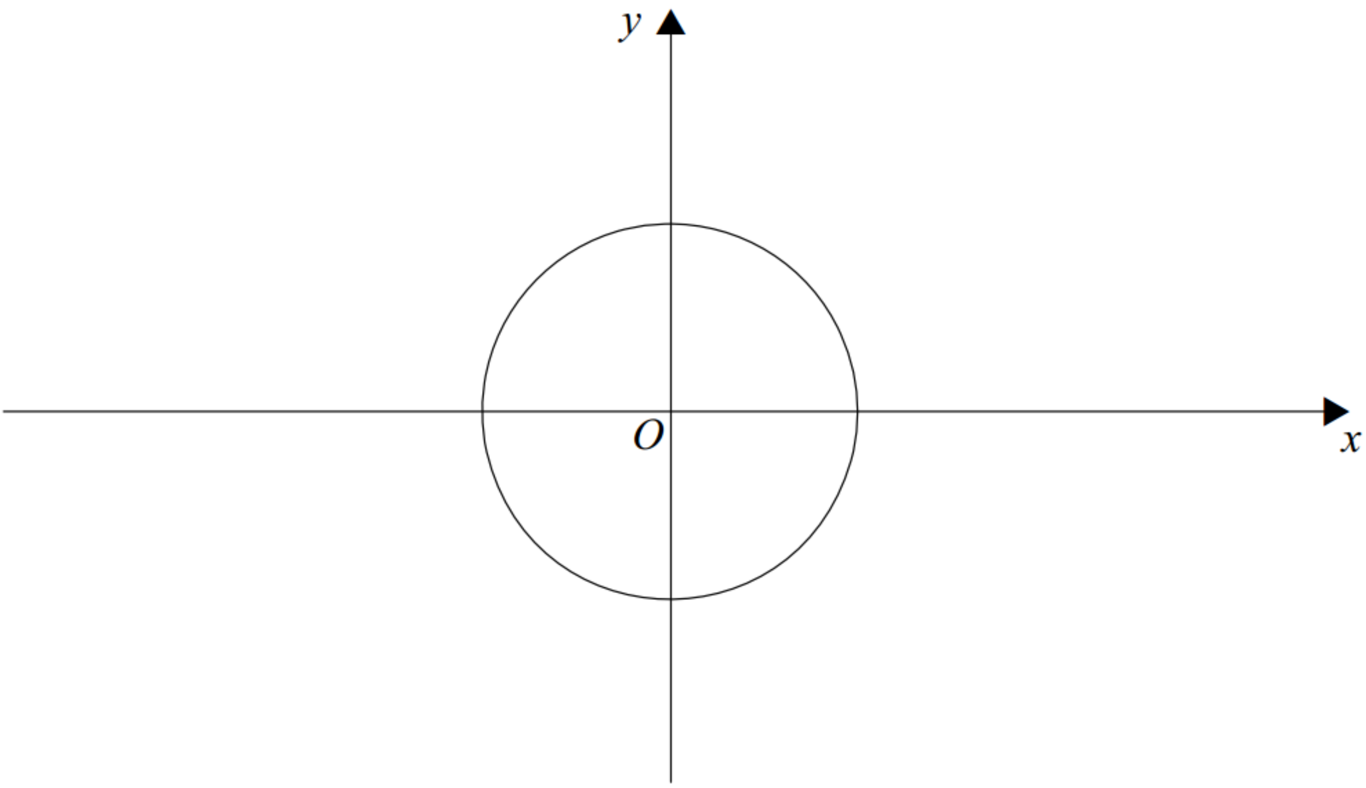
The circle C is now transformed by a stretch with scale factor 2 parallel to the x-axis. The image of C under this stretch is an ellipse E.

