# FP2: Complex Numbers

Past Paper Questions 2006 - 2013

Name:

January 2006

**3** The complex numbers $z_1$ and $z_2$ are given by

$$z_1 = \frac{1+i}{1-i} \qquad \text{and} \qquad z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

(a) Show that $z_1 = i$. *(2 marks)*

(b) Show that $|z_1| = |z_2|$. *(2 marks)*

(c) Express both $z_1$ and $z_2$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(3 marks)*

(d) Draw an Argand diagram to show the points representing $z_1$, $z_2$ and $z_1 + z_2$. *(2 marks)*

(e) Use your Argand diagram to show that

$$\tan \frac{5}{12}\pi = 2 + \sqrt{3}$$ *(3 marks)*

**5** The complex number $z$ satisfies the relation

$$|z + 4 - 4i| = 4$$

(a) Sketch, on an Argand diagram, the locus of $z$. *(3 marks)*

(b) Show that the greatest value of $|z|$ is $4(\sqrt{2} + 1)$. *(3 marks)*

(c) Find the value of $z$ for which

$$\arg(z + 4 - 4i) = \tfrac{1}{6}\pi$$

Give your answer in the form $a + ib$. *(3 marks)*

June 2006

**4** (a) On one Argand diagram, sketch the locus of points satisfying:

(i) $|z - 3 + 2i| = 4$; *(3 marks)*

(ii) $\arg(z - 1) = -\frac{1}{4}\pi$. *(3 marks)*

(b) Indicate on your sketch the set of points satisfying both

$$|z - 3 + 2i| \leqslant 4$$

$$\text{and} \qquad \arg(z - 1) = -\tfrac{1}{4}\pi$$ *(1 mark)*

February 2007

**2** (a) Sketch on one diagram:

(i) the locus of points satisfying $|z - 4 + 2\text{i}| = 2$; *(3 marks)*

(ii) the locus of points satisfying $|z| = |z - 3 - 2\text{i}|$. *(3 marks)*

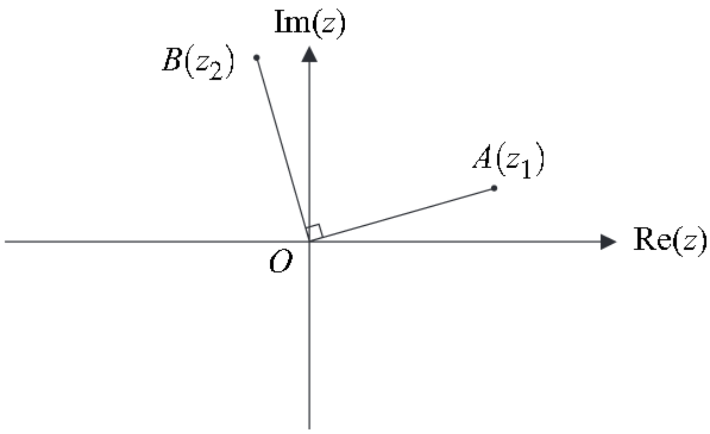
(b) Shade on your sketch the region in which

both $|z - 4 + 2\text{i}| \leqslant 2$

and $|z| \leqslant |z - 3 - 2\text{i}|$ *(2 marks)*

June 2007

**5** The sketch shows an Argand diagram. The points $A$ and $B$ represent the complex numbers $z_1$ and $z_2$ respectively. The angle $AOB = 90°$ and $OA = OB$.




(a) Explain why $z_2 = \text{i}z_1$. *(2 marks)*

(b) On a **single** copy of the diagram, draw:

(i) the locus $L_1$ of points satisfying $|z - z_2| = |z - z_1|$; *(2 marks)*

(ii) the locus $L_2$ of points satisfying $\arg(z - z_2) = \arg z_1$. *(3 marks)*

(c) Find, in terms of $z_1$, the complex number representing the point of intersection of $L_1$ and $L_2$. *(2 marks)*

January 2008

**3** A circle $C$ and a half-line $L$ have equations

$$|z - 2\sqrt{3} - \mathrm{i}| = 4$$

and $$\arg(z + \mathrm{i}) = \frac{\pi}{6}$$

respectively.

