FP4: Vector and Scalar Products

Past Paper Questions
2006 - 2013

Name:

January 2006

4 The vectors **a**, **b** and **c** are given by

$$\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \mathbf{c} = 4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

(a) (i) Evaluate $\begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & -1 \\ 4 & -1 & 5 \end{vmatrix}$. *(2 marks)*

(ii) Hence determine whether **a**, **b** and **c** are linearly dependent or independent. *(1 mark)*

(b) (i) Evaluate $\mathbf{b} \cdot \mathbf{c}$. *(2 marks)*

(ii) Show that $\mathbf{b} \times \mathbf{c}$ can be expressed in the form $m\mathbf{a}$, where m is a scalar. *(2 marks)*

(iii) Use these results to describe the geometrical relationship between **a**, **b** and **c**. *(1 mark)*

(c) The points A, B and C have position vectors **a**, **b** and **c** respectively relative to an origin O. The points O, A, B and C are four of the eight vertices of a cuboid. Determine the volume of this cuboid. *(2 marks)*

January 2007

3 The points P, Q and R have position vectors **p**, **q** and **r** respectively relative to an origin O, where

$$\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \ \mathbf{q} = \begin{bmatrix} -3 \\ 4 \\ 20 \end{bmatrix} \text{ and } \mathbf{r} = \begin{bmatrix} 9 \\ 2 \\ 4 \end{bmatrix}$$

(a) (i) Determine $\mathbf{p} \times \mathbf{q}$. *(2 marks)*

(ii) Find the area of triangle OPQ. *(3 marks)*

(b) Use the scalar triple product to show that **p**, **q** and **r** are linearly dependent, and interpret this result geometrically. *(3 marks)*

June 2007

1 Given that $\mathbf{a} \times \mathbf{b} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ and that $\mathbf{a} \times \mathbf{c} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, determine:

(a) $\mathbf{c} \times \mathbf{a}$; *(1 mark)*

(b) $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$; *(2 marks)*

(c) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{c})$; *(2 marks)*

(d) $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{c})$. *(1 mark)*

3 Three points, A, B and C, have position vectors

$$\mathbf{a} = \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

respectively.

(a) Using the scalar triple product, or otherwise, show that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar. *(2 marks)*

(b) (i) Calculate $(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$. *(3 marks)*

(ii) Hence find, to three significant figures, the area of the triangle ABC. *(3 marks)*

January 2008

2 It is given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + 4\mathbf{j} + 28\mathbf{k}$.

(a) Determine:

(i) $\mathbf{a} \cdot \mathbf{b}$; *(1 mark)*

(ii) $\mathbf{a} \times \mathbf{b}$; *(2 marks)*

(iii) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. *(2 marks)*

(b) Describe the geometrical relationship between the vectors:

(i) $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$; *(1 mark)*

(ii) $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. *(1 mark)*

June 2008

2 The vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are given by

$$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \mathbf{c} = -2\mathbf{i} + t\mathbf{j} + 6\mathbf{k}$$

where t is a scalar constant.

(a) Determine, in terms of t where appropriate:

(i) $\mathbf{a} \times \mathbf{b}$; *(2 marks)*

(ii) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$; *(2 marks)*

(iii) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$. *(2 marks)*

(b) Find the value of t for which $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are linearly dependent. *(2 marks)*

(c) Find the value of t for which $\mathbf{c}$ is parallel to $\mathbf{a} \times \mathbf{b}$. *(2 marks)*

January 2009

3 The points X, Y and Z have position vectors

$$\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 5 \\ 7 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{z} = \begin{bmatrix} -8 \\ 1 \\ a \end{bmatrix}$$

respectively, relative to the origin O.

(a) Find:

(i) $\mathbf{x} \times \mathbf{y}$; *(2 marks)*

(ii) $(\mathbf{x} \times \mathbf{y}) \cdot \mathbf{z}$. *(2 marks)*

(b) Using these results, or otherwise, find:

(i) the area of triangle OXY; *(2 marks)*

(ii) the value of a for which $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ are linearly dependent. *(2 marks)*

June 2010

1 The position vectors of the points P, Q and R are, respectively,

$$\mathbf{p} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

(a) Show that $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are linearly dependent. *(2 marks)*

(b) Determine the area of triangle PQR. *(4 marks)*

January 2011

2 The non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ have magnitudes a and b respectively.

Let $c = |\mathbf{a} \times \mathbf{b}|$ and $d = |\mathbf{a} \cdot \mathbf{b}|$.

By considering the definitions of the vector and scalar products, or otherwise, show that

$$c^2 + d^2 = a^2 b^2$$ *(3 marks)*

June 2011

3 Given the vectors $\mathbf{p} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix}$, where t is a scalar parameter, determine the value of t in each of the following cases:

(a) $\mathbf{p} \times \mathbf{q}$ is parallel to $\mathbf{r}$; *(3 marks)*

(b) $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are linearly dependent. *(3 marks)*

January 2012

1	The vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a}.\mathbf{b} = 21$, $\lvert\mathbf{a}\rvert = 5\sqrt{2}$ and $\lvert\mathbf{b}\rvert = 3$. Determine the exact value of $\lvert\mathbf{a} \times \mathbf{b}\rvert$.	*(5 marks)*

8	For $n \neq 1$, the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are such that $\mathbf{a} = \begin{bmatrix} 1 \\ n \\ n^2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2n \\ 2n^2 + n \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} n-1 \\ n^2-1 \\ 1-n^2 \end{bmatrix}$ Determine the value of n for which $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are linearly dependent.	*(9 marks)*

June 2012

1	Find the value of the constant p for which the vectors $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + p\mathbf{k}, \quad \mathbf{v} = 7\mathbf{i} - \mathbf{j} + 6\mathbf{k} \quad \text{and} \quad \mathbf{w} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ are linearly dependent.	*(3 marks)*