Core 3: Functions

Past Paper Questions
2006 - 2013

Name:

January 2006

8 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 4$. *(4 marks)*

(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

June 2006

4 (a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; *(1 mark)*

(ii) $y = |2x - 4|$. *(2 marks)*

(b) (i) Solve the equation $|x| = |2x - 4|$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. *(2 marks)*

8 A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x.

(a) Find the range of f. *(2 marks)*

(b) Show that $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x+1}{2}\right)$. *(3 marks)*

(c) Find the gradient of the curve $y = f^{-1}(x)$ when $x = 0$. *(4 marks)*

January 2007

3 The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \quad \text{for real values of } x, \ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

(i) Find $h(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

June 2007

5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x-2} \quad \text{for } x \geqslant 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \ x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 1$. *(3 marks)*

(c) The inverse of f is f^{-1}. Find $f^{-1}(x)$. *(3 marks)*

January 2008

4 The functions f and g are defined with their respective domains by

$$f(x) = x^3, \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x-3}, \qquad \text{for real values of } x,\ x \neq 3$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 64$. *(3 marks)*

(c) (i) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 - 5$. *(4 marks)*

(b) Sketch the graph of $y = |4x^2 - 5|$, indicating the coordinates of the point where the curve crosses the y-axis. *(3 marks)*

(c) (i) Solve the equation $|4x^2 - 5| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4x^2 - 5| \geqslant 4$. *(2 marks)*

June 2008

4 The functions f and g are defined with their respective domains by

$$f(x) = x^2, \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x-3}, \qquad \text{for real values of } x,\ x \neq \frac{3}{2}$$

(a) State the range of f. *(1 mark)*

(b) (i) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) Solve the equation $fg(x) = 9$. *(3 marks)*

January 2009

5 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = 2 - x^4 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x-4} \qquad \text{for real values of } x,\ x \neq 4$$

(a) State the range of f. *(2 marks)*

(b) Explain why the function f does not have an inverse. *(1 mark)*

(c) (i) Write down an expression for $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = -14$. *(3 marks)*

June 2009

2 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x+5}, \qquad \text{for real values of } x,\ x \geqslant -2.5$$

$$\mathrm{g}(x) = \frac{1}{4x+1}, \qquad \text{for real values of } x,\ x \neq -0.25$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the domain of f^{-1}. *(1 mark)*

(c) The composite function fg is denoted by h.

(i) Find an expression for $\mathrm{h}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{h}(x) = 3$. *(3 marks)*

4 (a) Sketch the graph of $y = |50 - x^2|$, indicating the coordinates of the point where the graph crosses the y-axis. *(3 marks)*

(b) Solve the equation $|50 - x^2| = 14$. *(3 marks)*

(c) Hence, or otherwise, solve the inequality $|50 - x^2| > 14$. *(2 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 50 - x^2$. *(4 marks)*

January 2010

4 (a) Sketch the graph of $y = |8 - 2x|$. *(2 marks)*

(b) Solve the equation $|8 - 2x| = 4$. *(2 marks)*

(c) Solve the inequality $|8 - 2x| > 4$. *(2 marks)*

6 The functions f and g are defined with their respective domains by

$$f(x) = e^{2x} - 3, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{3x + 4}, \quad \text{for real values of } x, \ x \neq -\frac{4}{3}$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $f^{-1}(x) = 0$. *(2 marks)*

(c) (i) Find an expression for $gf(x)$. *(1 mark)*

(ii) Solve the equation $gf(x) = 1$, giving your answer in an exact form. *(3 marks)*

January 2011

4 The functions f and g are defined with their respective domains by

$$f(x) = 3\cos\tfrac{1}{2}x, \quad \text{for } 0 \leqslant x \leqslant 2\pi$$

$$g(x) = |x|, \quad \text{for all real values of } x$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $f^{-1}(x) = 1$, giving your answer in an exact form. *(2 marks)*

(c) (i) Write down an expression for $gf(x)$. *(1 mark)*

(ii) Sketch the graph of $y = gf(x)$ for $0 \leqslant x \leqslant 2\pi$. *(3 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos x$ onto the graph of $y = 3\cos\tfrac{1}{2}x$. *(3 marks)*

June 2011

5 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x+1} \qquad \text{for real values of } x, \quad x \neq -0.5$$

(a) Explain why f does not have an inverse. *(1 mark)*

(b) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(c) State the range of g^{-1}. *(1 mark)*

(d) Solve the equation $fg(x) = g(x)$. *(3 marks)*

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7 (a) On separate diagrams:

