FP4: Invariant Lines

Past Paper Questions
2006 - 2013

Name:

January 2006

5 The transformation T maps (x, y) to (x', y'), where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) Describe the difference between *an invariant line* and *a line of invariant points* of T. *(1 mark)*

(b) Evaluate the determinant of the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and describe the geometrical significance of the result in relation to T. *(2 marks)*

(c) Show that T has a line of invariant points, and find a cartesian equation for this line. *(2 marks)*

(d) (i) Find the image of the point $(x, -x + c)$ under T. *(2 marks)*

(ii) Hence show that all lines of the form $y = -x + c$, where c is an arbitrary constant, are invariant lines of T. *(2 marks)*

(e) Describe the transformation T geometrically. *(3 marks)*

June 2006

4 The plane transformation T maps points (x, y) to points (x', y') such that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{where } \mathbf{A} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

(a) (i) State the line of invariant points of T. *(1 mark)*

(ii) Give a full geometrical description of T. *(2 marks)*

(b) Find $\mathbf{A}^2$, and hence give a full geometrical description of the single plane transformation given by the matrix $\mathbf{A}^2$. *(3 marks)*

January 2007

7 The transformation S is a shear with matrix $\mathbf{M} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$. Points (x, y) are mapped under S to image points (x', y') such that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) Find the equation of the line of invariant points of S. *(2 marks)*

(b) Show that all lines of the form $y = x + c$, where c is a constant, are invariant lines of S. *(3 marks)*

(c) Evaluate $\det \mathbf{M}$, and state the property of shears which is indicated by this result. *(2 marks)*

(d) Calculate, to the nearest degree, the acute angle between the line $y = -x$ and its image under S. *(3 marks)*

January 2008

3 A shear S is represented by the matrix $\mathbf{A} = \begin{bmatrix} p & q \\ -q & r \end{bmatrix}$, where p, q and r are constants.

(a) By considering one of the geometrical properties of a shear, explain why $pr + q^2 = 1$. *(2 marks)*

(b) Given that $p = 4$ and that the image of the point $(-1, 2)$ under S is $(2, -1)$, find:

(i) the value of q and the value of r; *(3 marks)*

(ii) the equation of the line of invariant points of S. *(3 marks)*

June 2009

4 (a) Show that the system of equations

$$\begin{aligned} 3x - y + 3z &= 11 \\ 4x + y - 5z &= 17 \\ 5x - 4y + 14z &= 16 \end{aligned}$$

does not have a unique solution and is consistent.

(You are not required to find any solutions to this system of equations.) *(4 marks)*

(b) A transformation T of three-dimensional space maps points (x, y, z) onto image points (x', y', z') such that

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x - y + 3z - 2 \\ 2x + 6y - 4z + 12 \\ 4x + 11y + 4z - 30 \end{bmatrix}$$

Find the coordinates of the invariant point of T. *(8 marks)*

January 2011

8 The plane transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$.

(a) The quadrilateral $ABCD$ has image $A'B'C'D'$ under T.

Evaluate $\det \mathbf{M}$ and describe the geometrical significance of both its sign and its magnitude in relation to $ABCD$ and $A'B'C'D'$. *(3 marks)*

(b) The line $y = px$ is a line of invariant points of T, and the line $y = qx$ is an invariant line of T.

Show that $p = \frac{1}{2}$ and determine the value of q. *(5 marks)*

(c) (i) Find the 2×2 matrix $\mathbf{R}$ which represents a reflection in the line $y = \frac{1}{2}x$. *(2 marks)*

(ii) Given that T is the composition of a shear, with matrix $\mathbf{S}$, followed by a reflection in the line $y = \frac{1}{2}x$, determine the matrix $\mathbf{S}$ and describe the shear as fully as possible. *(5 marks)*

June 2013

6 The plane transformation T is defined by

$$\mathrm{T}: \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) A shape has an area of 3 square units. Find the area of the shape after being transformed by T. *(2 marks)*

(b) (i) Find the equations of all the invariant lines of T. *(5 marks)*

(ii) State the equation of the line of invariant points of T. *(1 mark)*