FP4: Systems of Equations

Past Paper Questions
2006 - 2013

Name:

January 2006

6 (a) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$ *(5 marks)*

(b) (i) Hence, or otherwise, show that the system of equations

$$\begin{aligned} x + y + z &= p \\ 3x + 3y + 5z &= q \\ 15x + 15y + 9z &= r \end{aligned}$$

has no unique solution whatever the values of p, q and r. *(2 marks)*

(ii) Verify that this system is consistent when $24p - 3q - r = 0$. *(2 marks)*

(iii) Find the solution of the system in the case where $p = 1$, $q = 8$ and $r = 0$. *(5 marks)*

June 2006

5 A set of three planes is given by the system of equations

$$\begin{aligned} x + 3y - z &= 10 \\ 2x + ky + z &= -4 \\ 3x + 5y + (k-2)z &= k+4 \end{aligned}$$

where k is a constant.

(a) Show that $\begin{vmatrix} 1 & 3 & -1 \\ 2 & k & 1 \\ 3 & 5 & k-2 \end{vmatrix} = k^2 - 5k + 6$. *(2 marks)*

(b) In each of the following cases, determine the **number** of solutions of the given system of equations.

(i) $k = 1$.

(ii) $k = 2$.

(iii) $k = 3$. *(7 marks)*

(c) Give a geometrical interpretation of the significance of each of the three results in part (b) in relation to the three planes. *(3 marks)*

January 2007

1 Show that the system of equations

$$\begin{aligned} x + 2y - \ \ z &= 0 \\ 3x - \ \ y + 4z &= 7 \\ 8x + \ \ y + 7z &= 30 \end{aligned}$$

is inconsistent. *(4 marks)*

June 2007

4 Consider the following system of equations, where k is a real constant:

$$\begin{aligned} kx + 2y + z &= 5 \\ x + (k+1)y - 2z &= 3 \\ 2x - ky + 3z &= -11 \end{aligned}$$

(a) Show that the system does not have a unique solution when $k^2 = 16$. *(3 marks)*

(b) In the case when $k = 4$, show that the system is inconsistent. *(4 marks)*

(c) In the case when $k = -4$:

(i) solve the system of equations; *(5 marks)*

(ii) interpret this result geometrically. *(1 mark)*

January 2008

5 A system of equations is given by

$$\begin{aligned} x + 3y + 5z &= -2 \\ 3x - 4y + 2z &= 7 \\ ax + 11y + 13z &= b \end{aligned}$$

where a and b are constants.

(a) Find the unique solution of the system in the case when $a = 3$ and $b = 2$. *(5 marks)*

(b) (i) Determine the value of a for which the system does not have a unique solution. *(3 marks)*

(ii) For this value of a, find the value of b such that the system of equations is consistent. *(4 marks)*

June 2008

6 Three planes have equations

$$\begin{aligned} x + y - 3z &= b \\ 2x + y + 4z &= 3 \\ 5x + 2y + az &= 4 \end{aligned}$$

where a and b are constants.

(a) Find the coordinates of the single point of intersection of these three planes in the case when $a = 16$ and $b = 6$. *(5 marks)*

(b) (i) Find the value of a for which the three planes do not meet at a single point. *(3 marks)*

(ii) For this value of a, determine the value of b for which the three planes share a common line of intersection. *(5 marks)*

January 2009

7 Two fixed planes have equations

$$\begin{aligned} x - 2y + z &= -1 \\ -x + y + 3z &= 3 \end{aligned}$$

(a) The point P, whose z-coordinate is λ, lies on the line of intersection of these two planes. Find the x- and y-coordinates of P in terms of λ. *(3 marks)*

(b) The point P also lies on the variable plane with equation $5x + ky + 17z = 1$. Show that

$$(k + 13)(2\lambda - 1) = 0$$ *(3 marks)*

(c) For the system of equations

$$\begin{aligned} x - 2y + z &= -1 \\ -x + y + 3z &= 3 \\ 5x + ky + 17z &= 1 \end{aligned}$$

determine the solution(s), if any, of the system, and their geometrical significance in relation to the three planes, in the cases:

(i) $k = -13$;

(ii) $k \neq -13$. *(6 marks)*

June 2009

4 (a) Show that the system of equations

$$\begin{aligned} 3x - y + 3z &= 11 \\ 4x + y - 5z &= 17 \\ 5x - 4y + 14z &= 16 \end{aligned}$$

does not have a unique solution and is consistent.