(i) sketch the curve with equation $y = |3x + 3|$; *(2 marks)*

(ii) sketch the curve with equation $y = |x^2 - 1|$. *(3 marks)*

(b) (i) Solve the equation $|3x + 3| = |x^2 - 1|$. *(5 marks)*

(ii) Hence solve the inequality $|3x + 3| < |x^2 - 1|$. *(2 marks)*

June 2012

5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{2x - 5}, \qquad \text{for } x \geqslant 2.5$$

$$g(x) = \frac{10}{x}, \qquad \text{for real values of } x, \quad x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

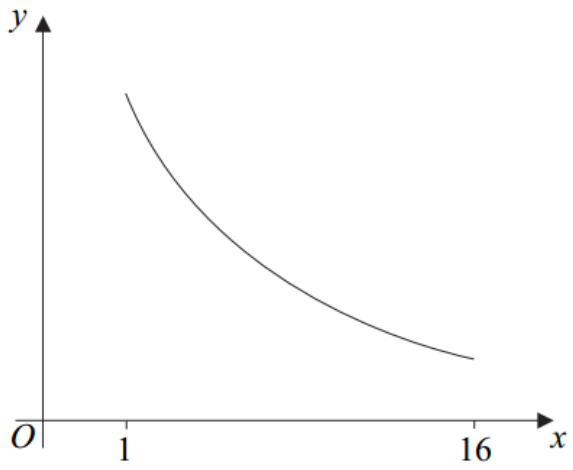
(ii) Solve the equation $fg(x) = 5$. *(2 marks)*

(c) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $f^{-1}(x) = 7$. *(2 marks)*

2 The curve with equation $y = \dfrac{63}{4x - 1}$ is sketched below for $1 \leqslant x \leqslant 16$.



The function f is defined by $\mathrm{f}(x) = \dfrac{63}{4x - 1}$ for $1 \leqslant x \leqslant 16$.

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 1$. *(2 marks)*

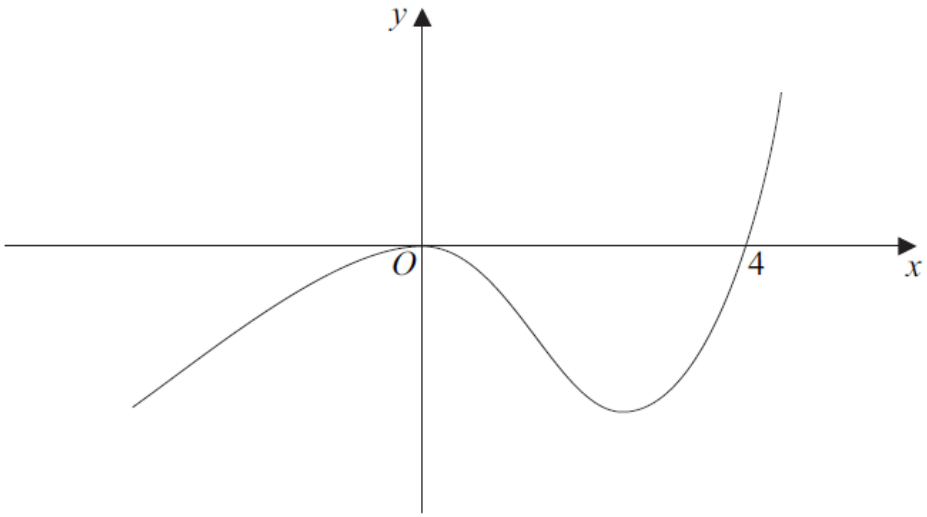
(c) The function g is defined by $\mathrm{g}(x) = x^2$ for $-4 \leqslant x \leqslant -1$.

(i) Write down an expression for $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 1$. *(3 marks)*

January 2013

4 The diagram shows a sketch of the curve with equation $y = f(x)$.



(a) On the axes below, sketch the curve with equation $y = |f(x)|$. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = f(2x - 1)$. *(4 marks)*

5 The function f is defined by

$$f(x) = \frac{x^2 - 4}{3}, \quad \text{for real values of } x, \textbf{ where } \boldsymbol{x \leqslant 0}$$

(a) State the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

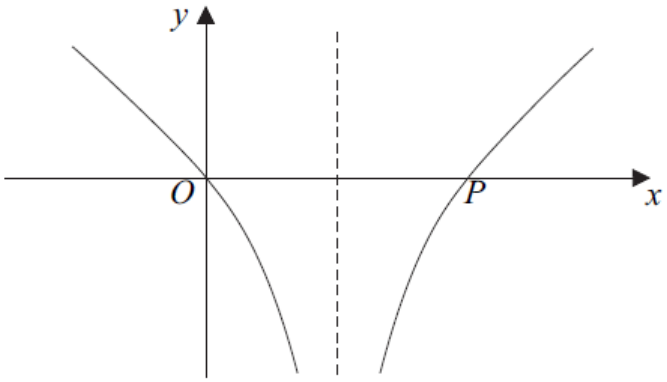
(i) Write down the domain of f^{-1}. *(1 mark)*

(ii) Find an expression for $f^{-1}(x)$. *(3 marks)*

(c) The function g is defined by

$$g(x) = \ln|3x - 1|, \quad \text{for real values of } x, \text{ where } x \neq \frac{1}{3}$$

The curve with equation $y = g(x)$ is sketched below.



