FP1: Numerical Methods

Past Paper Questions
2006 - 2013

Name:

January 2006

1 (a) Show that the equation

$$x^3 + 2x - 2 = 0$$

has a root between 0.5 and 1. *(2 marks)*

(b) Use linear interpolation once to find an estimate of this root. Give your answer to two decimal places. *(3 marks)*

June 2006

2 A curve satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \log_{10} x$$

Starting at the point (2, 3) on the curve, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 2.4$. Give your answer to three decimal places. *(6 marks)*

January 2007

7 The function f is defined for all real numbers by

$$\mathrm{f}(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $\mathrm{f}(x) = 0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$\mathrm{g}(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $\mathrm{g}(x)$ gives a good approximation to $\mathrm{f}(x)$ for small values of x.

(i) Show that $\mathrm{g}(0.05)$ and $\mathrm{f}(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = \mathrm{g}(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $\mathrm{g}'(0)$. *(1 mark)*

June 2007

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. *(3 marks)*

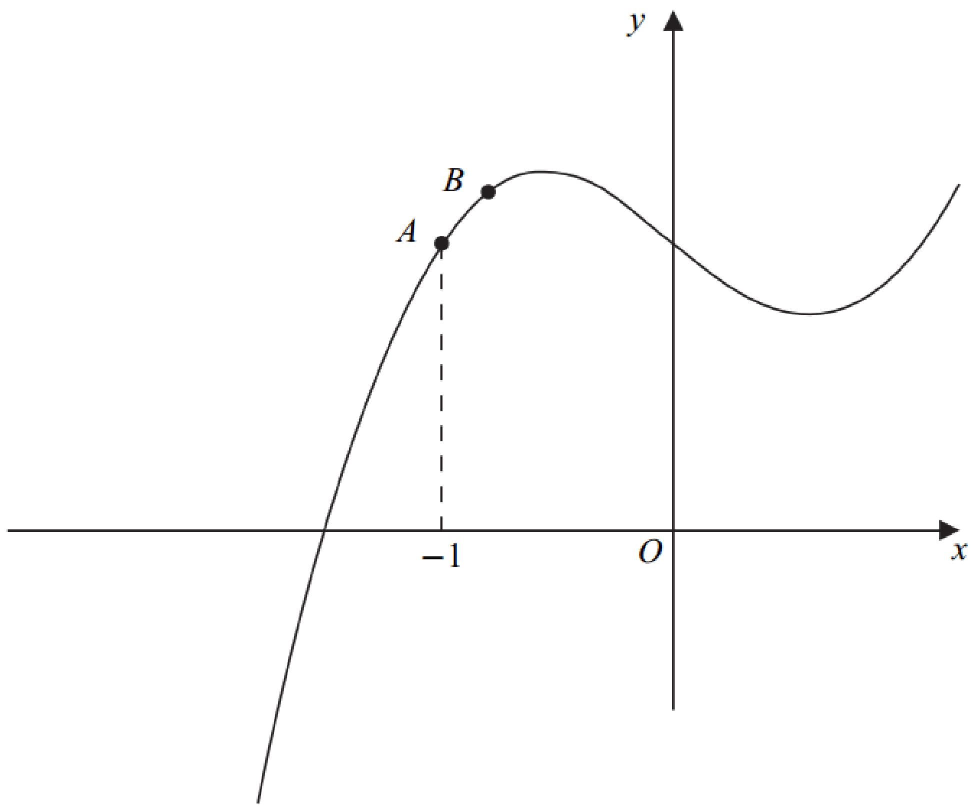
(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. *(4 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x-coordinates -1 and $-1 + h$ respectively.



(a) (i) Show that the y-coordinate of the point B is

$$1 + 2h - 3h^2 + h^3$$ *(3 marks)*

(ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(3 marks)*

(iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A. State the value of this gradient. *(2 marks)*

(b) The equation $x^3 - x + 1 = 0$ has one real root, α.

(i) Taking $x_1 = -1$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α. *(2 marks)*

(ii) On **Figure 1**, draw a straight line to illustrate the Newton-Raphson method as used in part (b)(i). Show the points $(x_2, 0)$ and $(\alpha, 0)$ on your diagram. *(2 marks)*

January 2009

1 A curve passes through the point $(0,\ 1)$ and satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{1 + x^2}$$

Starting at the point $(0,\ 1)$, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 0.4$. Give your answer to five decimal places. *(5 marks)*

June 2009

2 A curve has equation

$$y = x^2 - 6x + 5$$

The points A and B on the curve have x-coordinates 2 and $2 + h$ respectively.

(a) Find, in terms of h, the gradient of the line AB, giving your answer in its simplest form. *(5 marks)*

(b) Explain how the result of part **(a)** can be used to find the gradient of the curve at A. State the value of this gradient. *(3 marks)*

January 2010

7 A curve C has equation $y = \dfrac{1}{(x-2)^2}$.

(a) (i) Write down the equations of the asymptotes of the curve C. *(2 marks)*

(ii) Sketch the curve C. *(2 marks)*

(b) The line $y = x - 3$ intersects the curve C at a point which has x-coordinate α.

(i) Show that α lies within the interval $3 < x < 4$. *(2 marks)*

(ii) Starting from the interval $3 < x < 4$, use interval bisection twice to obtain an interval of width 0.25 within which α must lie. *(3 marks)*

June 2010

1 A curve passes through the point $(1,\ 3)$ and satisfies the differential equation

$$\frac{dy}{dx} = 1 + x^3$$

Starting at the point $(1,\ 3)$, use a step-by-step method with a step length of 0.1 to estimate the y-coordinate of the point on the curve for which $x = 1.3$. Give your answer to three decimal places.

(No credit will be given for methods involving integration.) *(6 marks)*

June 2011

1 A curve passes through the point (2, 3) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{\sqrt{2+x}}$$

Starting at the point (2, 3), use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 3$. Give your answer to four decimal places. *(5 marks)*

January 2012

5 The diagram below (not to scale) shows a part of a curve $y = f(x)$ which passes through the points $A(2, -10)$ and $B(5, 22)$.

(a) (i) On the diagram, draw a line which illustrates the method of linear interpolation for solving the equation $f(x) = 0$. The point of intersection of this line with the x-axis should be labelled P. *(1 mark)*

(ii) Calculate the x-coordinate of P. Give your answer to one decimal place. *(3 marks)*

(b) (i) On the same diagram, draw a line which illustrates the Newton–Raphson method for solving the equation $f(x) = 0$, with initial value $x_1 = 2$. The point of intersection of this line with the x-axis should be labelled Q. *(1 mark)*

(ii) Given that the gradient of the curve at A is 8, calculate the x-coordinate of Q. Give your answer as an exact decimal. *(2 marks)*

June 2012

7 The equation

$$24x^3 + 36x^2 + 18x - 5 = 0$$

has one real root, α.

(a) Show that α lies in the interval $0.1 < x < 0.2$. *(2 marks)*

(b) Starting from the interval $0.1 < x < 0.2$, use interval bisection **twice** to obtain an interval of width 0.025 within which α must lie. *(3 marks)*

(c) Taking $x_1 = 0.2$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to four decimal places. *(4 marks)*

January 2013

1 A curve passes through the point (1, 3) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{x}{1+x^3}$$

Starting at the point (1, 3), use a step-by-step method with a step length of 0.1 to estimate the value of y at $x = 1.2$. Give your answer to four decimal places. *(5 marks)*

1 The equation

$$x^3 - x^2 + 4x - 900 = 0$$

has exactly one real root, α.

Taking $x_1 = 10$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to four significant figures.

(3 marks)