FP4: Determinants

Past Paper Questions
2006 - 2013

Name:

January 2006

6 (a) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$ *(5 marks)*

(b) (i) Hence, or otherwise, show that the system of equations

$$\begin{aligned} x + y + z &= p \\ 3x + 3y + 5z &= q \\ 15x + 15y + 9z &= r \end{aligned}$$

has no unique solution whatever the values of p, q and r. *(2 marks)*

(ii) Verify that this system is consistent when $24p - 3q - r = 0$. *(2 marks)*

(iii) Find the solution of the system in the case where $p = 1$, $q = 8$ and $r = 0$. *(5 marks)*

June 2006

3 Express the determinant $\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$ as the product of four linear factors. *(6 marks)*

January 2007

2 (a) Show that $(a-b)$ is a factor of the determinant

$$\Delta = \begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ bc & ca & ab \end{vmatrix}$$ *(2 marks)*

(b) Factorise Δ completely into linear factors. *(5 marks)*

June 2007

2 Factorise completely the determinant $\begin{vmatrix} y & x & x+y-1 \\ x & y & 1 \\ y+1 & x+1 & 2 \end{vmatrix}$. *(6 marks)*

June 2008

8 By considering the determinant

$$\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$$

show that $(x+y+z)$ is a factor of $x^3 + y^3 + z^3 - kxyz$ for some value of the constant k to be determined. *(3 marks)*

January 2009

5 (a) Expand the determinant

$$D = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ z & x & y \end{vmatrix}$$ *(2 marks)*

(b) Show that $(x+y+z)$ is a factor of the determinant

$$\Delta = \begin{vmatrix} x & y & z \\ y-z & z-x & x-y \\ x+z & y+x & z+y \end{vmatrix}$$ *(2 marks)*

(c) Show that $\Delta = k(x+y+z)D$ for some integer k. *(3 marks)*

June 2009

8 (a) Matrix $\mathbf{M} = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$. Without attempting to factorise, expand fully $\det \mathbf{M}$.

(2 marks)

(b) Matrix $\mathbf{N} = \begin{bmatrix} d & e & f \\ f & d & e \\ e & f & d \end{bmatrix}$. Find the product matrix $\mathbf{MN}$. *(3 marks)*

(c) Prove that the product

$$(a^3 + b^3 + c^3 - 3abc)(d^3 + e^3 + f^3 - 3def)$$

can be written in the form $x^3 + y^3 + z^3 - 3xyz$, stating clearly each of x, y and z in terms of a, b, c, d, e and f. *(2 marks)*

January 2010

7 (a) It is given that $\Delta = \begin{vmatrix} 16-q & 5 & 7 \\ -12 & -1-q & -7 \\ 6 & 6 & 10-q \end{vmatrix}$.

(i) By using row operations on the first two rows of Δ, show that $(4-q)$ is a factor of Δ. *(2 marks)*

(ii) Express Δ as the product of three linear factors. *(4 marks)*

(b) It is given that $\mathbf{M} = \begin{bmatrix} 16 & 5 & 7 \\ -12 & -1 & -7 \\ 6 & 6 & 10 \end{bmatrix}$.

(i) Verify that $\begin{bmatrix} 2 \\ 5 \\ -7 \end{bmatrix}$ is an eigenvector of $\mathbf{M}$ and state its corresponding eigenvalue. *(3 marks)*

(ii) For each of the other two eigenvalues of $\mathbf{M}$, find a corresponding eigenvector. *(7 marks)*

(c) The transformation T has matrix $\mathbf{M}$. Write down cartesian equations for any one of the invariant lines of T. *(2 marks)*

June 2010

5 Factorise fully the determinant $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$. *(8 marks)*

January 2011

1 Let $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$.

(a) Use a row operation to show that $(x+y+z)$ is a factor of Δ. *(2 marks)*

(b) Hence, or otherwise, express Δ as a product of linear factors. *(2 marks)*

June 2011

7 Let $\Delta = \begin{vmatrix} n(n+1) & n+1 & -1 \\ 0 & 1 & n \\ 1 & -(n+1) & 1 \end{vmatrix}$.

(a) (i) Show that (n^2+n+1) is a factor of Δ. *(2 marks)*

(ii) Hence, or otherwise, express Δ in factorised form. *(2 marks)*

(b) By expanding Δ directly, show that

$$\Delta = [n(n+1)]^2 + \mathrm{f}(n)$$

where $\mathrm{f}(n)$ can be expressed as the sum of two squares. *(2 marks)*

(c) Hence express the number 12 321 as the sum of three squares. *(2 marks)*

June 2012

3 Let $\Delta = \begin{vmatrix} yz & xz & xy \\ x & y & z \\ x^2 & -y^2 & z^2 \end{vmatrix}$.

(a) Show that $(y+z)$ is a factor of Δ. *(2 marks)*

(b) Factorise Δ as completely as possible. *(4 marks)*

January 2013

2 It is given that **A** and **B** are 3×3 matrices such that

$$\det(\mathbf{AB}) = 24 \quad \text{and} \quad \det(\mathbf{A}^{-1}) = -3$$

(a) State the value of $\det \mathbf{A}$. *(1 mark)*

(b) A three-dimensional shape S, with volume $20\,\text{cm}^3$, is transformed using matrix **B**.

Find the volume of the image of S. *(3 marks)*

June 2013

3 The determinant Δ is given by

$$\Delta = \begin{vmatrix} x^2-x & y^2-y & z^2-z \\ x & y & z \\ x^2+y^2+z^2 & x^2+y^2+z^2 & x^2+y^2+z^2 \end{vmatrix}$$

where x, y and z are distinct real numbers.

(a) Express Δ as a product of one quadratic factor and three linear factors. *(6 marks)*

(b) Deduce that $\Delta \neq 0$. *(2 marks)*