(a) **On the diagram below**, sketch the ellipse E, indicating the coordinates of the points where it intersects the coordinate axes. *(4 marks)*

(b) Find equations of:

(i) the ellipse E; *(2 marks)*

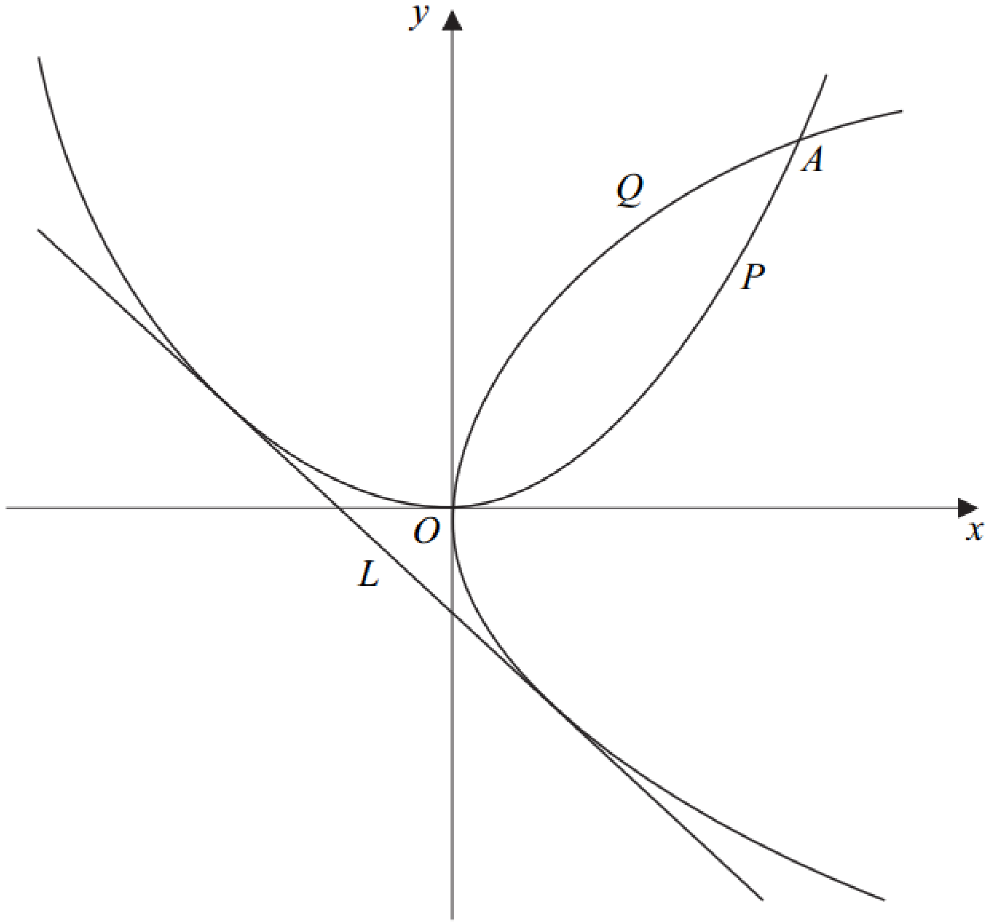
(ii) the tangent to E at the point $(2, 1)$. *(2 marks)*



9 The diagram shows a parabola P which has equation $y = \frac{1}{8}x^2$, and another parabola Q which is the image of P under a reflection in the line $y = x$.

The parabolas P and Q intersect at the origin and again at a point A.

The line L is a tangent to both P and Q.



(a) (i) Find the coordinates of the point A. *(2 marks)*

(ii) Write down an equation for Q. *(1 mark)*

(iii) Give a reason why the gradient of L must be -1. *(1 mark)*

(b) (i) Given that the line $y = -x + c$ intersects the parabola P at two distinct points, show that

$$c > -2$$ *(3 marks)*

(ii) Find the coordinates of the points at which the line L touches the parabolas P and Q.

(No credit will be given for solutions based on differentiation.) *(4 marks)*

9 A curve has equation

$$y = \frac{x}{x - 1}$$

(a) Find the equations of the asymptotes of this curve. *(2 marks)*

(b) Given that the line $y = -4x + c$ intersects the curve, show that the x-coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c + 3)x + c = 0$$ *(3 marks)*

(c) It is given that the line $y = -4x + c$ is a tangent to the curve.

(i) Find the two possible values of c.

