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# Core 4: Implicit Differentiation

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Past Paper Questions  
2006 - 2013

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Name:

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June 2006

**5** A curve is defined by the equation

$$y^2 - xy + 3x^2 - 5 = 0$$

- (a) Find the  $y$ -coordinates of the two points on the curve where  $x = 1$ . *(3 marks)*
- (b) (i) Show that  $\frac{dy}{dx} = \frac{y - 6x}{2y - x}$ . *(6 marks)*
- (ii) Find the gradient of the curve at each of the points where  $x = 1$ . *(2 marks)*
- (iii) Show that, at the two stationary points on the curve,  $33x^2 - 5 = 0$ . *(3 marks)*

June 2007

**5** The point  $P(1, a)$ , where  $a > 0$ , lies on the curve  $y + 4x = 5x^2y^2$ .

- (a) Show that  $a = 1$ . *(2 marks)*
- (b) Find the gradient of the curve at  $P$ . *(7 marks)*
- (c) Find an equation of the tangent to the curve at  $P$ . *(1 mark)*

January 2008

**6** A curve has equation  $3xy - 2y^2 = 4$ .

Find the gradient of the curve at the point  $(2, 1)$ . *(5 marks)*

January 2009

**6** A curve is defined by the equation  $x^2y + y^3 = 2x + 1$ .

- (a) Find the gradient of the curve at the point  $(2, 1)$ . *(6 marks)*
- (b) Show that the  $x$ -coordinate of any stationary point on this curve satisfies the equation

$$\frac{1}{x^3} = x + 1 \quad \text{span style="float: right;">*(4 marks)*$$

June 2009

**5** A curve is defined by the equation  $4x^2 + y^2 = 4 + 3xy$ .

Find the gradient at the point  $(1, 3)$  on this curve. *(5 marks)*

January 2010

**5** A curve is defined by the equation

$$x^2 + xy = e^y$$

Find the gradient at the point  $(-1, 0)$  on this curve.

*(5 marks)*

June 2010

**6** A curve has equation  $x^3y + \cos(\pi y) = 7$ .

**(a)** Find the exact value of the  $x$ -coordinate at the point on the curve where  $y = 1$ .

*(2 marks)*

**(b)** Find the gradient of the curve at the point where  $y = 1$ .

*(5 marks)*

June 2011

**6** A curve is defined by the equation  $2y + e^{2x}y^2 = x^2 + C$ , where  $C$  is a constant.

The point  $P\left(1, \frac{1}{e}\right)$  lies on the curve.

**(a)** Find the exact value of  $C$ .

*(1 mark)*

**(b)** Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

*(7 marks)*

**(c)** Verify that  $P\left(1, \frac{1}{e}\right)$  is a stationary point on the curve.

*(2 marks)*

June 2012

**6** A curve is defined by the equation  $9x^2 - 6xy + 4y^2 = 3$ .

Find the coordinates of the two stationary points of this curve.

*(8 marks)*

January 2013

**4 (a)** A curve is defined by the equation  $x^2 - y^2 = 8$ .

**(i)** Show that at any point  $(p, q)$  on the curve, where  $q \neq 0$ , the gradient of the curve is given by  $\frac{dy}{dx} = \frac{p}{q}$ .

*(2 marks)*

**(ii)** Show that the tangents at the points  $(p, q)$  and  $(p, -q)$  intersect on the  $x$ -axis.

*(4 marks)*