FP1: Matrices

Past Paper Questions
2006 - 2013

Name:

January 2006

7 (a) The transformation T is defined by the matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Describe the transformation T geometrically. *(2 marks)*

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

(b) The matrix **B** is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

June 2006

5 The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(b) Describe fully the geometrical transformation represented by **M**. *(2 marks)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

January 2007

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

June 2007

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix **M** represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

January 2008

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix $\mathbf{M}$ can be written in the form

$$q\begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

(ii) the equation of the mirror line of the reflection. *(1 mark)*

(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

June 2008

6 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(b) Show that $\mathbf{A}^2$ is of the form $k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. *(3 marks)*

January 2009

5 The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix $\mathbf{A}$ represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

June 2009

7 (a) Using surd forms where appropriate, find the matrix which represents:

(i) a rotation about the origin through 30° anticlockwise; *(2 marks)*

(ii) a reflection in the line $y = \frac{1}{\sqrt{3}}x$. *(2 marks)*

(b) The matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(2 marks)*

(c) The transformation represented by $\mathbf{A}$ is followed by the transformation represented by $\mathbf{B}$, where

$$\mathbf{B} = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

Find the matrix of the combined transformation and give a full geometrical description of this combined transformation. *(5 marks)*

January 2010

4 It is given that

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

and that $\mathbf{I}$ is the 2×2 identity matrix.

(a) Show that $(\mathbf{A} - \mathbf{I})^2 = k\mathbf{I}$ for some integer k. *(3 marks)*

(b) Given further that

$$\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$$

find the integer p such that

$$(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2$$ *(4 marks)*

June 2010

6 The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

(a) $\mathbf{A}$; *(2 marks)*

(b) $\mathbf{B}$; *(2 marks)*

(c) $\mathbf{A}^2$; *(2 marks)*

(d) $\mathbf{B}^2$; *(2 marks)*

(e) $\mathbf{AB}$. *(3 marks)*

January 2011

3 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a rotation about the origin through $90°$ clockwise; *(1 mark)*

(ii) a rotation about the origin through $180°$. *(1 mark)*

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

(i) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(ii) Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, for some integer k. *(3 marks)*

(c) Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

(iii) $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

June 2011

7 The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

(a) (i) Calculate the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) Show that $\mathbf{A}^3 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(b) Describe the single geometrical transformation, or combination of two geometrical transformations, corresponding to each of the matrices:

(i) $\mathbf{A}^3$; *(2 marks)*

(ii) $\mathbf{A}$. *(3 marks)*

8 The diagram below shows a rectangle R_1 which has vertices (0, 0), (3, 0), (3, 2) and (0, 2).

(a) On the diagram, draw:

(i) the image R_2 of R_1 under a rotation through 90° clockwise about the origin; *(1 mark)*

(ii) the image R_3 of R_2 under the transformation which has matrix

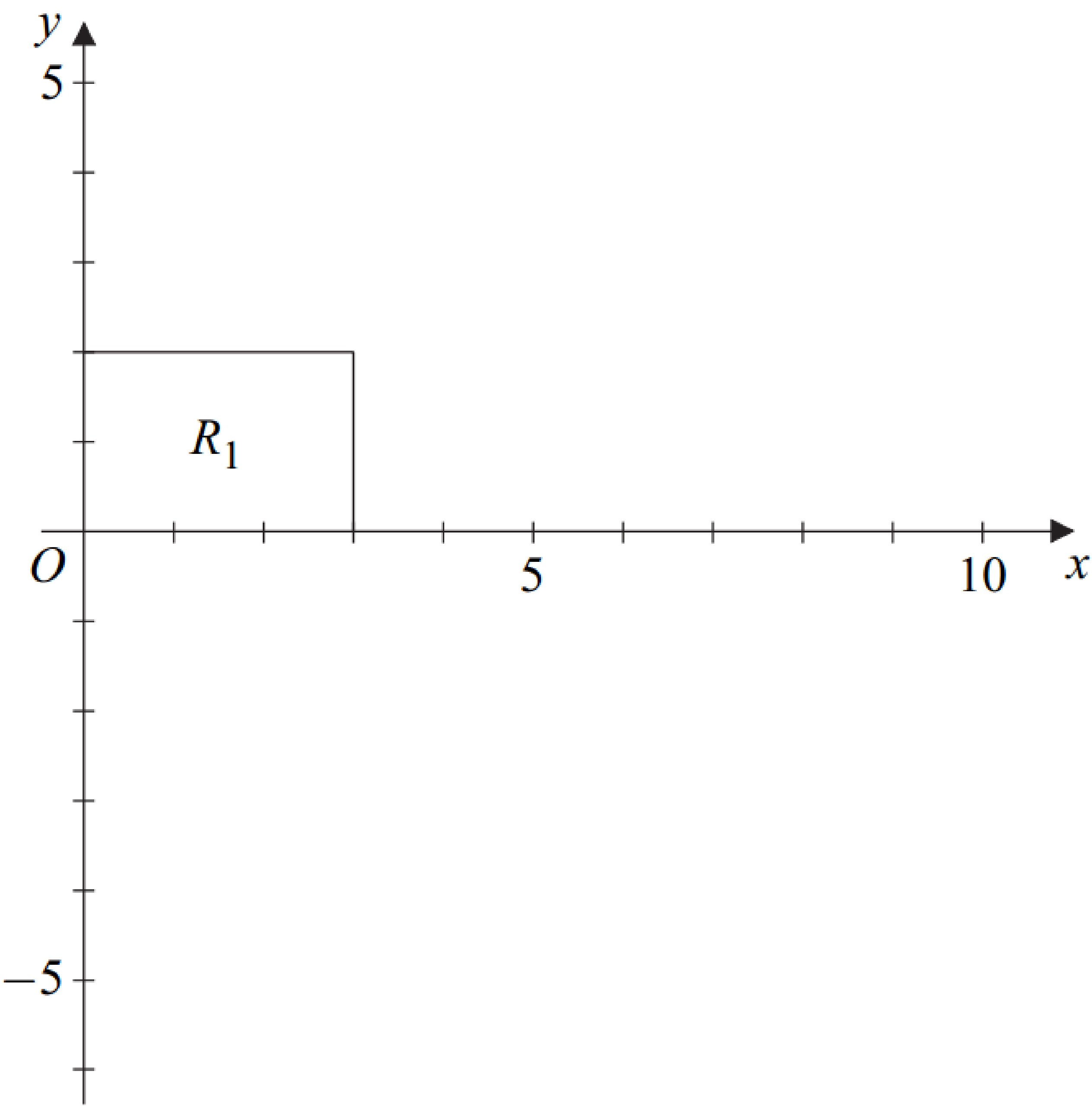
$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

