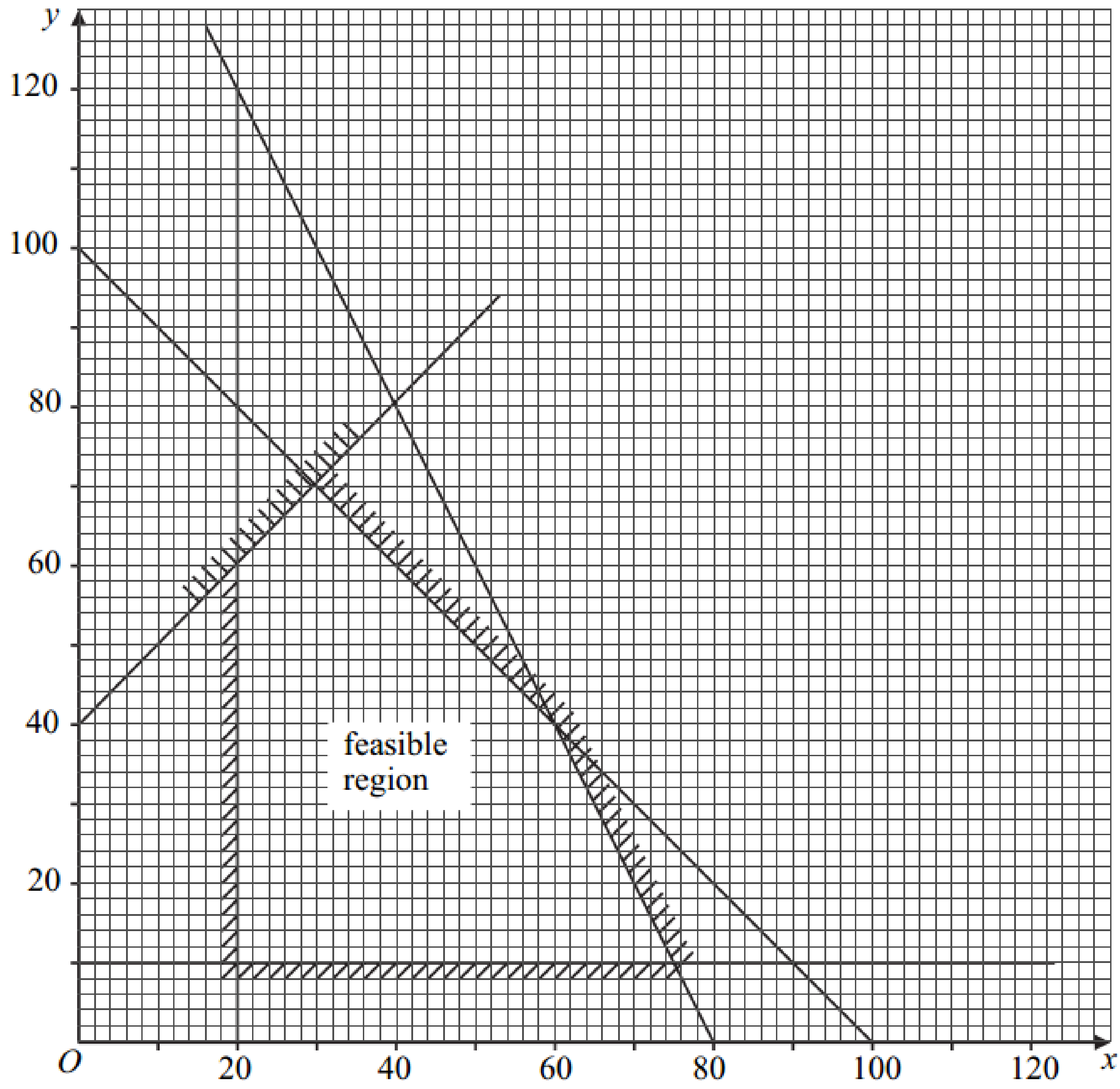
Decision 1: Linear Programming

Past Paper Questions
2006 - 2013

Name:

4 The diagram shows the feasible region of a linear programming problem.



(a) On the feasible region, find:

(i) the maximum value of $2x + 3y$; *(2 marks)*

(ii) the maximum value of $3x + 2y$; *(2 marks)*

(iii) the minimum value of $-2x + y$. *(2 marks)*

(b) Find the 5 inequalities that define the feasible region. *(6 marks)*

9 A factory makes three different types of widget: plain, bland and ordinary. Each widget is made using three different machines: A, B and C.

Each plain widget needs 5 minutes on machine A, 12 minutes on machine B and 24 minutes on machine C.

Each bland widget needs 4 minutes on machine A, 8 minutes on machine B and 12 minutes on machine C.

Each ordinary widget needs 3 minutes on machine A, 10 minutes on machine B and 18 minutes on machine C.

Machine A is available for 3 hours a day, machine B for 4 hours a day and machine C for 9 hours a day.

The factory must make:

more plain widgets than bland widgets;

more bland widgets than ordinary widgets.

At least 40% of the total production must be plain widgets.

Each day, the factory makes x plain, y bland and z ordinary widgets.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, writing your answers with simplified integer coefficients. *(8 marks)*

6 [**Figure 3**, printed on the insert, is provided for use in this question.]

Ernesto is to plant a garden with two types of tree: palms and conifers.

He is to plant at least 10, but not more than 80 palms.

He is to plant at least 5, but not more than 40 conifers.

He cannot plant more than 100 trees in total.

Each palm needs 20 litres of water each day and each conifer needs 60 litres of water each day. There are 3000 litres of water available each day.

Ernesto makes a profit of £2 on each palm and £1 on each conifer that he plants and he wishes to maximise his profit.

Ernesto plants x palms and y conifers.

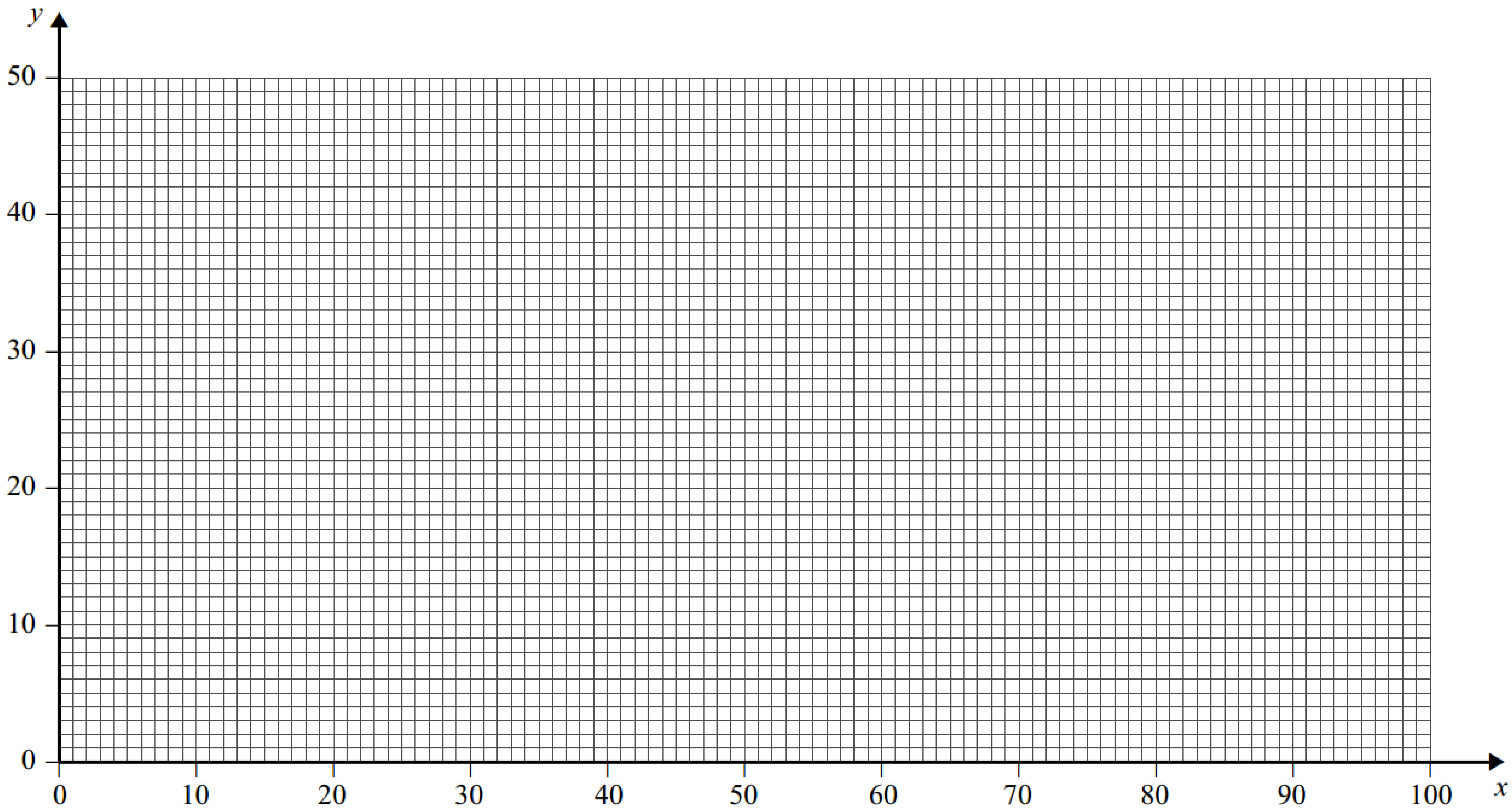
(a) Formulate Ernesto's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 3**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(7 marks)*

(c) Find the maximum profit for Ernesto. *(2 marks)*

(d) Ernesto introduces a new pricing structure in which he makes a profit of £1 on each palm and £4 on each conifer.

Find Ernesto's new maximum profit and the number of each type of tree that he should plant to obtain this maximum profit. *(2 marks)*


y
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
x

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

Dino is to have a rectangular swimming pool at his villa.

