Core 3: Integration

Past Paper Questions
2006 - 2013

Name:

Integration

(+ constant; $a > 0$ where relevant)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathrm{d}x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$
$\sec^2 kx$	$\dfrac{1}{k}\tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a + x}{a - x}\right\rvert = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x - a}{x + a}\right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\int \frac{2x^3 + 1}{x^4 + 2x}\,dx$. *(2 marks)*

(b) (i) Use the substitution $u = 2x + 1$ to show that

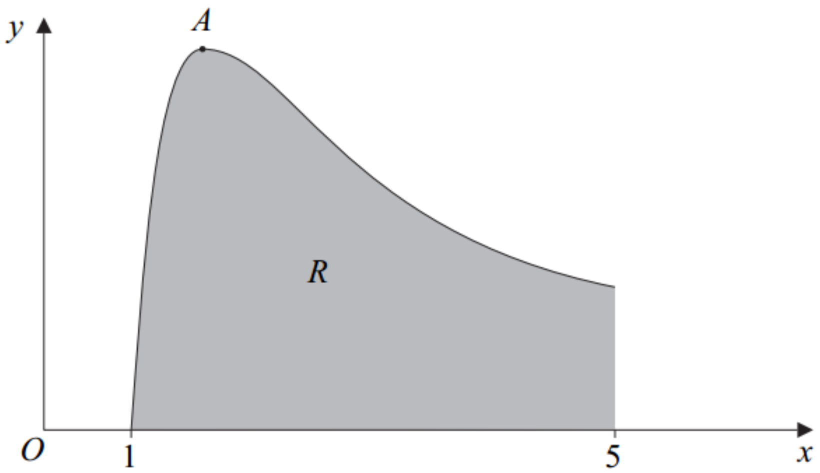
$$\int x\sqrt{2x+1}\,dx = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$ *(3 marks)*

(ii) Hence show that $\int_0^4 x\sqrt{2x+1}\,dx = 19.9$ correct to three significant figures. *(4 marks)*

9 (a) Given that $y = x^{-2}\ln x$, show that $\frac{dy}{dx} = \frac{1 - 2\ln x}{x^3}$. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2}\ln x\,dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2}\ln x$.



(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

June 2006

2 Use the substitution $u = 2x + 1$ to find $\int x(2x+1)^8 \, dx$, giving your answer in terms of x. *(4 marks)*

6 (i) Given that $y = x \ln x$, find $\frac{dy}{dx}$. *(2 marks)*

(ii) Hence, or otherwise, find $\int \ln x \, dx$. *(2 marks)*

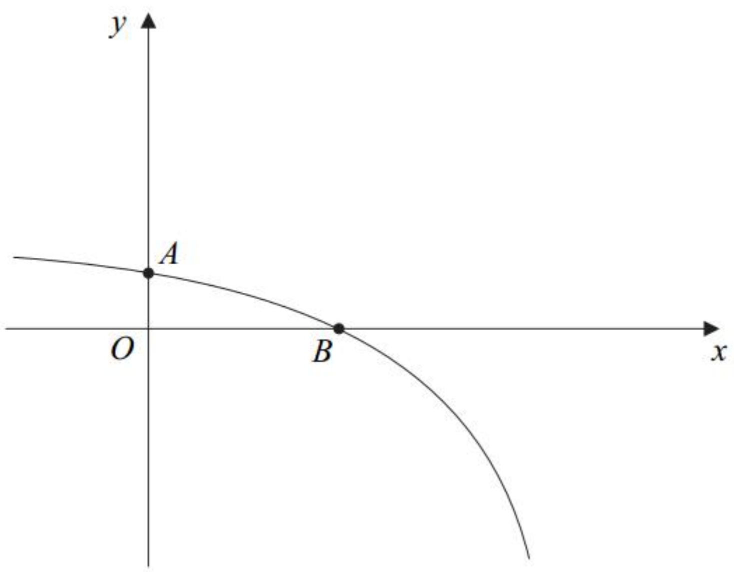
(iii) Find the exact value of $\int_1^5 \ln x \, dx$. *(2 marks)*

January 2007

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2+5} \, dx$. *(4 marks)*

9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.



(a) (i) Find $\int (4 - e^{2x}) dx$. *(2 marks)*

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x}) dx = 4 \ln 2 - \frac{3}{2}$. *(2 marks)*

June 2007

6 (a) Use integration by parts to find $\int xe^{5x}\,dx$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx = \int \frac{2}{1+u}\,du$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$. *(3 marks)*

8 (a) Write down $\int \sec^2 x\,dx$. *(1 mark)*

(b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. *(3 marks)*

(d) Hence find $\int_{0.5}^{1} (\tan x + \cot x)^2\,dx$, giving your answer to two significant figures. *(4 marks)*

January 2008

5 (a) (i) Given that $y = 2x^2 - 8x + 3$, find $\frac{dy}{dx}$. *(1 mark)*

(ii) Hence, or otherwise, find

$$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\,dx$$

giving your answer in the form $k \ln 3$, where k is a rational number. *(4 marks)*

(b) Use the substitution $u = 3x - 1$ to find $\int x\sqrt{3x-1}\,dx$, giving your answer in terms of x. *(4 marks)*

