Mechanics 1: Kinematics in 2 Dimensions

Past Paper Questions
2006 - 2013

Name:

January 2006

2 A particle P moves with acceleration $(-3\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially the velocity of P is $4\mathbf{i}\,\mathrm{m\,s^{-1}}$.

(a) Find the velocity of P at time t seconds. *(2 marks)*

(b) Find the speed of P when $t = 0.5$. *(3 marks)*

June 2006

6 The points A and B have position vectors $(3\mathbf{i} + 2\mathbf{j})$ metres and $(6\mathbf{i} - 4\mathbf{j})$ metres respectively. The vectors $\mathbf{i}$ and $\mathbf{j}$ are in a horizontal plane.

(a) A particle moves from A to B with constant velocity $(\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. Calculate the time that the particle takes to move from A to B. *(3 marks)*

(b) The particle then moves from B to a point C with a constant acceleration of $2\mathbf{j}\,\mathrm{m\,s^{-2}}$. It takes 4 seconds to move from B to C.

(i) Find the position vector of C. *(4 marks)*

(ii) Find the distance AC. *(2 marks)*

January 2007

8 A particle is initially at the origin, where it has velocity $(5\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. It moves with a constant acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ for 10 seconds to the point with position vector $75\mathbf{i}$ metres.

(a) Show that $\mathbf{a} = 0.5\mathbf{i} + 0.4\mathbf{j}$. *(3 marks)*

(b) Find the position vector of the particle 8 seconds after it has left the origin. *(3 marks)*

(c) Find the position vector of the particle when it is travelling parallel to the unit vector $\mathbf{i}$. *(6 marks)*

June 2007

8 A boat is initially at the origin, heading due east at $5\,\text{m s}^{-1}$. It then experiences a constant acceleration of $(-0.2\mathbf{i} + 0.25\mathbf{j})\,\text{m s}^{-2}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) State the initial velocity of the boat as a vector. *(1 mark)*

(b) Find an expression for the velocity of the boat t seconds after it has started to accelerate. *(2 marks)*

(c) Find the value of t when the boat is travelling due north. *(3 marks)*

(d) Find the bearing of the boat from the origin when the boat is travelling due north. *(6 marks)*

January 2008

8 A Jet Ski is at the origin and is travelling due north at $5\,\text{m s}^{-1}$ when it begins to accelerate uniformly. After accelerating for 40 seconds, it is travelling due east at $4\,\text{m s}^{-1}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) Show that the acceleration of the Jet Ski is $(0.1\,\mathbf{i} - 0.125\,\mathbf{j})\,\text{m s}^{-2}$. *(4 marks)*

(b) Find the position vector of the Jet Ski at the end of the 40 second period. *(3 marks)*

(c) The Jet Ski is travelling southeast t seconds after it leaves the origin.

(i) Find t. *(5 marks)*

(ii) Find the velocity of the Jet Ski at this time. *(2 marks)*

June 2008

5 The unit vectors **i** and **j** are directed east and north respectively. A helicopter moves horizontally with a constant acceleration of $(-0.4\mathbf{i} + 0.5\mathbf{j})\,\text{m s}^{-2}$. At time $t = 0$, the helicopter is at the origin and has velocity $20\mathbf{i}\,\text{m s}^{-1}$.

(a) Write down an expression for the velocity of the helicopter at time t seconds. *(2 marks)*

(b) Find the time when the helicopter is travelling due north. *(3 marks)*

(c) Find an expression for the position vector of the helicopter at time t seconds. *(2 marks)*

(d) When $t = 100$:

(i) show that the helicopter is due north of the origin; *(3 marks)*

(ii) find the speed of the helicopter. *(3 marks)*

June 2009

7 A particle moves on a smooth horizontal plane. It is initially at the point A, with position vector $(9\mathbf{i} + 7\mathbf{j})$ m, and has velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. The particle moves with a constant acceleration of $(0.25\mathbf{i} + 0.3\mathbf{j})\,\mathrm{m\,s^{-2}}$ for 20 seconds until it reaches the point B. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the velocity of the particle at the point B. *(3 marks)*

(b) Find the velocity of the particle when it is travelling due north. *(4 marks)*

(c) Find the position vector of the point B. *(3 marks)*

(d) Find the average velocity of the particle as it moves from A to B. *(2 marks)*

January 2010

5 ~~The constant forces $\mathbf{F}_1 = (8\mathbf{i} + 12\mathbf{j})$ newtons and $\mathbf{F}_2 = (4\mathbf{i} - 4\mathbf{j})$ newtons act on a particle. No other forces act on the particle.~~

~~(a) Find the resultant force acting on the particle. *(2 marks)*~~

~~(b) Given that the mass of the particle is 4 kg, show that~~ the acceleration of the particle is $(3\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-2}}$. ~~*(2 marks)*~~

(c) At time t seconds, the velocity of the particle is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

(i) When $t = 20$, $\mathbf{v} = 40\mathbf{i} + 32\mathbf{j}$.