He wants its width to be at least 2 metres and its length to be at least 5 metres.

He wants its length to be at least twice its width.

He wants its length to be no more than three times its width.

Each metre of the width of the pool costs £1000 and each metre of the length of the pool costs £500.

He has £9000 available.

Let the width of the pool be x metres and the length of the pool be y metres.

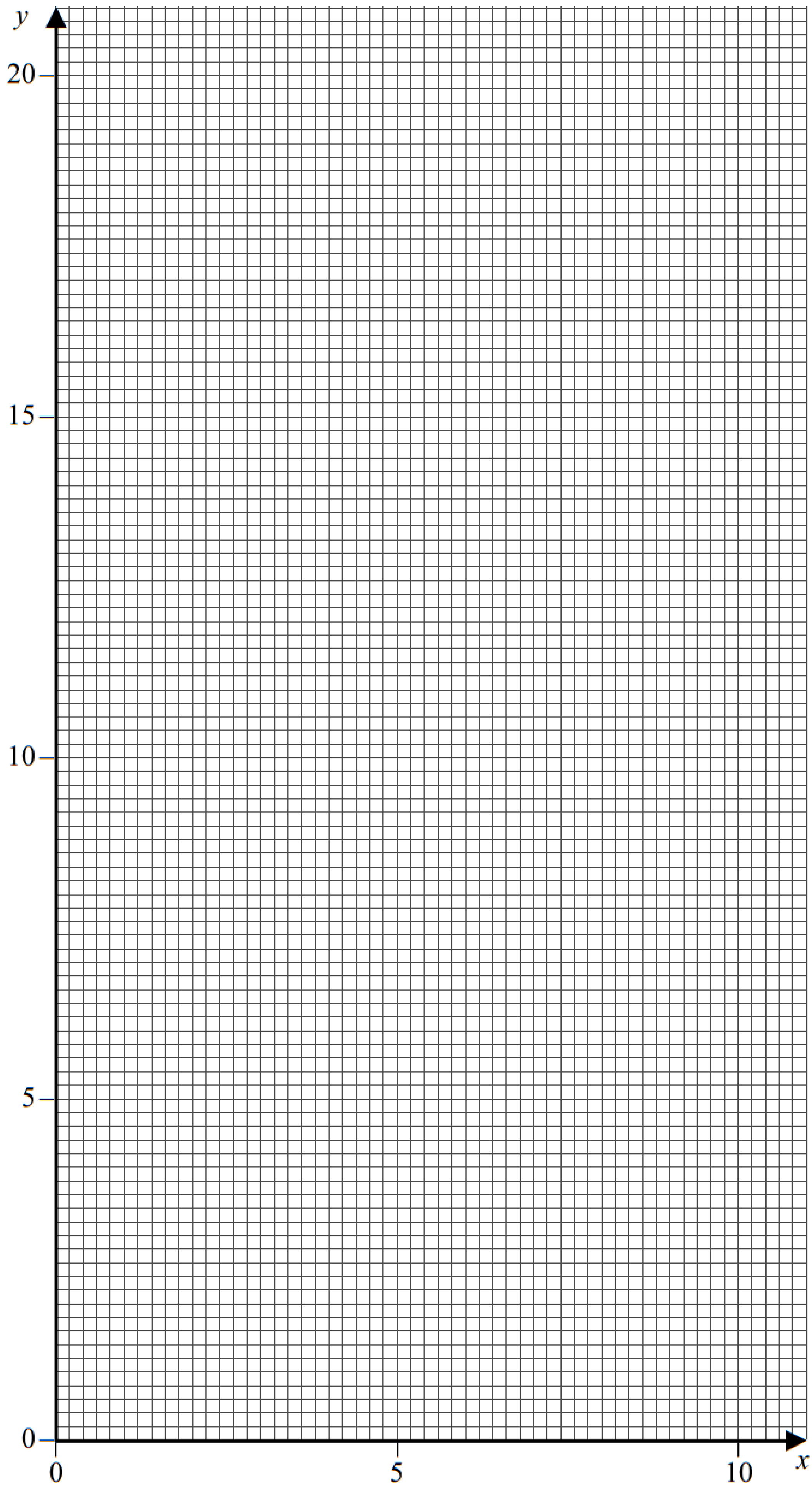
(a) Show that one of the constraints leads to the inequality

$$2x + y \leqslant 18$$ *(1 mark)*

(b) Find four further inequalities. *(3 marks)*

(c) On **Figure 1**, draw a suitable diagram to show the feasible region. *(6 marks)*

(d) Use your diagram to find the maximum width of the pool. State the corresponding length of the pool. *(3 marks)*



y
20
15
10
5
0
0
5
10
x

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

The Jolly Company sells two types of party pack: excellent and luxury.

Each excellent pack has five balloons and each luxury pack has ten balloons.

Each excellent pack has 32 sweets and each luxury pack has 8 sweets.

The company has 1500 balloons and 4000 sweets available.

The company sells at least 50 of each type of pack and at least 140 packs in total.

The company sells x excellent packs and y luxury packs.

(a) Show that the above information can be modelled by the following inequalities.

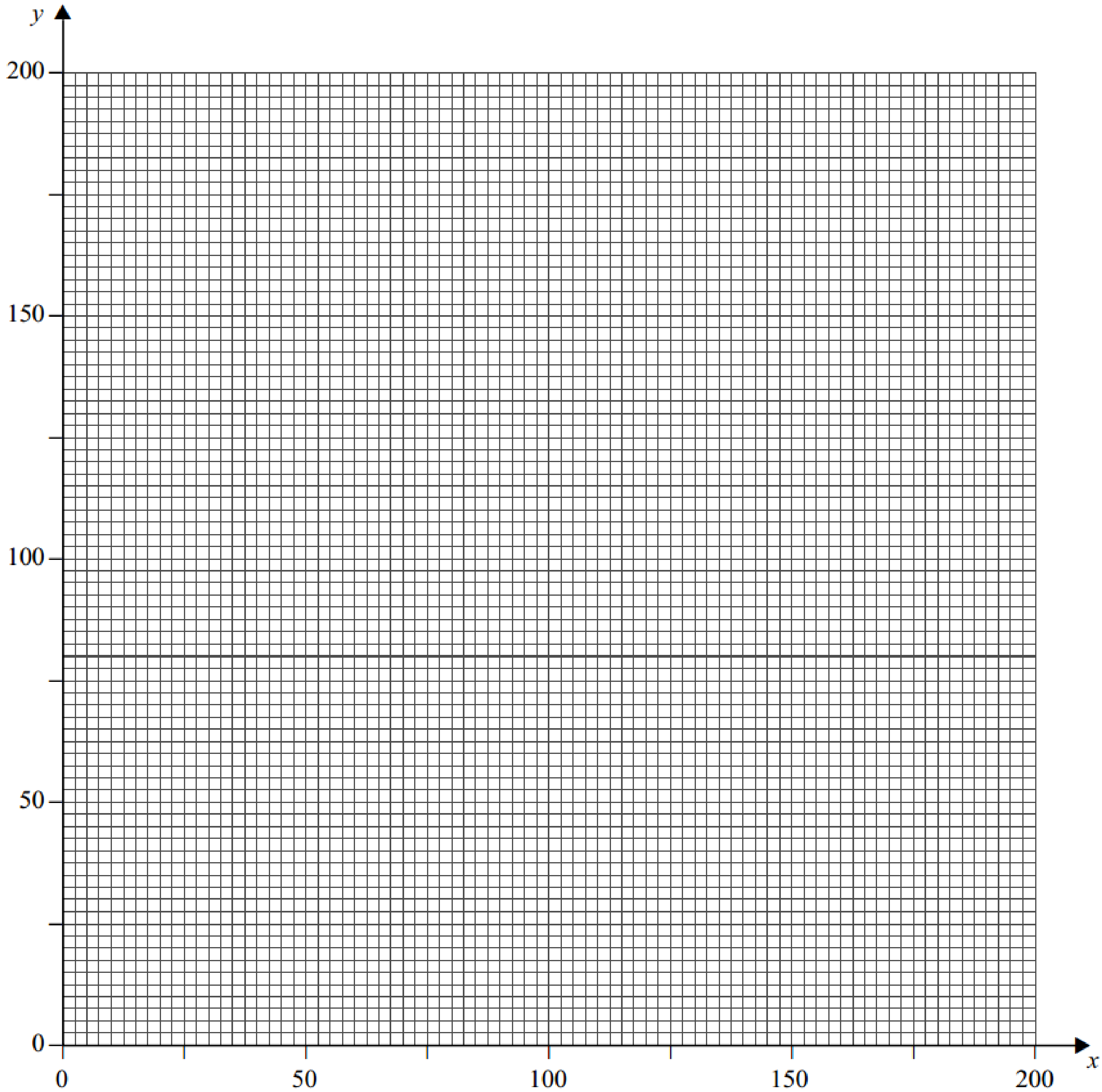
$x + 2y \leqslant 300$, $4x + y \leqslant 500$, $x \geqslant 50$, $y \geqslant 50$, $x + y \geqslant 140$ *(4 marks)*

(b) The company sells each excellent pack for 80p and each luxury pack for £1.20. The company needs to find its minimum and maximum total income.

