Decision 1: Graphs

Past Paper Questions
2006 - 2013

Name:

June 2006

7 A connected graph **G** has m vertices and n edges.

(a) (i) Write down the number of edges in a minimum spanning tree of **G**. *(1 mark)*

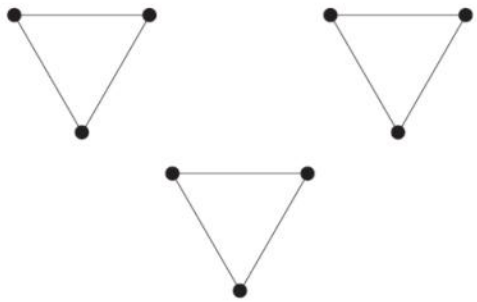
(ii) Hence write down an inequality relating m and n. *(2 marks)*

(b) The graph **G** contains a Hamiltonian cycle. Write down the number of edges in this cycle. *(1 mark)*

(c) In the case where **G** is Eulerian, draw a graph of **G** for which $m = 6$ and $n = 12$. *(2 marks)*

January 2007

8 (a) The diagram shows a graph **G** with 9 vertices and 9 edges.



(i) State the minimum number of edges that need to be added to **G** to make a connected graph. Draw an example of such a graph. *(2 marks)*

(ii) State the minimum number of edges that need to be added to **G** to make the graph Hamiltonian. Draw an example of such a graph. *(2 marks)*

(iii) State the minimum number of edges that need to be added to **G** to make the graph Eulerian. Draw an example of such a graph. *(2 marks)*

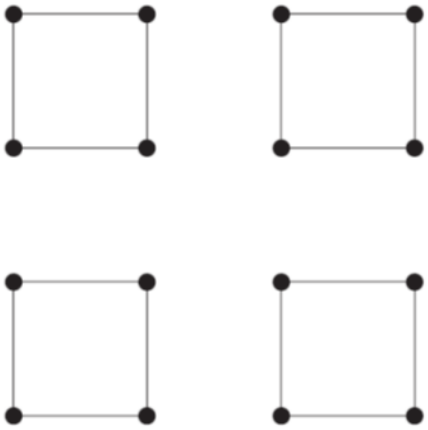
(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

(ii) In addition, the number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(1 mark)*

June 2009

7 (a) The diagram shows a graph with 16 vertices and 16 edges.



(i) On **Figure 1** below, add the minimum number of edges to make a connected graph. *(1 mark)*

(ii) On **Figure 2** opposite, add the minimum number of edges to make the graph Hamiltonian. *(2 marks)*

(iii) On **Figure 3** opposite, add the minimum number of edges to make the graph Eulerian. *(2 marks)*

Figure 1

Connected Graph

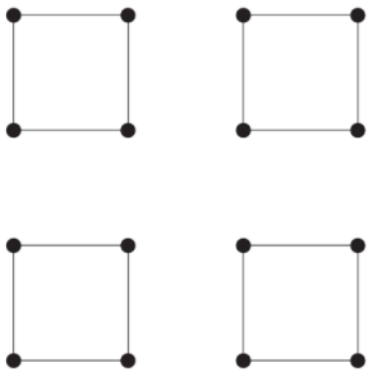


Figure 2

Hamiltonian Graph

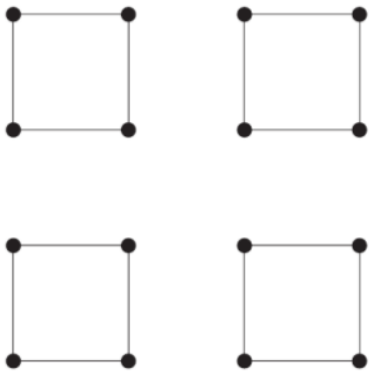
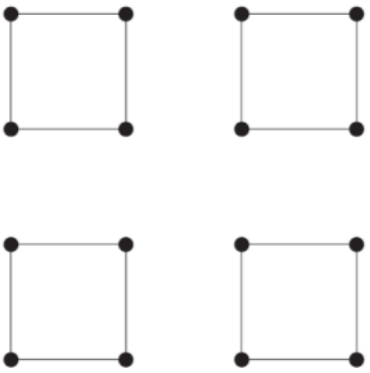


Figure 3

Eulerian Graph



June 2010

8 A simple connected graph has six vertices.

(a) One vertex has degree x. State the greatest and least possible values of x. *(2 marks)*

(b) The six vertices have degrees

$$x-2,\ x-2,\ x,\ 2x-4,\ 2x-4,\ 4x-12$$

Find the value of x, justifying your answer. *(2 marks)*

January 2011

6 (a) The complete graph K_n has every one of its n vertices connected to each of the other vertices by a single edge.

(i) Find the total number of edges in the graph K_5. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for the graph K_5. *(1 mark)*

(iii) State the number of edges in a Hamiltonian cycle for the graph K_5. *(1 mark)*

(b) A simple graph G has six vertices and nine edges, and G is Eulerian. Draw a sketch to show a possible graph G. *(2 marks)*

June 2012

6 The complete graph K_n $(n > 1)$ has every one of its n vertices connected to each of the other vertices by a single edge.

(a) Draw the complete graph K_4. *(1 mark)*

(b) (i) Find the total number of edges for the graph K_8.

(ii) Give a reason why K_8 is not Eulerian. *(2 marks)*

(c) For the graph K_n, state in terms of n:

(i) the total number of edges;

(ii) the number of edges in a minimum spanning tree;

(iii) the condition for K_n to be Eulerian;

(iv) the condition for the number of edges of a Hamiltonian cycle to be equal to the number of edges of an Eulerian cycle. *(4 marks)*

7 (a) A simple connected graph X has eight vertices.

(i) State the minimum number of edges of the graph.

(ii) Find the maximum number of edges of the graph. *(2 marks)*

(b) A simple connected graph Y has n vertices.

(i) State the minimum number of edges of the graph.

(ii) Find the maximum number of edges of the graph. *(2 marks)*

(c) A simple graph Z has six vertices and each of the vertices has the same degree d.

(i) State the possible values of d.

(ii) If Z is connected, state the possible values of d.

(iii) If Z is Eulerian, state the possible values of d. *(4 marks)*