(No credit will be given for methods involving differentiation.) *(3 marks)*

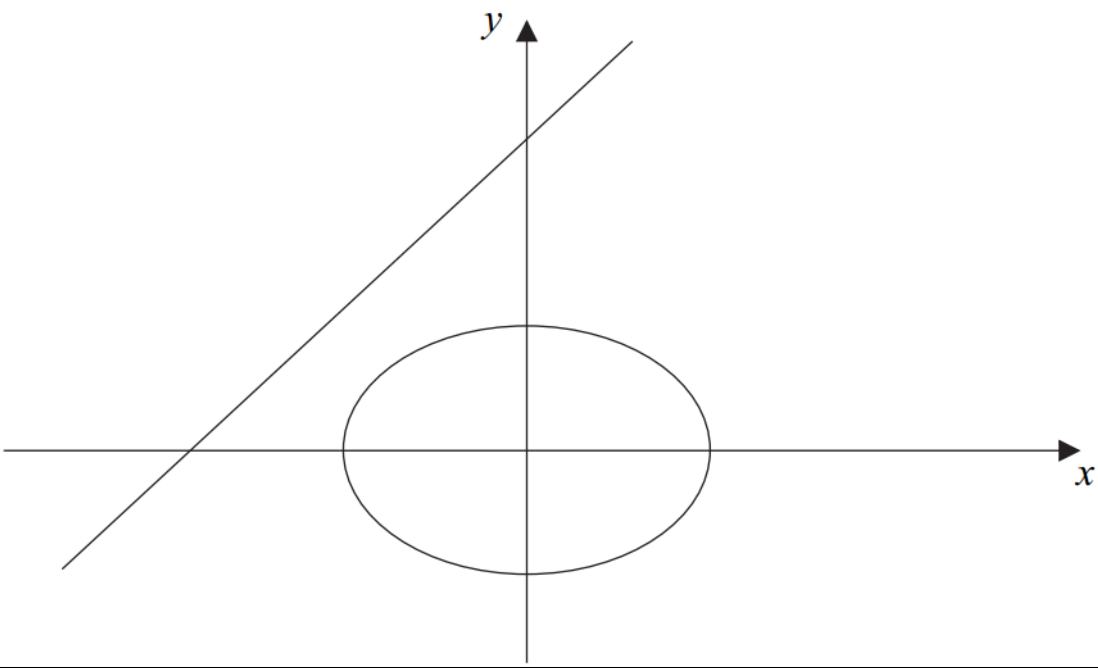
(ii) For each of the two values found in part **(c)(i)**, find the coordinates of the point where the line touches the curve. *(4 marks)*

8 The diagram shows the ellipse E with equation

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

and the straight line L with equation

$$y = x + 4$$



(a) Write down the coordinates of the points where the ellipse E intersects the coordinate axes. *(2 marks)*

(b) The ellipse E is translated by the vector $\begin{bmatrix} p \\ 0 \end{bmatrix}$, where p is a constant. Write down the equation of the translated ellipse. *(2 marks)*

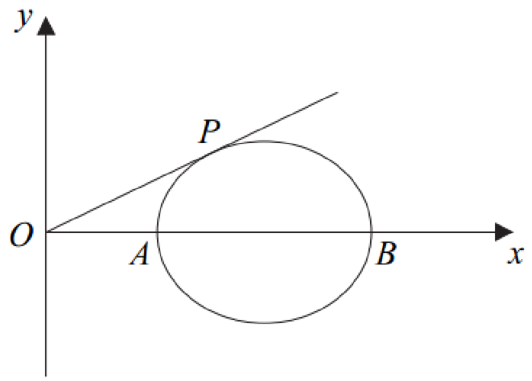
(c) Show that, if the translated ellipse intersects the line L, the x-coordinates of the points of intersection must satisfy the equation

$$9x^2 - (8p - 40)x + (4p^2 + 60) = 0$$ *(3 marks)*

(d) Given that the line L is a tangent to the translated ellipse, find the coordinates of the two possible points of contact.

(No credit will be given for solutions based on differentiation.) *(8 marks)*

9 An ellipse is shown below.



The ellipse intersects the x-axis at the points A and B. The equation of the ellipse is

$$\frac{(x-4)^2}{4} + y^2 = 1$$

(a) Find the x-coordinates of A and B. *(2 marks)*

(b) The line $y = mx$ $(m > 0)$ is a tangent to the ellipse, with point of contact P.

(i) Show that the x-coordinate of P satisfies the equation

$$(1 + 4m^2)x^2 - 8x + 12 = 0$$ *(3 marks)*

(ii) Hence find the exact value of m. *(4 marks)*

(iii) Find the coordinates of P. *(4 marks)*