(i) On **Figure 2**, draw a suitable diagram to enable this linear programming problem to be solved graphically, indicating the feasible region and an objective line. *(8 marks)*

(ii) Find the company's maximum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

(iii) Find the company's minimum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*



y
200
150
100
50
0
0
50
100
150
200
x

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by

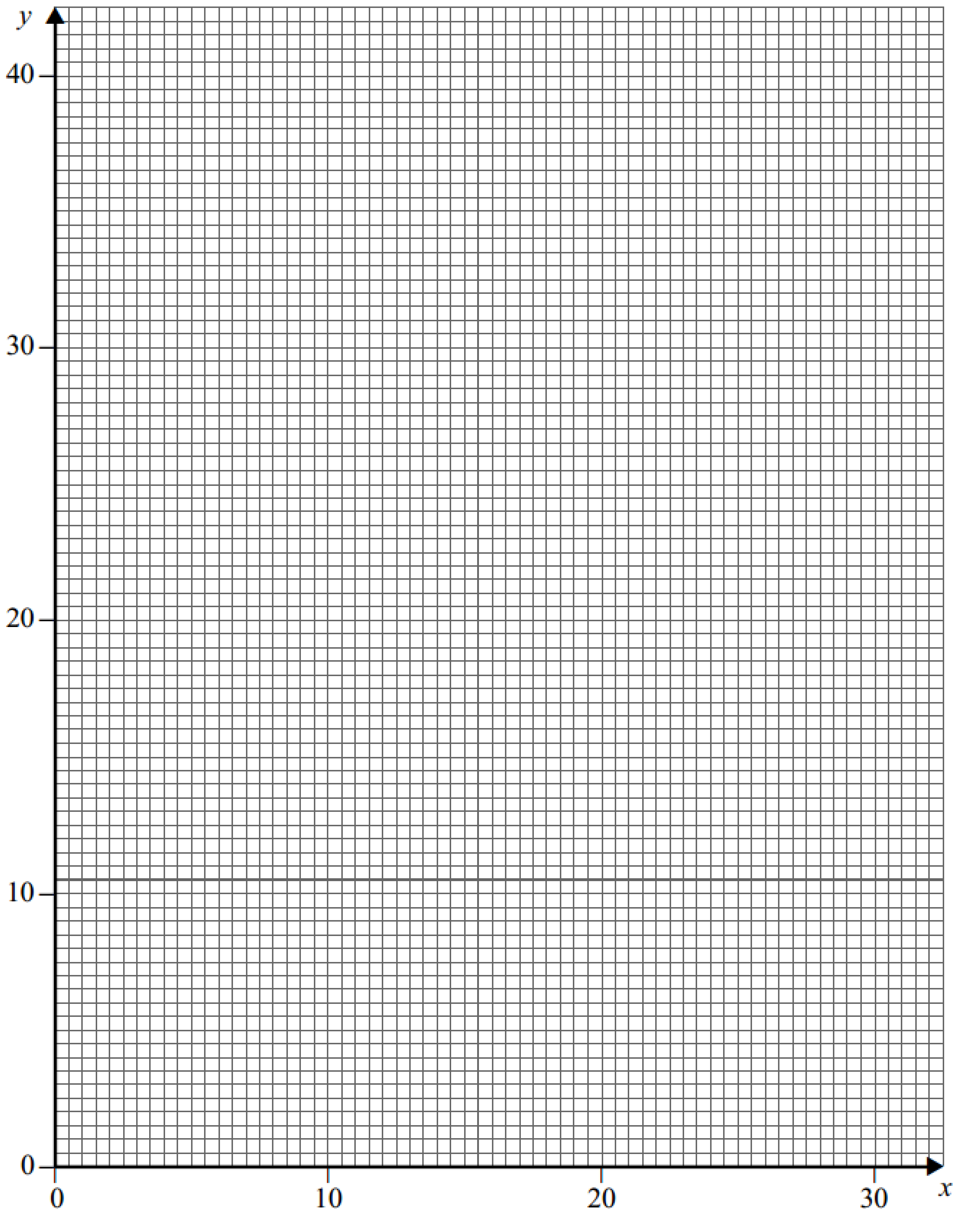
$$x + y \leqslant 30$$
$$2x + y \leqslant 40$$
$$y \geqslant 5$$
$$x \geqslant 4$$
$$y \geqslant \tfrac{1}{2}x$$

(a) On **Figure 1**, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) Use your diagram to find the maximum value of F, on the feasible region, in the case where:

(i) $F = 3x + y$; *(2 marks)*

(ii) $F = x + 3y$. *(2 marks)*


y
40
30
20
10
0
0
10
20
30
x

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

A factory makes two types of lock, standard and large, on a particular day.

On that day:

the maximum number of standard locks that the factory can make is 100;
the maximum number of large locks that the factory can make is 80;
the factory must make at least 60 locks in total;
the factory must make more large locks than standard locks.

Each standard lock requires 2 screws and each large lock requires 8 screws, and on that day the factory must use at least 320 screws.

On that day, the factory makes x standard locks and y large locks.

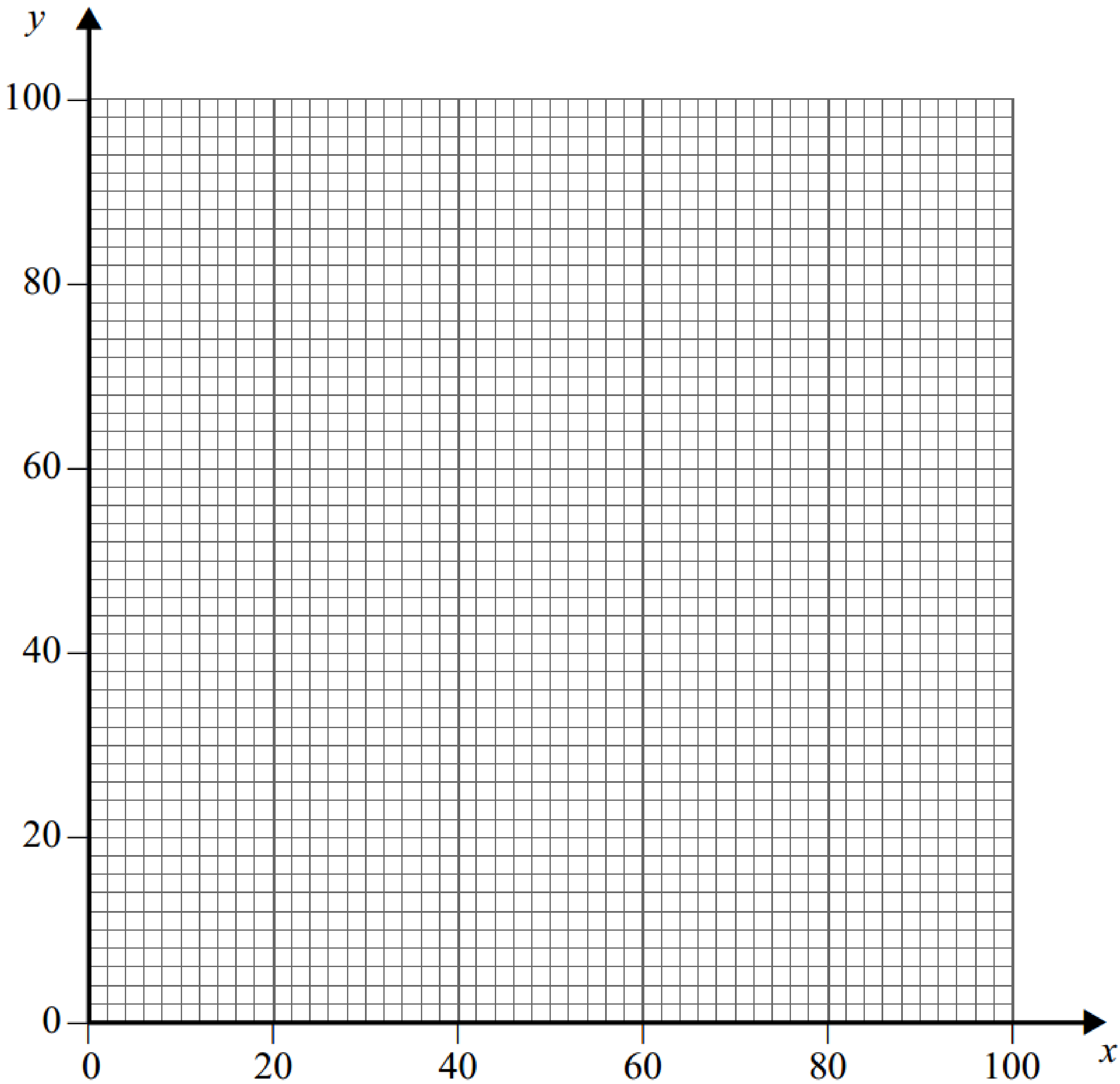
Each standard lock costs £1.50 to make and each large lock costs £3 to make.

The manager of the factory wishes to minimise the cost of making the locks.

(a) Formulate the manager's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 1**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*

(c) Find the values of x and y that correspond to the minimum cost. Hence find this minimum cost. *(4 marks)*



y
100
80
60
40
20
0
0
20
40
60
80
100
x

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

Each year, farmer Giles buys some goats, pigs and sheep.

He must buy at least 110 animals.
He must buy at least as many pigs as goats.
The total of the number of pigs and the number of sheep that he buys must not be greater than 150.
Each goat costs £16, each pig costs £8 and each sheep costs £24.
He has £3120 to spend on the animals.

At the end of the year, Giles sells all of the animals. He makes a profit of £70 on each goat, £30 on each pig and £50 on each sheep. Giles wishes to maximize his total profit, £P.

Each year, Giles buys x goats, y pigs and z sheep.

(a) Formulate Giles's situation as a linear programming problem. *(5 marks)*

(b) One year, Giles buys 30 sheep.