8 (a) Given that $e^{-2x} = 3$, find the exact value of x. *(2 marks)*

(b) Use integration by parts to find $\int xe^{-2x}\,dx$. *(4 marks)*

June 2008

3 Use integration by parts to find $\int_0^{0.5} x\cos 2x \, dx$, giving your answer to three significant figures. *(5 marks)*

7 Use the substitution $x = \sin\theta$ to find $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} \, dx$, giving your answer in terms of x. *(5 marks)*

January 2009

9 (i) Use the substitution $u = 4x - 3$ to find $\int \frac{4x}{4x-3} \, dx$, giving your answer in terms of x. *(4 marks)*

(ii) By using integration by parts, or otherwise, find $\int \ln(4x-3) \, dx$. *(4 marks)*

June 2009

7 (a) Use integration by parts to find $\int (t-1)\ln t \, dt$. *(4 marks)*

(b) Use the substitution $t = 2x + 1$ to show that $\int 4x\ln(2x+1) \, dx$ can be written as $\int (t-1)\ln t \, dt$. *(3 marks)*

(c) Hence find the exact value of $\int_0^1 4x\ln(2x+1) \, dx$. *(3 marks)*

January 2010

8 (a) Using integration by parts, find $\int x\sin(2x-1) \, dx$. *(5 marks)*

(b) Use the substitution $u = 2x - 1$ to find $\int \frac{x^2}{2x-1} \, dx$, giving your answer in terms of x. *(6 marks)*

June 2010

7 (a) Use integration by parts to find:

(i) $\int x \cos 4x \, dx$; *(4 marks)*

(ii) $\int x^2 \sin 4x \, dx$. *(4 marks)*

January 2011

6 (a) Use the substitution $u = 3x + 1$ to find the exact value of $\int_0^1 x\sqrt{3x+1} \, dx$.

(6 marks)

5 (a) Find $\int \frac{1}{3+2x} \, dx$. *(2 marks)*

(b) By using integration by parts, find $\int x \sin \frac{x}{2} \, dx$. *(4 marks)*

June 2011

8 Use the substitution $u = 1 + 2\tan x$ to find

$$\int \frac{1}{(1 + 2\tan x)^2 \cos^2 x} \, dx$$

(5 marks)

9 (a) Use integration by parts to find $\int x \ln x \, dx$. *(3 marks)*

January 2012

3 (a) Given that $y = 4x^3 - 6x + 1$, find $\frac{dy}{dx}$. *(1 mark)*

(b) Hence find $\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1}\,dx$, giving your answer in the form $p \ln q$, where p and q are rational numbers. *(5 marks)*

6 (a) Given that $x = \frac{1}{\sin\theta}$, use the quotient rule to show that $\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$. *(3 marks)*

(b) Use the substitution $x = \operatorname{cosec}\theta$ to find $\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2 - 1}}\,dx$, giving your answer to three significant figures. *(9 marks)*

June 2012

4 (a) By using integration by parts, find $\int x\,e^{6x}\,dx$. *(4 marks)*

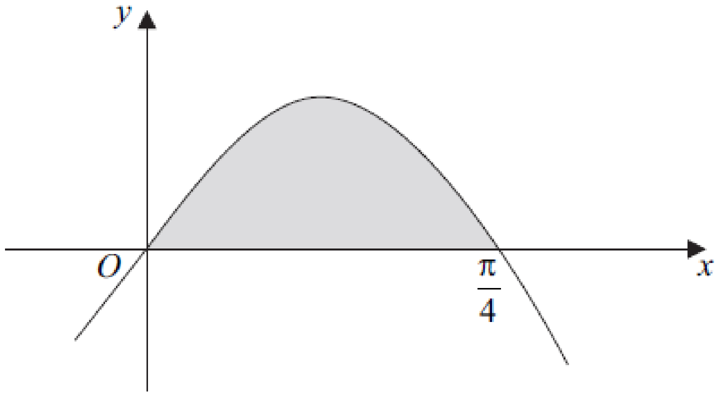
6 Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4 + 2)^2}\,dx$, giving your answer in the form $p \ln q + r$, where p, q and r are rational numbers. *(6 marks)*

January 2013

7 A curve has equation $y = 4x\cos 2x$.

(a) Find an exact equation of the tangent to the curve at the point on the curve where $x = \frac{\pi}{4}$. *(5 marks)*

(b) The region shaded on the diagram below is bounded by the curve $y = 4x\cos 2x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{4}$.



By using integration by parts, find the exact value of the area of the shaded region. *(5 marks)*

8 (a) Show that

$$\int_0^{\ln 2} e^{1-2x}\,dx = \frac{3}{8}e$$ *(4 marks)*

(b) Use the substitution $u = \tan x$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\,dx$$ *(8 marks)*

June 2013

10 (a) (i) By writing $\ln x$ as $(\ln x)\times 1$, use integration by parts to find $\int \ln x\,dx$. *(4 marks)*

(ii) Find $\int (\ln x)^2\,dx$. *(4 marks)*

(b) Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x+\sqrt{x}}\,dx$$ *(7 marks)*