(a) Show that:

(i) the circle $C$ passes through the point where $z = -\mathrm{i}$; *(2 marks)*

(ii) the half-line $L$ passes through the centre of $C$. *(3 marks)*

(b) On one Argand diagram, sketch $C$ and $L$. *(4 marks)*

(c) Shade on your sketch the set of points satisfying both

$$|z - 2\sqrt{3} - \mathrm{i}| \leqslant 4$$

and $$0 \leqslant \arg(z + \mathrm{i}) \leqslant \frac{\pi}{6}$$ *(2 marks)*

June 2008

**4** (a) A circle $C$ in the Argand diagram has equation

$$|z + 5 - \mathrm{i}| = \sqrt{2}$$

Write down its radius and the complex number representing its centre. *(2 marks)*

(b) A half-line $L$ in the Argand diagram has equation

$$\arg(z + 2\mathrm{i}) = \frac{3\pi}{4}$$

Show that $z_1 = -4 + 2\mathrm{i}$ lies on $L$. *(2 marks)*

(c) (i) Show that $z_1 = -4 + 2\mathrm{i}$ also lies on $C$. *(1 mark)*

(ii) Hence show that $L$ touches $C$. *(3 marks)*

(iii) Sketch $L$ and $C$ on one Argand diagram. *(2 marks)*

(d) The complex number $z_2$ lies on $C$ and is such that $\arg(z_2 + 2\mathrm{i})$ has as great a value as possible.

Indicate the position of $z_2$ on your sketch. *(2 marks)*

January 2009

| | | | |
|---|---|---|---|
| **2** | (a) | Indicate on an Argand diagram the region for which $\lvert z - 4\mathrm{i} \rvert \leqslant 2$. | *(4 marks)* |
| | (b) | The complex number $z$ satisfies $\lvert z - 4\mathrm{i} \rvert \leqslant 2$. Find the range of possible values of $\arg z$. | *(4 marks)* |

June 2009

| | | | |
|---|---|---|---|
| **6** | (a) | Two points, $A$ and $B$, on an Argand diagram are represented by the complex numbers $2 + 3\mathrm{i}$ and $-4 - 5\mathrm{i}$ respectively. Given that the points $A$ and $B$ are at the ends of a diameter of a circle $C_1$, express the equation of $C_1$ in the form $\lvert z - z_0 \rvert = k$. | *(4 marks)* |
| | (b) | A second circle, $C_2$, is represented on the Argand diagram by the equation $\lvert z - 5 + 4\mathrm{i} \rvert = 4$. Sketch on one Argand diagram both $C_1$ and $C_2$. | *(3 marks)* |
| | (c) | The points representing the complex numbers $z_1$ and $z_2$ lie on $C_1$ and $C_2$ respectively and are such that $\lvert z_1 - z_2 \rvert$ has its maximum value. Find this maximum value, giving your answer in the form $a + b\sqrt{5}$. | *(5 marks)* |

January 2010

**2** (a) On the same Argand diagram, draw:

(i) the locus of points satisfying $\lvert z - 4 + 2\mathrm{i} \rvert = 4$; *(3 marks)*

(ii) the locus of points satisfying $\lvert z \rvert = \lvert z - 2\mathrm{i} \rvert$. *(3 marks)*

(b) Indicate on your sketch the set of points satisfying both

$$\lvert z - 4 + 2\mathrm{i} \rvert \leqslant 4$$

and

$$\lvert z \rvert \geqslant \lvert z - 2\mathrm{i} \rvert$$

*(2 marks)*

June 2010

**3** Two loci, $L_1$ and $L_2$, in an Argand diagram are given by

$$L_1 : |z + 1 + 3\mathrm{i}| = |z - 5 - 7\mathrm{i}|$$

$$L_2 : \arg z = \frac{\pi}{4}$$

**(a)** Verify that the point represented by the complex number $2 + 2\mathrm{i}$ is a point of intersection of $L_1$ and $L_2$. *(2 marks)*