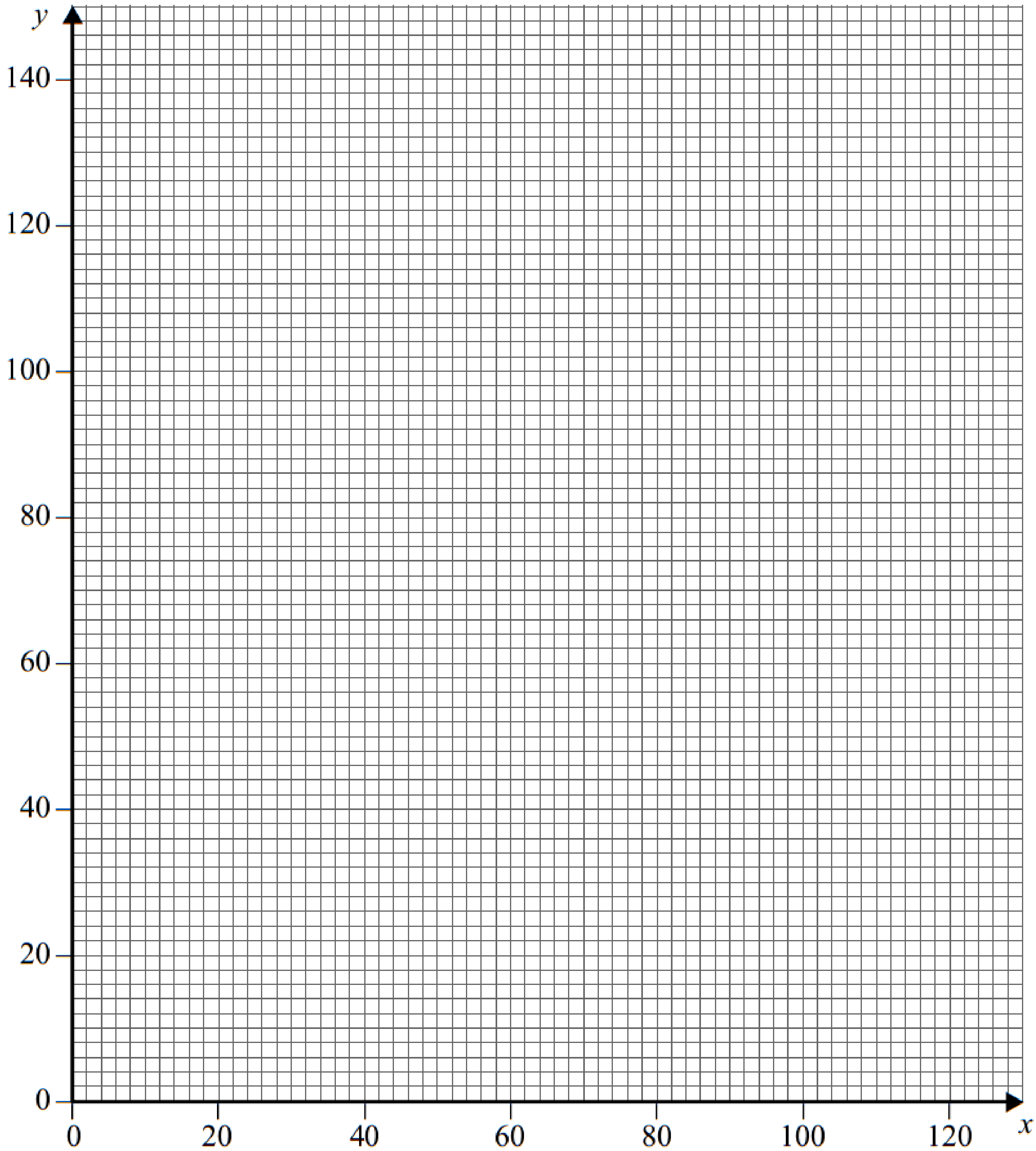
(i) Show that the constraints for Giles's situation for this year can be modelled by

$$y \geqslant x, \qquad 2x + y \leqslant 300, \qquad x + y \geqslant 80, \qquad y \leqslant 120$$

(2 marks)

(ii) On **Figure 2**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(8 marks)*

(iii) Find Giles's maximum profit for this year and the number of each animal that he must buy to obtain this maximum profit. *(3 marks)*



y
140
120
100
80
60
40
20
0
0
20
40
60
80
100
120
x

6 Each day, a factory makes three types of widget: basic, standard and luxury. The widgets produced need three different components: type A, type B and type C.

Basic widgets need 6 components of type A, 6 components of type B and 12 components of type C.
Standard widgets need 4 components of type A, 3 components of type B and 18 components of type C.
Luxury widgets need 2 components of type A, 9 components of type B and 6 components of type C.
Each day, there are 240 components of type A available, 300 of type B and 900 of type C.
Each day, the factory must use at least twice as many components of type C as type B.

Each day, the factory makes x basic widgets, y standard widgets and z luxury widgets.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find four inequalities in x, y and z that model the above constraints, simplifying each inequality. *(8 marks)*

(b) Each day, the factory makes the maximum possible number of widgets. On a particular day, the factory must make the same number of luxury widgets as basic widgets.

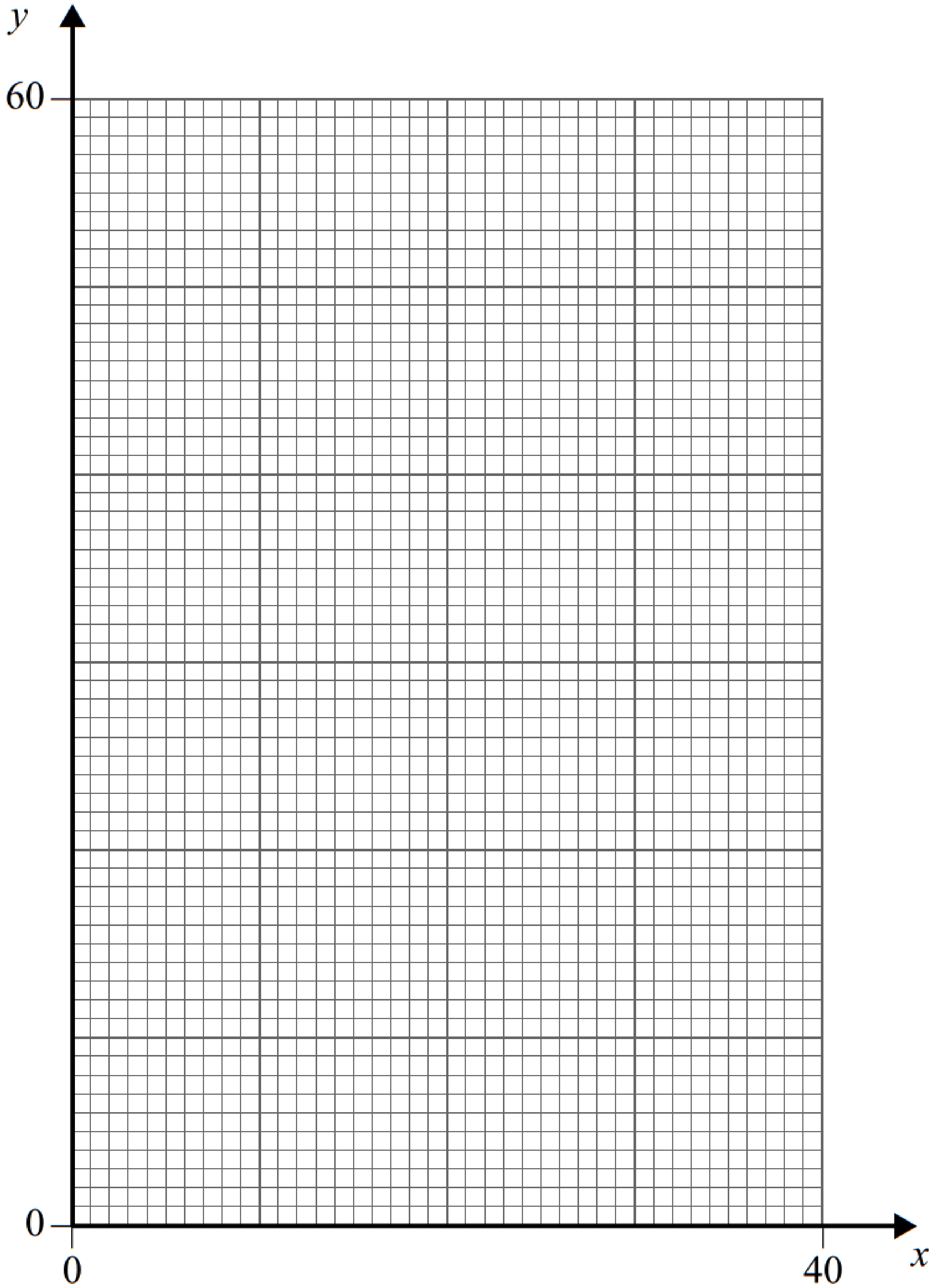
(i) Show that your answers in part **(a)** become

$$2x + y \leqslant 60, \quad 5x + y \leqslant 100, \quad x + y \leqslant 50, \quad y \geqslant x$$ *(3 marks)*

(ii) Using the axes opposite, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region. *(5 marks)*

(iii) Find the total number of widgets made on that day. *(2 marks)*

(iv) Find all possible combinations of the number of each type of widget made that correspond to this maximum number. *(3 marks)*



y
60
0
0
40
x

8 A factory packs three different kinds of novelty box: red, blue and green. Each box contains three different types of toy: A, B and C.

Each red box has 2 type A toys, 3 type B toys and 4 type C toys.

Each blue box has 3 type A toys, 1 type B toy and 3 type C toys.

Each green box has 4 type A toys, 5 type B toys and 2 type C toys.

Each day, the maximum number of each type of toy available to be packed is 360 type A, 300 type B and 400 type C.

Each day, the factory must pack more type A toys than type B toys.

Each day, the total number of type A and type B toys that are packed must together be at least as many as the number of type C toys that are packed.

Each day, at least 40% of the total toys that are packed must be type C toys.