Show that $\mathbf{v} = -20\mathbf{i} - 8\mathbf{j}$ when $t = 0$. *(3 marks)*

(ii) Write down an expression for $\mathbf{v}$ at time t. *(1 mark)*

(iii) Find the times when the speed of the particle is $8\,\mathrm{m\,s^{-1}}$. *(6 marks)*

June 2010

7 A particle, of mass 10 kg, moves on a smooth horizontal surface. A single horizontal force, $(9\mathbf{i} + 12\mathbf{j})$ newtons, acts on the particle.

The unit vectors **i** and **j** are directed east and north respectively.

(a) Find the acceleration of the particle. *(2 marks)*

(b) At time t seconds, the velocity of the particle is $\mathbf{v}\,\mathrm{m\,s^{-1}}$. When $t = 0$, the velocity of the particle is $(2.2\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$ and the particle is at the origin.

(i) Find the distance between the particle and the origin when $t = 5$. *(4 marks)*

(ii) Express **v** in terms of t. *(2 marks)*

(iii) Find t when the particle is travelling north-east. *(3 marks)*

January 2011

5 A particle moves with constant acceleration $(-0.4\mathbf{i} + 0.2\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially, it has velocity $(4\mathbf{i} + 0.5\mathbf{j})\,\mathrm{m\,s^{-1}}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find an expression for the velocity of the particle at time t seconds. *(2 marks)*

(b) (i) Find the velocity of the particle when $t = 22.5$. *(2 marks)*

(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the time when the speed of the particle is $5\,\mathrm{m\,s^{-1}}$. *(6 marks)*

June 2011

7 A helicopter is initially hovering above a lighthouse. It then sets off so that its acceleration is $(0.5\mathbf{i} + 0.375\mathbf{j})\,\mathrm{m\,s^{-2}}$. The helicopter does not change its height above sea level as it moves. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find the speed of the helicopter 20 seconds after it leaves its position above the lighthouse. *(4 marks)*

(b) Find the bearing on which the helicopter is travelling, giving your answer to the nearest degree. *(3 marks)*

(c) The helicopter stops accelerating when it is 500 metres from its initial position.

Find the time that it takes for the helicopter to travel from its initial position to the point where it stops accelerating. *(5 marks)*

January 2012

7 A helicopter is initially at rest on the ground at the origin when it begins to accelerate in a vertical plane. Its acceleration is $(4.2\mathbf{i} + 2.5\mathbf{j})\,\mathrm{m\,s^{-2}}$ for the first 20 seconds of its motion. The unit vectors **i** and **j** are horizontal and vertical respectively.

Assume that the helicopter moves over horizontal ground.

(a) Find the height of the helicopter above the ground at the end of the 20 seconds. *(3 marks)*

(b) Find the velocity of the helicopter at the end of the 20 seconds. *(2 marks)*

(c) Find the speed of the helicopter when it is at a height of 180 metres above the ground. *(7 marks)*

June 2012

7 A particle moves with a constant acceleration of $(0.1\mathbf{i} - 0.2\mathbf{j})\,\mathrm{m\,s^{-2}}$. It is initially at the origin where it has velocity $(-\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find an expression for the position vector of the particle t seconds after it has left the origin. *(2 marks)*

(b) Find the time that it takes for the particle to reach the point where it is due east of the origin. *(3 marks)*

(c) Find the speed of the particle when it is travelling south-east. *(6 marks)*

January 2013

7 A particle is initially at the point A, which has position vector $13.6\mathbf{i}$ metres, with respect to an origin O. At the point A, the particle has velocity $(6\mathbf{i} + 2.4\mathbf{j})\,\mathrm{m\,s^{-1}}$, and in its subsequent motion, it has a constant acceleration of $(-0.8\mathbf{i} + 0.1\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find an expression for the velocity of the particle t seconds after it leaves A. *(2 marks)*

(b) Find an expression for the position vector of the particle, with respect to the origin O, t seconds after it leaves A. *(3 marks)*

(c) Find the distance of the particle from the origin O when it is travelling in a north-westerly direction. *(7 marks)*

June 2013

8 A helicopter travels at a constant height above the sea. It passes directly over a lighthouse with position vector $(500\mathbf{i} + 200\mathbf{j})$ metres relative to the origin, with a velocity of $(-17.5\mathbf{i} - 27\mathbf{j})\,\mathrm{m\,s^{-1}}$. The helicopter moves with a constant acceleration of $(0.5\mathbf{i} + 0.6\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors **i** and **j** are directed east and north respectively.

(a) Find the position vector of the helicopter t seconds after it has passed over the lighthouse. *(3 marks)*

(b) The position vector of a rock is $(200\mathbf{i} - 400\mathbf{j})$ metres relative to the origin. Show that the helicopter passes directly over the rock, and state the time that it takes for the helicopter to move from the lighthouse to the rock. *(7 marks)*

(c) Find the average velocity of the helicopter as it moves from the lighthouse to the rock. *(3 marks)*

(d) Is the magnitude of the average velocity equal to the average speed of the helicopter? Give a reason for your answer. *(2 marks)*