**(b)** Sketch $L_1$ and $L_2$ on one Argand diagram. *(5 marks)*

**(c)** Shade on your Argand diagram the region satisfying

both $$|z + 1 + 3\mathrm{i}| \leqslant |z - 5 - 7\mathrm{i}|$$

and $$\frac{\pi}{4} \leqslant \arg z \leqslant \frac{\pi}{2}$$ *(2 marks)*

January 2011

**1 (a)** Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 4 + 3\mathrm{i}| = 5$$ *(3 marks)*

**(b) (i)** Indicate on your diagram the point $P$ representing $z_1$, where both

$$|z_1 - 4 + 3\mathrm{i}| = 5 \quad \text{and} \quad \arg z_1 = 0$$ *(1 mark)*

**(ii)** Find the value of $|z_1|$. *(1 mark)*

June 2011

**1 (a)** Draw on the same Argand diagram:

**(i)** the locus of points for which

$$|z - 2 - 5\mathrm{i}| = 5$$ *(3 marks)*

**(ii)** the locus of points for which

$$\arg(z + 2\mathrm{i}) = \frac{\pi}{4}$$ *(3 marks)*

**(b)** Indicate on your diagram the set of points satisfying both

$$|z - 2 - 5\mathrm{i}| \leqslant 5$$

and $$\arg(z + 2\mathrm{i}) = \frac{\pi}{4}$$ *(2 marks)*

January 2012

**2 (a)** Draw on an Argand diagram the locus $L$ of points satisfying the equation $\arg z = \frac{\pi}{6}$.
*(1 mark)*

**(b) (i)** A circle $C$, of radius 6, has its centre lying on $L$ and touches the line $\mathrm{Re}(z) = 0$. Draw $C$ on your Argand diagram from part **(a)**. *(2 marks)*

**(ii)** Find the equation of $C$, giving your answer in the form $|z - z_0| = k$. *(3 marks)*

**(iii)** The complex number $z_1$ lies on $C$ and is such that $\arg z_1$ has its least possible value. Find $\arg z_1$, giving your answer in the form $p\pi$, where $-1 < p \leqslant 1$. *(2 marks)*

June 2012

**2 (a)** Draw on the Argand diagram below:

**(i)** the locus of points for which

$$|z - 2 - 3\mathrm{i}| = 2$$ *(3 marks)*

**(ii)** the locus of points for which

$$|z + 2 - \mathrm{i}| = |z - 2|$$ *(3 marks)*

**(b)** Indicate on your diagram the points satisfying both

$$|z - 2 - 3\mathrm{i}| = 2$$

and $$|z + 2 - \mathrm{i}| \leqslant |z - 2|$$ *(1 mark)*

January 2013

**2** Two loci, $L_1$ and $L_2$, in an Argand diagram are given by

$$L_1 : |z + 6 - 5\mathrm{i}| = 4\sqrt{2}$$

$$L_2 : \quad \arg(z + \mathrm{i}) = \frac{3\pi}{4}$$

The point $P$ represents the complex number $-2 + \mathrm{i}$.

**(a)** Verify that the point $P$ is a point of intersection of $L_1$ and $L_2$. *(2 marks)*

**(b)** Sketch $L_1$ and $L_2$ on one Argand diagram. *(6 marks)*

**(c)** The point $Q$ is also a point of intersection of $L_1$ and $L_2$. Find the complex number that is represented by $Q$. *(2 marks)*

**1 (a)** Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 6\mathrm{i}| = 3$$

*(3 marks)*

**(b)** It is given that $z$ satisfies the equation $|z - 6\mathrm{i}| = 3$.

**(i)** Write down the greatest possible value of $|z|$. *(1 mark)*

**(ii)** Find the greatest possible value of $\arg z$, giving your answer in the form $p\pi$, where $-1 < p \leqslant 1$. *(3 marks)*