(3 marks)

(b) Find the matrix of:

(i) the rotation which maps R_1 onto R_2; *(1 mark)*

(ii) the combined transformation which maps R_1 onto R_3. *(3 marks)*



June 2012

6 (a) Using surd forms, find the matrix of a rotation about the origin through 135° anticlockwise. *(2 marks)*

(b) The matrix **M** is defined by $\mathbf{M} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$.

(i) Given that **M** represents an enlargement followed by a rotation, find the scale factor of the enlargement and the angle of the rotation. *(3 marks)*

(ii) The matrix $\mathbf{M}^2$ also represents an enlargement followed by a rotation. State the scale factor of the enlargement and the angle of the rotation. *(2 marks)*

(iii) Show that $\mathbf{M}^4 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(iv) Deduce that $\mathbf{M}^{2012} = -2^n\mathbf{I}$ for some positive integer n. *(2 marks)*

January 2013

6 (a) The matrix **X** is defined by $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$.

(i) Given that $\mathbf{X}^2 = \begin{bmatrix} m & 2 \\ 3 & 6 \end{bmatrix}$, find the value of m. *(1 mark)*

(ii) Show that $\mathbf{X}^3 - 7\mathbf{X} = n\mathbf{I}$, where n is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(4 marks)*

(b) It is given that $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

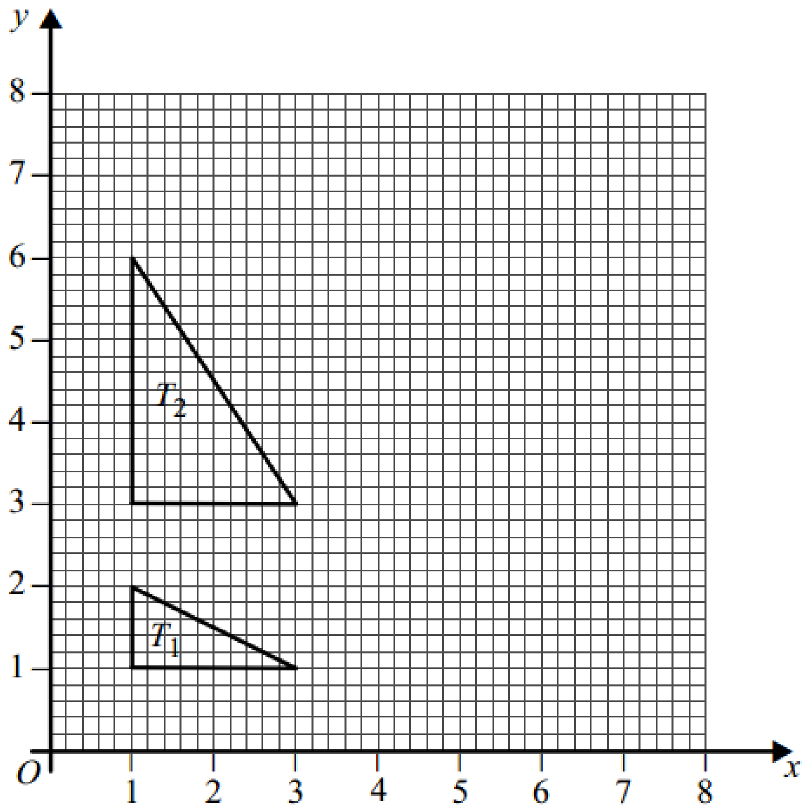
(i) Describe the geometrical transformation represented by **A**. *(1 mark)*

(ii) The matrix **B** represents an anticlockwise rotation through 45° about the origin.

Show that $\mathbf{B} = k\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, where k is a surd. *(2 marks)*

(iii) Find the image of the point $P(-1, 2)$ under an anticlockwise rotation through 45° about the origin, followed by the transformation represented by **A**. *(4 marks)*

8 The diagram shows two triangles, T_1 and T_2.



(a) Find the matrix which represents the stretch that maps triangle T_1 onto triangle T_2.
(2 marks)

(b) The triangle T_2 is reflected in the line $y = \sqrt{3}x$ to give a third triangle, T_3. Find, using surd forms where appropriate:

(i) the matrix which represents the reflection that maps triangle T_2 onto triangle T_3;
(2 marks)

(ii) the matrix which represents the combined transformation that maps triangle T_1 onto triangle T_3.
(2 marks)