FP4: Roots of Quadratics

Past Paper Questions
2006 - 2013

Name:

June 2006

1 The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots α and β.

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) (i) Expand $(\alpha + \beta)^3$. *(1 mark)*

(ii) Show that $\alpha^3 + \beta^3 = 4$. *(3 marks)*

(c) Find a quadratic equation with roots α^3 and β^3, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers. *(3 marks)*

January 2007

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

June 2007

4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{4}$. *(2 marks)*

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta}$$ *(3 marks)*

January 2008

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + i\sqrt{3})^3 = (1 - i\sqrt{3})^3$$ *(2 marks)*

June 2008

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. *(2 marks)*

June 2009

1 The equation

$$2x^2 + x - 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $4\alpha^2$ and $4\beta^2$. Give your answer in the form $x^2 + px + q = 0$, where p and q are integers. *(3 marks)*

January 2010

1 The quadratic equation

$$3x^2 - 6x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^3 + \beta^3 = 6$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. *(4 marks)*

June 2010

8 The quadratic equation

$$x^2 - 4x + 10 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{5}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\alpha + \frac{2}{\beta}$ and $\beta + \frac{2}{\alpha}$. *(6 marks)*

January 2011

1 The quadratic equation $x^2 - 6x + 18 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find a quadratic equation, with integer coefficients, which has roots α^2 and β^2. *(4 marks)*

(c) Hence find the values of α^2 and β^2. *(1 mark)*

June 2011

2 The equation

$$4x^2 + 6x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{3}{4}$. *(2 marks)*

(c) Find an equation, with integer coefficients, which has roots

$$3\alpha - \beta \text{ and } 3\beta - \alpha$$ *(5 marks)*

January 2012

1 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{17}{4}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$ *(5 marks)*

June 2012

1 The quadratic equation

$$5x^2 - 7x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{39}{5}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha + \frac{1}{\alpha} \quad \text{and} \quad \beta + \frac{1}{\beta}$$ *(5 marks)*

January 2013

5 The roots of the quadratic equation

$$x^2 + 2x - 5 = 0$$

are α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Calculate the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $\alpha^3\beta + 1$ and $\alpha\beta^3 + 1$. *(5 marks)*

June 2013

6 The equation

$$2x^2 + 3x - 6 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Hence show that $\alpha^3 + \beta^3 = -\frac{135}{8}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, whose roots are $\alpha + \frac{\alpha}{\beta^2}$ and $\beta + \frac{\beta}{\alpha^2}$. *(6 marks)*