Each day, the factory packs x red boxes, y blue boxes and z green boxes.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, simplifying your answers. *(8 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by the following:

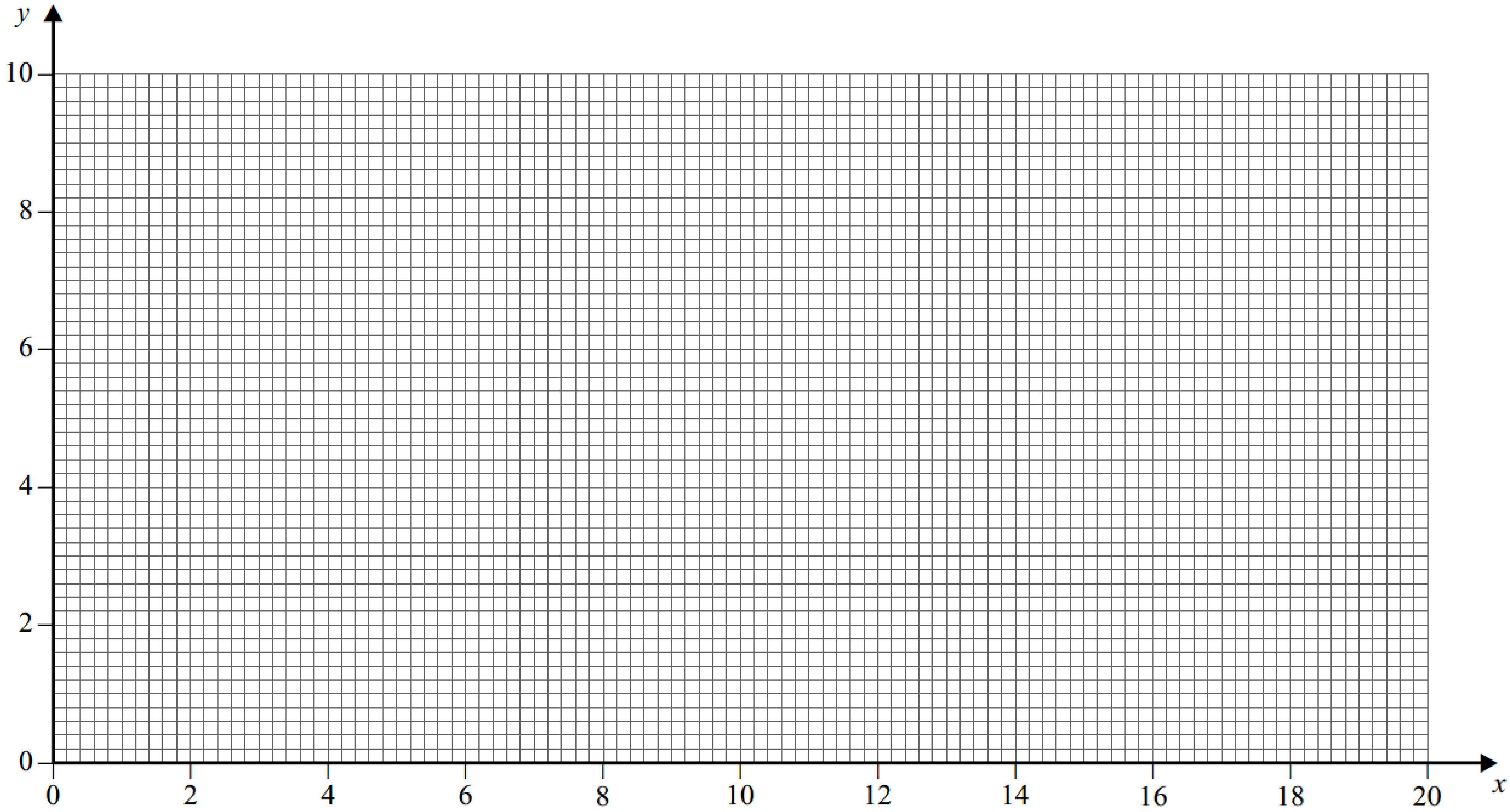
$$x \geqslant 0, y \geqslant 0$$
$$x + 4y \leqslant 36$$
$$4x + y \leqslant 68$$
$$y \leqslant 2x$$
$$y \geqslant \frac{1}{4}x$$

(a) On **Figure 1**, draw a suitable diagram to represent the inequalities and indicate the feasible region. *(6 marks)*

(b) Use your diagram to find the maximum value of P, stating the corresponding coordinates, on the feasible region, in the case where:

(i) $P = x + 5y$; *(2 marks)*

(ii) $P = 5x + y$. *(2 marks)*


y
10
8
6
4
2
0
0
2
4
6
8
10
12
14
16
18
20
x

6 Phil is to buy some squash balls for his club. There are three different types of ball that he can buy: slow, medium and fast.

He must buy at least 190 slow balls, at least 50 medium balls and at least 50 fast balls.

He must buy at least 300 balls in total.

Each slow ball costs £2.50, each medium ball costs £2.00 and each fast ball costs £2.00.

He must spend no more than £1000 in total.

At least 60% of the balls that he buys must be slow balls.

Phil buys x slow balls, y medium balls and z fast balls.

(a) Find six inequalities that model Phil's situation. *(4 marks)*

(b) Phil decides to buy the same number of medium balls as fast balls.

(i) Show that the inequalities found in part **(a)** simplify to give

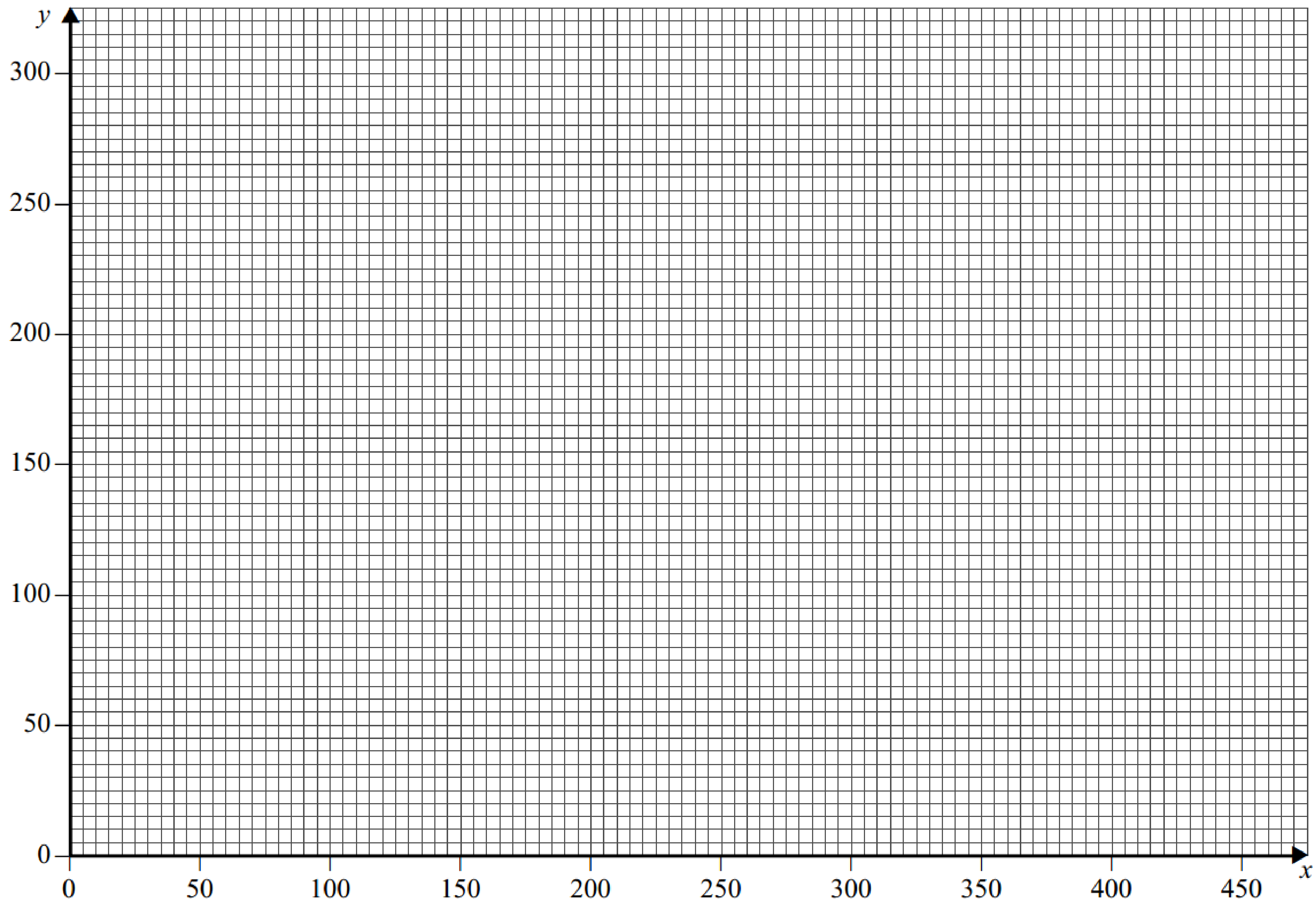
$$x \geqslant 190, \; y \geqslant 50, \; x + 2y \geqslant 300, \; 5x + 8y \leqslant 2000, \; y \leqslant \tfrac{1}{3}x$$ *(2 marks)*

(ii) Phil sells all the balls that he buys to members of the club. He sells each slow ball for £3.00, each medium ball for £2.25 and each fast ball for £2.25. He wishes to maximise his profit.

On **Figure 1** on page 14, draw a diagram to enable this problem to be solved graphically, indicating the feasible region and the direction of an objective line. *(7 marks)*

(iii) Find Phil's maximum possible profit and state the number of each type of ball that he must buy to obtain this maximum profit. *(4 marks)*

$$x \geqslant 190, \quad y \geqslant 50, \quad x + 2y \geqslant 300, \quad 5x + 8y \leqslant 2000, \quad y \leqslant \frac{1}{3}x$$



9 Herman is packing some hampers. Each day, he packs three types of hamper: basic, standard and luxury.

Each basic hamper has 6 tins, 9 packets and 6 bottles.

Each standard hamper has 9 tins, 6 packets and 12 bottles.

Each luxury hamper has 9 tins, 9 packets and 18 bottles.

Each day, Herman has 600 tins and 600 packets available, and he must use at least 480 bottles.