(i) The curve $y = g(x)$ intersects the x-axis at the origin and at the point P.

Find the x-coordinate of P. *(2 marks)*

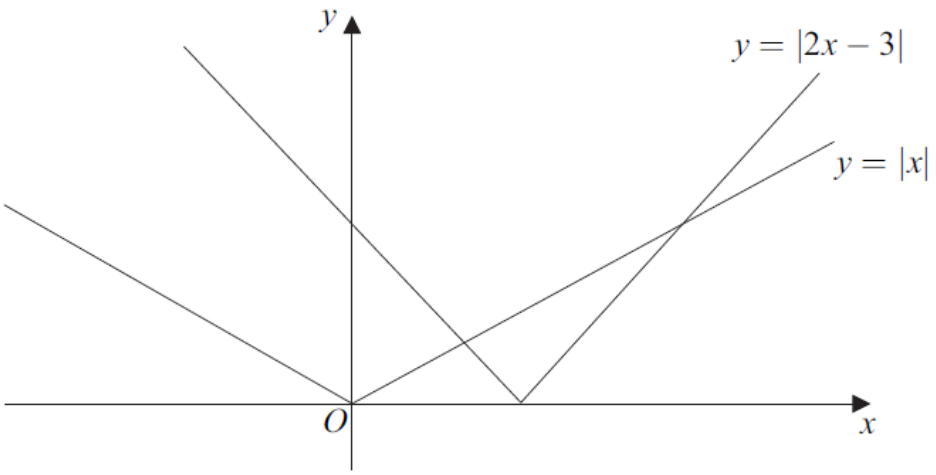
(ii) State whether the function g has an inverse. Give a reason for your answer. *(1 mark)*

(iii) Show that $gf(x) = \ln|x^2 - k|$, stating the value of the constant k. *(2 marks)*

(iv) Solve the equation $gf(x) = 0$. *(4 marks)*

June 2013

1 The diagram below shows the graphs of $y = |2x - 3|$ and $y = |x|$.

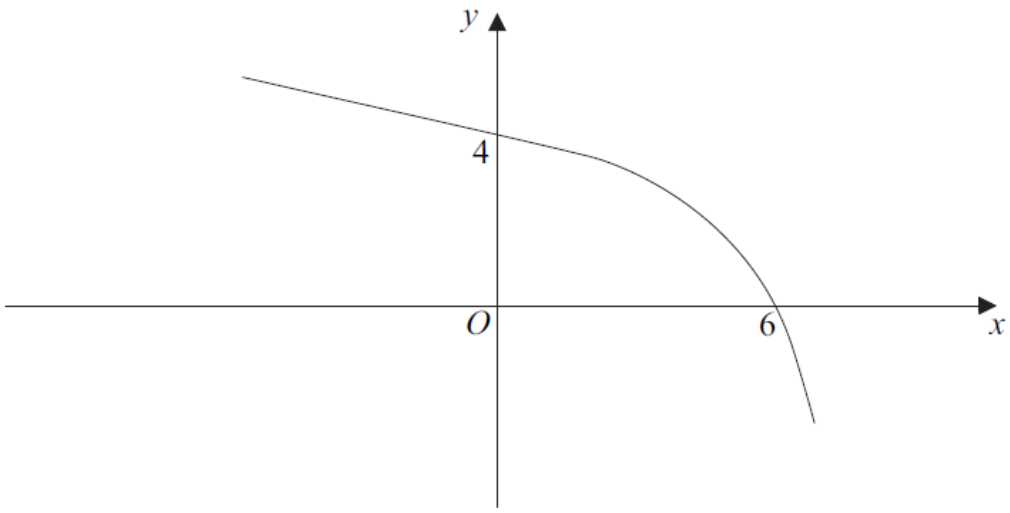


(a) Find the x-coordinates of the points of intersection of the graphs of $y = |2x - 3|$ and $y = |x|$. *(3 marks)*

(b) Hence, or otherwise, solve the inequality

$$|2x - 3| \geqslant |x| \qquad \textit{(2 marks)}$$

7 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



(a) sketch the curve with equation $y = -\mathrm{f}(3x)$, indicating the values where the curve cuts the coordinate axes. *(2 marks)*

(b) sketch the curve with equation $y = \mathrm{f}(|x|)$, indicating the values where the curve cuts the coordinate axes. *(3 marks)*

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}\left(-\frac{1}{2}x\right)$. *(4 marks)*