(You are not required to find any solutions to this system of equations.) *(4 marks)*

(b) A transformation T of three-dimensional space maps points (x, y, z) onto image points (x', y', z') such that

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x - y + 3z - 2 \\ 2x + 6y - 4z + 12 \\ 4x + 11y + 4z - 30 \end{bmatrix}$$

Find the coordinates of the invariant point of T. *(8 marks)*

January 2010

4 (a) Determine the two values of k for which the system of equations

$$\begin{aligned} x - 2y + kz &= 5 \\ (k+1)x + 3y &= k \\ 2x + y + (k-1)z &= 3 \end{aligned}$$

does not have a unique solution. *(4 marks)*

(b) Show that this system of equations is consistent for one of these values of k, but is inconsistent for the other.

(You are not required to find any solutions to this system of equations.) *(8 marks)*

June 2010

6 The line L and the plane Π have vector equations

$$\mathbf{r} = \begin{bmatrix} 7 \\ 8 \\ 50 \end{bmatrix} + t\begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} -2 \\ 0 \\ -25 \end{bmatrix} + \lambda\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} + \mu\begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$$

respectively.

(a) (i) Find direction cosines for L. *(2 marks)*

(ii) Show that L is perpendicular to Π. *(3 marks)*

(b) For the system of equations

$$\begin{aligned} 6p + 5q + r &= 9 \\ 2p + 3q + 6r &= 8 \\ -9p + 4q + 2r &= 75 \end{aligned}$$

form a pair of equations in p and q only, and hence find the unique solution of this system of equations. *(5 marks)*

(c) It is given that L meets Π at the point P.

(i) Demonstrate how the coordinates of P may be obtained from the system of equations in part **(b)**. *(2 marks)*

(ii) Hence determine the coordinates of P. *(2 marks)*

January 2011

3 (a) Find the values of t for which the system of equations

$$\begin{aligned} tx + 2y + 3z &= a \\ 2x + 3y - tz &= b \\ 3x + 5y + (t+1)z &= c \end{aligned}$$

does not have a unique solution. *(3 marks)*

(b) For the integer value of t found in part **(a)**, find the relationship between a, b and c such that this system of equations is consistent. *(3 marks)*

June 2011

4 The system of equations S is given in terms of the real parameters a and b by

$$\begin{aligned} 2x + y + 3z &= a + 1 \\ 5x - 2y + (a+1)z &= 3 \\ ax + 2y + 4z &= b \end{aligned}$$

(a) Find the two values of a for which S does not have a unique solution. *(4 marks)*

(b) In the case when $a = 2$, determine the value of b for which S has infinitely many solutions. *(4 marks)*

January 2012

5 (a) Determine the two values of the integer n for which the system of equations

$$\begin{aligned} 2x + ny + z &= 5 \\ 3x - y + nz &= 1 \\ -x + 7y + z &= n \end{aligned}$$

does not have a unique solution. *(4 marks)*

(b) For the positive value of n found in part **(a)**, determine whether the system is consistent or inconsistent, and interpret this result geometrically. *(6 marks)*

June 2012

6 The planes Π_1, Π_2 and Π_3 have cartesian equations

$$\begin{aligned} 2x + y - z &= 3 \\ 3x - 2y + z &= 5 \\ 12x - y - z &= 40 \end{aligned}$$

respectively.

(a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$, a vector equation for the line L which is the intersection of Π_1 and Π_2. *(5 marks)*

(b) (i) Determine whether L meets Π_3, and use your answer to decide whether the system given by the equations of these three planes is consistent or inconsistent. *(3 marks)*

(ii) Describe geometrically the arrangement of the three planes. *(1 mark)*

(c) (i) Find the coordinates of a common point of Π_2 and Π_3. *(3 marks)*

(ii) **Deduce** a vector equation for the line of intersection of Π_2 and Π_3. *(1 mark)*

January 2013

5 (a) By direct expansion, or otherwise, show that the value of $\begin{vmatrix} -2 & 1 & 2k \\ -1 & 1 & k+1 \\ 2 & k-1 & 1 \end{vmatrix}$ is independent of k. *(4 marks)*

(b) State, with a reason, whether the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

are linearly dependent or linearly independent. *(2 marks)*

(c) (i) State, with a reason, whether the equations

$$\begin{aligned} -2x + y + 6z &= 1 \\ -x + y + 4z &= 0 \\ 2x + 2y + z &= -1 \end{aligned}$$

are consistent or inconsistent. *(2 marks)*

(ii) The three equations given in part **(c)(i)** are the Cartesian equations of three planes.

State the geometrical configuration of these three planes. *(1 mark)*

June 2013

2 The system of equations

$$\begin{aligned} 2x - y - z &= 3 \\ x + 2y - 3z &= 4 \\ 2x + y + az &= b \end{aligned}$$

does not have a unique solution.

(a) Show that $a = -3$. *(3 marks)*

(b) Given further that the equations are inconsistent, find the possible values of b. *(2 marks)*

(c) State, with a reason, whether the vectors $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}$ are linearly dependent or linearly independent. *(1 mark)*