Each day, Herman packs x basic hampers, y standard hampers and z luxury hampers.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find three inequalities in x, y and z that model the above constraints, simplifying each inequality. *(4 marks)*

(b) On a particular day, Herman packs the same number of standard hampers as luxury hampers.

(i) Show that your answers in part **(a)** become

$$x + 3y \leqslant 100$$

$$3x + 5y \leqslant 200$$

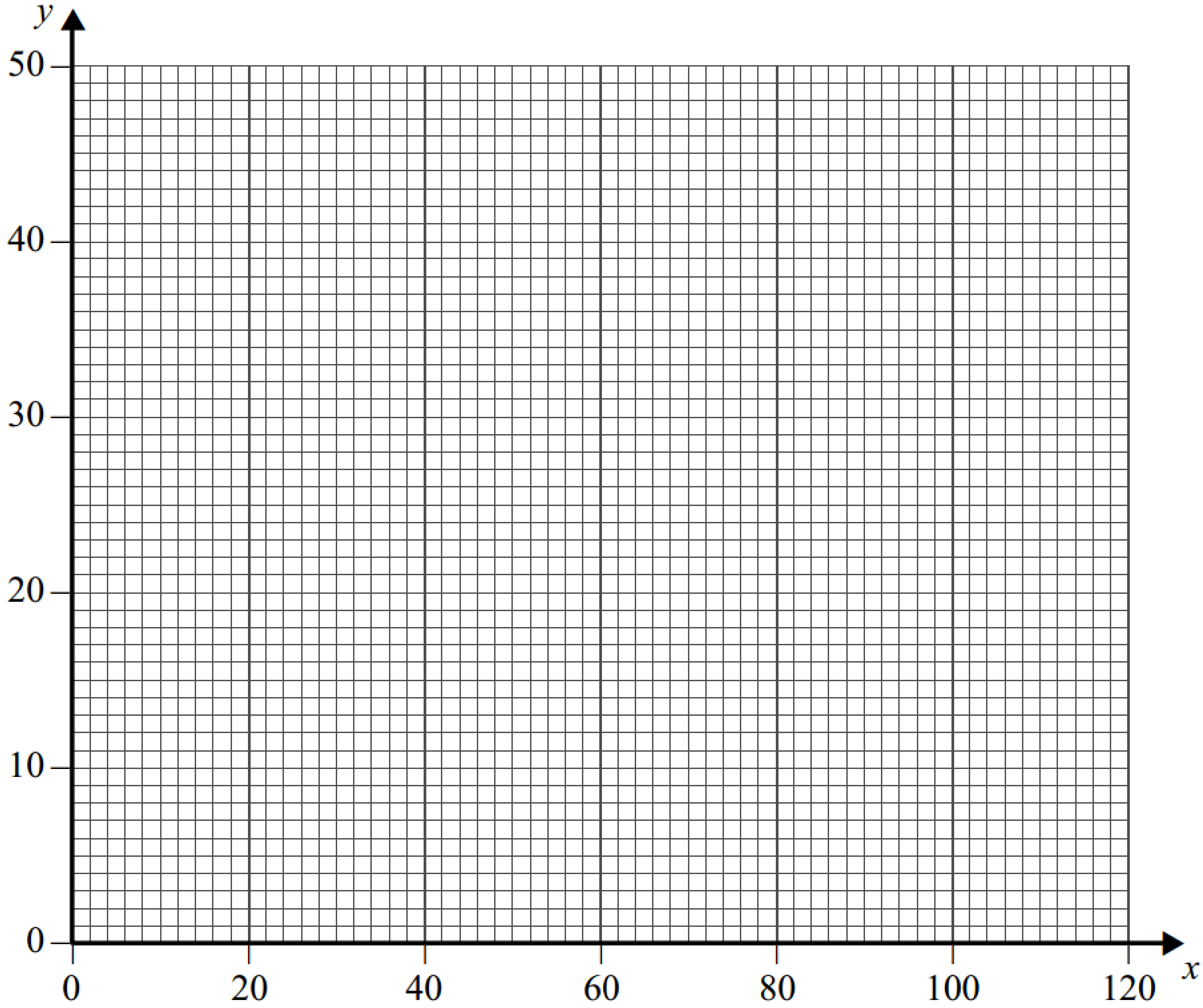
$$x + 5y \geqslant 80$$

(2 marks)

(ii) On the grid opposite, draw a suitable diagram to represent Herman's situation, indicating the feasible region. *(4 marks)*

(iii) Use your diagram to find the maximum total number of hampers that Herman can pack on that day. *(2 marks)*

(iv) Find the number of each type of hamper that Herman packs that corresponds to your answer to part **(b)(iii)**. *(1 mark)*



y
50
40
30
20
10
0
0
20
40
60
80
100
120
x

7 A builder needs some screws, nails and plugs. At the local store, there are packs containing a mixture of the three items.

A DIY pack contains 200 nails, 200 screws and 100 plugs.

A trade pack contains 1000 nails, 1500 screws and 2500 plugs.

A DIY pack costs £2.50 and a trade pack costs £15.

The builder needs at least 5000 nails, 6000 screws and 4000 plugs.

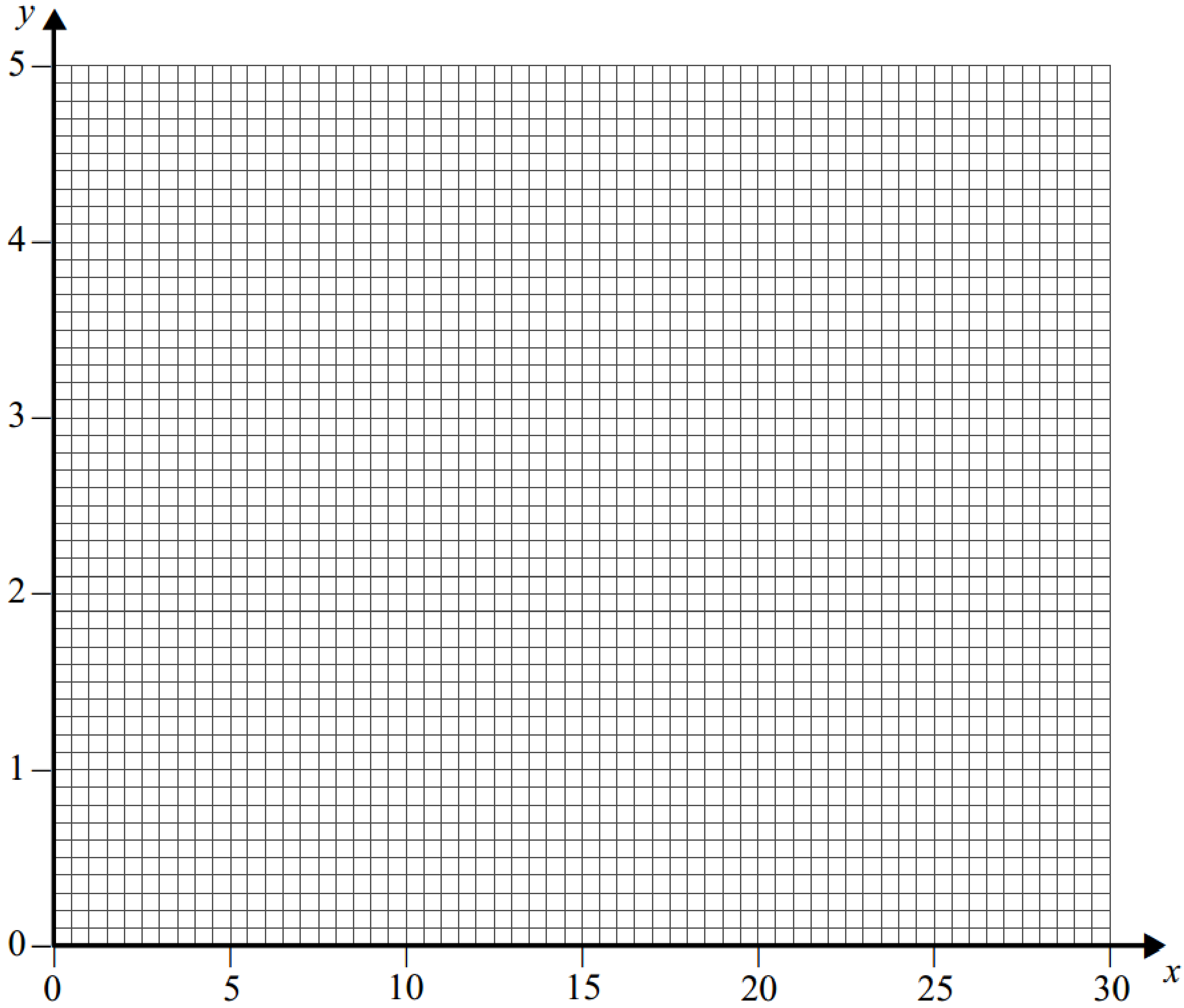
The builder buys x DIY packs and y trade packs and wishes to keep his total cost to a minimum.

