FP4: Matrix Transformations

Past Paper Questions 2006 - 2013

Name:

January 2006

1 Describe the geometrical transformation defined by the matrix

$$\begin{bmatrix} 0.6 & 0 & 0.8 \\ 0 & 1 & 0 \\ -0.8 & 0 & 0.6 \end{bmatrix}$$

(3 marks)

June 2006

2 A transformation is represented by the matrix $\mathbf{A} = \begin{bmatrix} 0.28 & -0.96 & 0 \\ 0.96 & 0.28 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Evaluate det **A**. *(1 mark)*

(b) State the invariant line of the transformation. *(1 mark)*

(c) Give a full geometrical description of this transformation. *(3 marks)*

January 2007

4 The matrices $\mathbf{M}_\text{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\mathbf{M}_\text{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ represent the transformations A and B respectively.

(a) Give a full geometrical description of each of A and B. *(5 marks)*

(b) Transformation C is obtained by carrying out A followed by B.

(i) Find $\mathbf{M}_\text{C}$, the matrix of C. *(2 marks)*

(ii) Hence give a full geometrical description of the single transformation C. *(2 marks)*

June 2007

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & t \end{bmatrix}$$

(a) Find, in terms of t, the matrices:

(i) **AB**; *(3 marks)*

(ii) **BA**. *(2 marks)*

(b) Explain why **AB** is singular for all values of t. *(1 mark)*

(c) In the case when $t = -2$, show that the transformation with matrix **BA** is the combination of an enlargement, E, and a second transformation, F. Find the scale factor of E and give a full geometrical description of F. *(6 marks)*

January 2008

1 Give a full geometrical description of the transformation represented by each of the following matrices:

(a) $\begin{bmatrix} 0.8 & 0 & -0.6 \\ 0 & 1 & 0 \\ 0.6 & 0 & 0.8 \end{bmatrix}$; *(3 marks)*

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. *(2 marks)*

Jan 2009

8 The plane transformation T has matrix $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, and maps points (x, y) onto image points (X, Y) such that

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) (i) Find $\mathbf{A}^{-1}$. *(2 marks)*

(ii) Hence express each of x and y in terms of X and Y. *(2 marks)*

(b) Give a full geometrical description of T. *(5 marks)*

(c) Any plane curve with equation of the form $\frac{x^2}{p} + \frac{y^2}{q} = 1$, where p and q are distinct positive constants, is an ellipse.

(i) Show that the curve E with equation $6x^2 + y^2 = 3$ is an ellipse. *(1 mark)*

(ii) Deduce that the image of the curve E under T has equation

$$2X^2 + 4XY + 5Y^2 = 15$$ *(2 marks)*

(iii) Explain why the curve with equation $2x^2 + 4xy + 5y^2 = 15$ is an ellipse. *(1 mark)*

June 2009

2 (a) Write down the 3×3 matrices which represent the transformations A and B, where:

(i) A is a reflection in the plane $y = x$; *(2 marks)*

(ii) B is a rotation about the z-axis through the angle θ, where $\theta = \frac{\pi}{2}$. *(1 mark)*

(b) (i) Find the matrix $\mathbf{R}$ which represents the composite transformation

'A followed by B' *(3 marks)*

(ii) Describe the single transformation represented by $\mathbf{R}$. *(2 marks)*

January 2010

1 The 2×2 matrix $\mathbf{M}$ represents the plane transformation T. Write down the value of $\det \mathbf{M}$ in each of the following cases:

(a) T is a rotation;

(b) T is a reflection;

(c) T is a shear;

(d) T is an enlargement with scale factor 3. *(4 marks)*

5 The plane transformations T_A and T_B are represented by the matrices $\mathbf{A}$ and $\mathbf{B}$ respectively, where $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 5 & 13 \end{bmatrix}$.

(a) Find the equation of the line which is the image of $y = 2x + 1$ under T_A. *(3 marks)*

(b) The rectangle $PQRS$, with area 4.5 cm^2, is mapped onto the parallelogram $P'Q'R'S'$ under T_B. Determine the area of $P'Q'R'S'$. *(2 marks)*

(c) The transformation T_C is the composition

'T_B followed by T_A'

By finding the matrix which represents T_C, give a full geometrical description of T_C. *(5 marks)*

June 2010

8 The matrix $\begin{bmatrix} 12 & 16 \\ -9 & 36 \end{bmatrix}$ represents the transformation which is the composition, in either order, of the two plane transformations

E: an enlargement, centre O and scale factor k $(k > 0)$

and

S: a shear parallel to the line l which passes through O

Show that $k = 24$ and find a cartesian equation for l. *(7 marks)*

June 2011

2 The plane transformation T is the composition of a reflection in the line $y = x \tan \alpha$ followed by an anticlockwise rotation about O through an angle β.

Determine the matrix which represents T, and hence describe T as a single transformation. *(6 marks)*

2 Describe the single transformation represented by each of the matrices:

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; *(2 marks)*

(b) $\begin{bmatrix} 0.6 & 0 & -0.8 \\ 0 & 1 & 0 \\ 0.8 & 0 & 0.6 \end{bmatrix}$. *(3 marks)*

7 The plane transformation T is a rotation through θ radians anticlockwise about O, and maps points (x, y) onto image points (X, Y) such that

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

where $c = \cos\theta$ and $s = \sin\theta$.

(a) Write down the inverse of the matrix $\begin{bmatrix} c & -s \\ s & c \end{bmatrix}$ and hence show that

$$x = cX + sY \quad \text{and} \quad y = -sX + cY$$ *(3 marks)*

(b) The curve C has equation $x^2 - 6xy - 7y^2 = 8$.

The image of C under T is the curve C' with equation $pX^2 + qXY + rY^2 = 8$.

(i) Use the results of part **(a)** to show that

$$q = 6s^2 + 16sc - 6c^2$$

and express p and r similarly in terms of c and s. *(4 marks)*

(ii) Given that θ is an acute angle, find the values of c and s for which $q = 0$ and hence in this case express the equation of C' in the form

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$$ *(8 marks)*

(iii) Hence explain why C is a hyperbola. *(1 mark)*

6 The linear transformations T_1 and T_2 are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } \mathbf{M}_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

respectively.

(a) Give a full geometrical description of the transformations:

(i) T_1 ; *(2 marks)*

(ii) T_2 . *(3 marks)*

(b) Find the matrix which represents the transformation T_1 followed by T_2 . *(2 marks)*

(c) The linear transformation T_3 is represented by the matrix

$$\mathbf{M}_3 = \begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

where k is a constant.

For one particular value of k, T_3 has a line L of invariant points.

(i) Find k.

(ii) Find the Cartesian equations of L in the form $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$. *(7 marks)*