(a) Formulate the builder's situation as a linear programming problem. *(4 marks)*

(b) (i) On the grid opposite, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of an objective line. *(6 marks)*

(ii) Use your diagram to find the number of each type of pack that the builder should buy in order to minimise his cost. *(1 mark)*

(iii) Find the builder's minimum cost. *(1 mark)*


y
5
4
3
2
1
0
0
5
10
15
20
25
30
x

5 The feasible region of a linear programming problem is determined by the following:

$$y \geqslant 20$$
$$x + y \geqslant 25$$
$$5x + 2y \leqslant 100$$
$$y \leqslant 4x$$
$$y \geqslant 2x$$

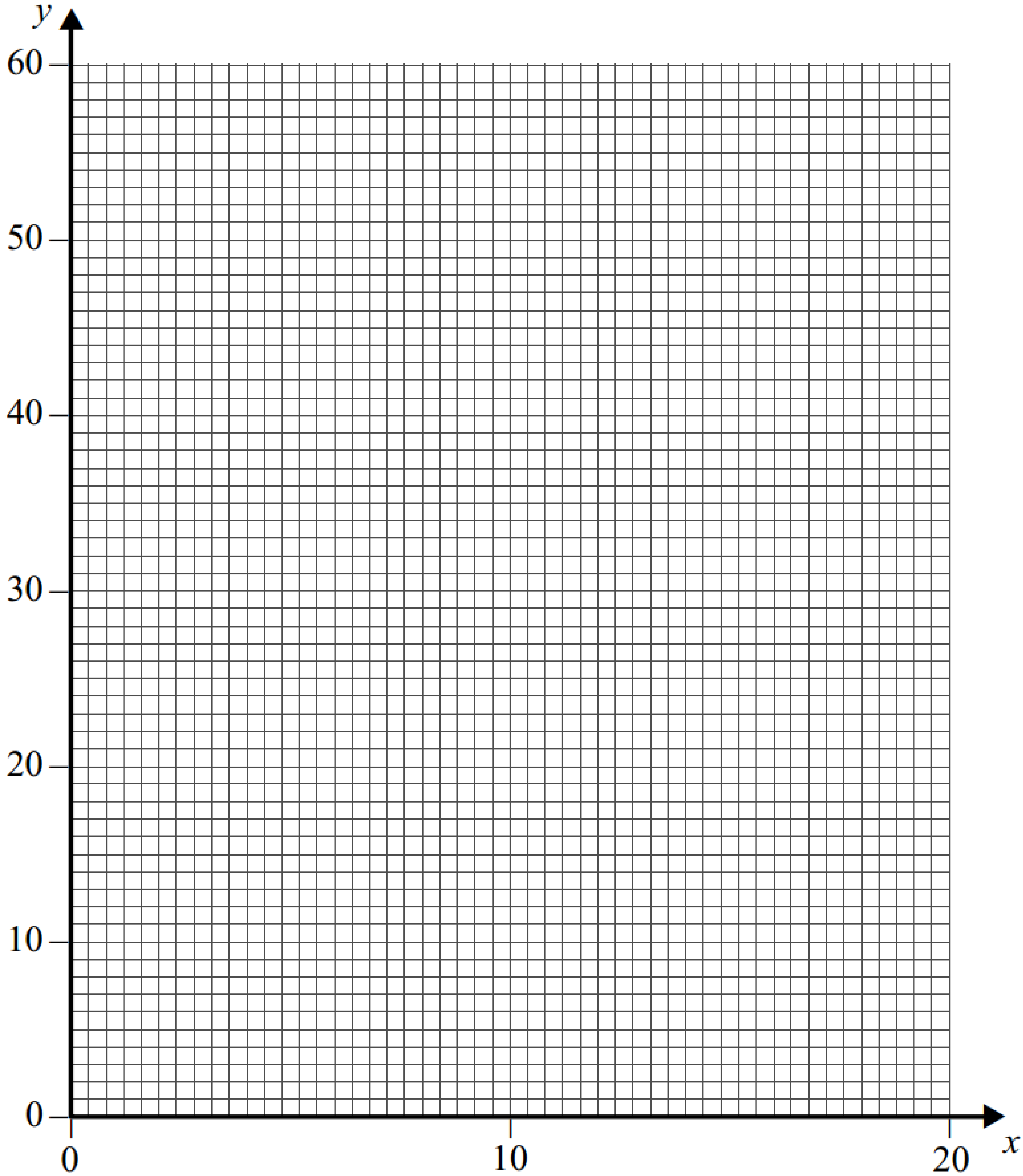
(a) On **Figure 1** opposite, draw a suitable diagram to represent the inequalities and indicate the feasible region. *(6 marks)*

(b) Use your diagram to find the minimum value of P, on the feasible region, in the case where:

(i) $P = x + 2y$;

(ii) $P = -x + y$.

In each case, state the corresponding values of x and y. *(4 marks)*



y
60
50
40
30
20
10
0
0
10
20
x

9 Ollyin is buying new pillows for his hotel. He buys three types of pillow: soft, medium and firm.

He must buy at least 100 soft pillows and at least 200 medium pillows.

He must buy at least 400 pillows in total.

Soft pillows cost £4 each. Medium pillows cost £3 each. Firm pillows cost £4 each.

He wishes to spend no more than £1800 on new pillows.

At least 40% of the new pillows must be medium pillows.

Ollyin buys x soft pillows, y medium pillows and z firm pillows.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find five inequalities in x, y and z that model the above constraints. *(3 marks)*

(b) Ollyin decides to buy twice as many soft pillows as firm pillows.

(i) Show that three of your answers in part **(a)** become

$$3x + 2y \geqslant 800$$

$$2x + y \leqslant 600$$

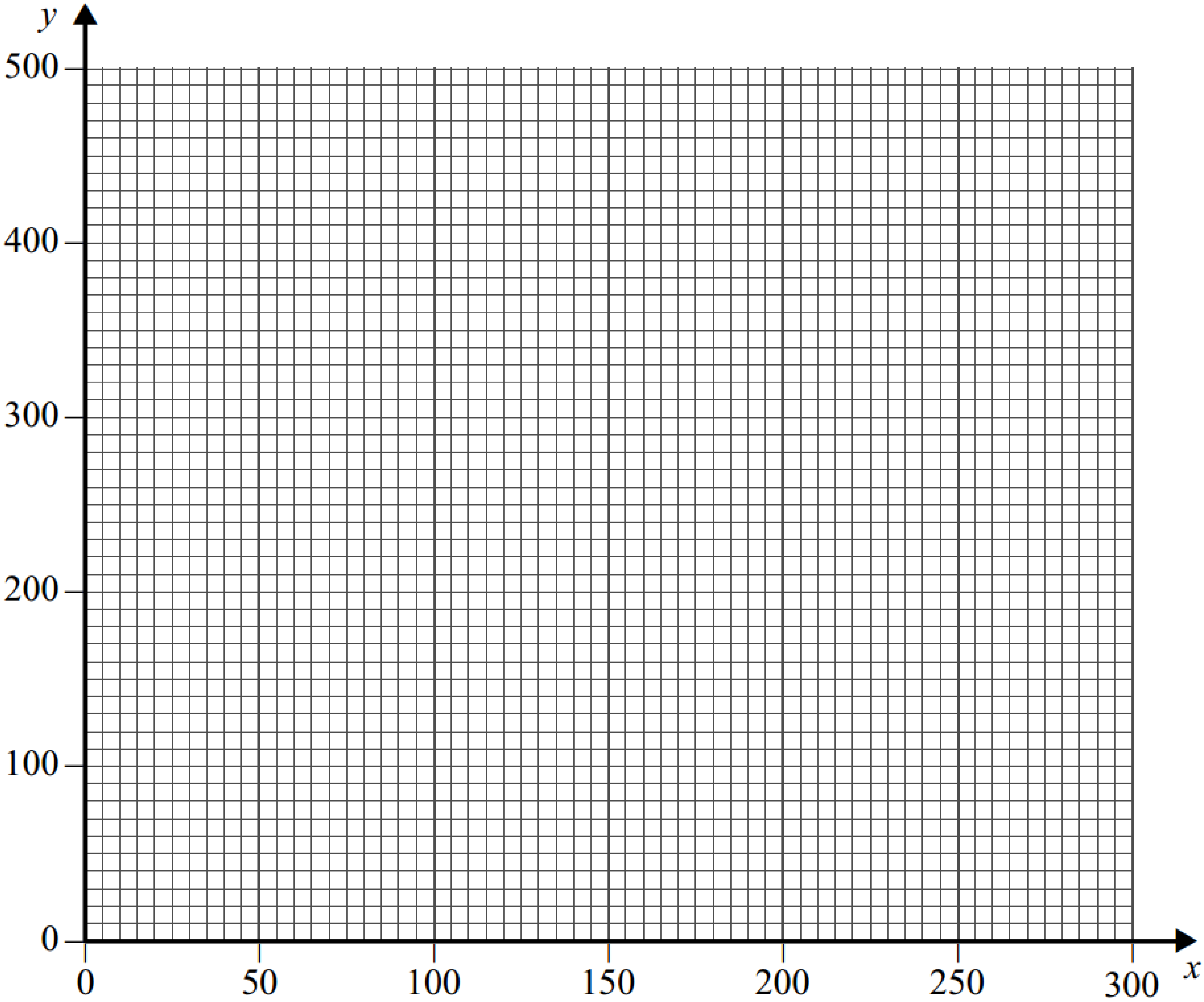
$$y \geqslant x$$

(3 marks)

(ii) On the grid opposite, draw a suitable diagram to represent Ollyin's situation, indicating the feasible region. *(5 marks)*

(iii) Use your diagram to find the maximum total number of pillows that Ollyin can buy. *(2 marks)*

(iv) Find the number of each type of pillow that Ollyin can buy that corresponds to your answer to part **(b)(iii)**. *(1 mark)*



y
500
400
300
200
100
0
0
50
100
150
200
250
300
x

9 A factory can make three different kinds of balloon pack: gold, silver and bronze. Each pack contains three different types of balloon: A, B and C.

Each gold pack has 2 type A balloons, 3 type B balloons and 6 type C balloons.

Each silver pack has 3 type A balloons, 4 type B balloons and 2 type C balloons.

Each bronze pack has 5 type A balloons, 3 type B balloons and 2 type C balloons.

Every hour, the maximum number of each type of balloon available is 400 type A, 400 type B and 400 type C.

Every hour, the factory must pack at least 1000 balloons.

Every hour, the factory must pack more type A balloons than type B balloons.

Every hour, the factory must ensure that no more than 40% of the total balloons packed are type C balloons.

Every hour, the factory makes x gold, y silver and z bronze packs.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, simplifying your answers. *(8 marks)*

5 The feasible region of a linear programming problem is defined by

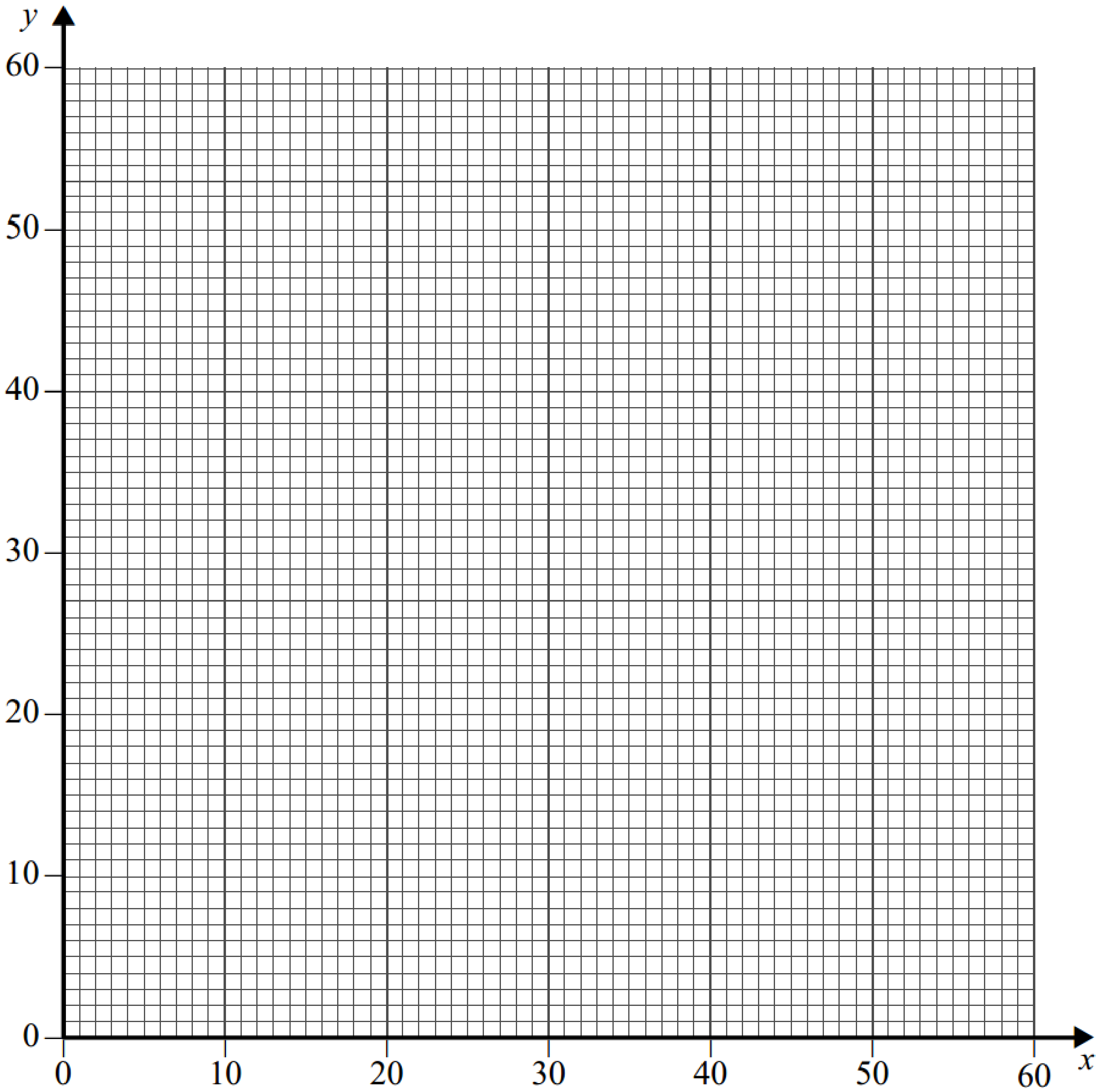
$$x + y \leqslant 60$$
$$2x + y \leqslant 80$$
$$y \geqslant 20$$
$$x \geqslant 15$$
$$y \geqslant x$$

(a) On the grid opposite, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) In each of the following cases, use your diagram to find the maximum value of P on the feasible region. In each case, state the corresponding values of x and y.

(i) $P = x + 4y$ *(2 marks)*

(ii) $P = 4x + y$ *(3 marks)*


y
60
50
40
30
20
10
0
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10
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30
40
50
60
x

7 Paul is a florist. Every day, he makes three types of floral bouquet: gold, silver and bronze.

Each gold bouquet has 6 roses, 6 carnations and 6 dahlias.

Each silver bouquet has 4 roses, 6 carnations and 4 dahlias.

Each bronze bouquet has 3 roses, 4 carnations and 4 dahlias.

Each day, Paul must use at least 420 roses and at least 480 carnations, but he can use at most 720 dahlias.

Each day, Paul makes x gold bouquets, y silver bouquets and z bronze bouquets.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find three inequalities in x, y and z that model the above constraints. *(3 marks)*

(b) On a particular day, Paul makes the same number of silver bouquets as bronze bouquets.

(i) Show that x and y must satisfy the following inequalities.

$$6x + 7y \geqslant 420$$
$$3x + 5y \geqslant 240$$
$$3x + 4y \leqslant 360$$

(2 marks)

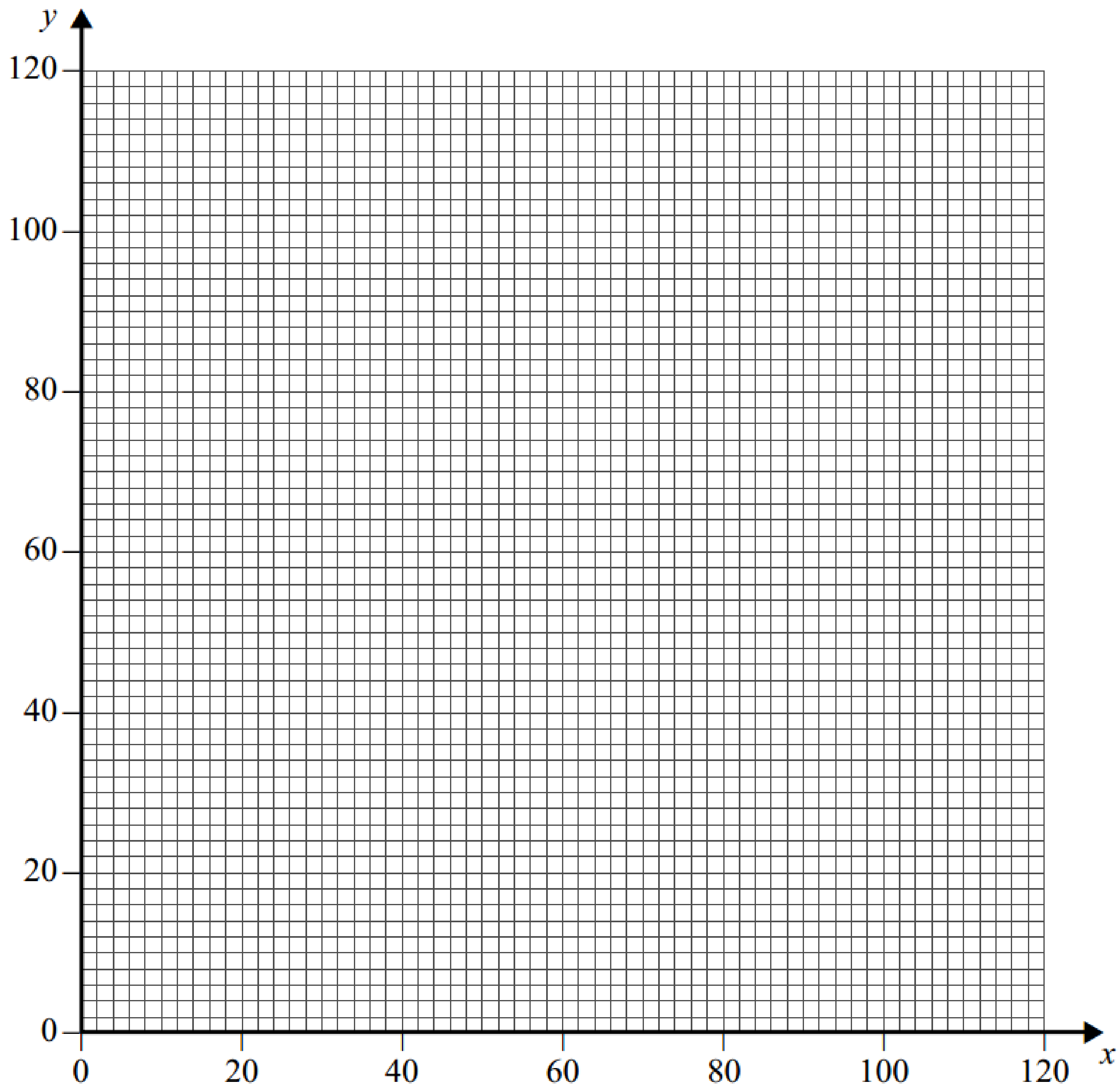
(ii) Paul makes a profit of £4 on each gold bouquet sold, a profit of £2.50 on each silver bouquet sold and a profit of £2.50 on each bronze bouquet sold. Each day, Paul sells all the bouquets he makes. Paul wishes to maximise his daily profit, £P.

Draw a suitable diagram, on the grid opposite, to enable this problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*

(iii) Use your diagram to find Paul's maximum daily profit and the number of each type of bouquet he must make to achieve this maximum. *(2 marks)*

(c) On another day, Paul again makes the same number of silver bouquets as bronze bouquets, but he makes a profit of £2 on each gold bouquet sold, a profit of £6 on each silver bouquet sold and a profit of £6 on each bronze bouquet sold.

Find Paul's maximum daily profit, and the number of each type of bouquet he must make to achieve this maximum. *(3 marks)*


y
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