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# Core 2: Logarithms

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Past Paper Questions  
2006 - 2013

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**Logarithms and exponentials**

$$a^x = e^{x \ln a}$$

January 2006

- 3 (a) Use logarithms to solve the equation  $0.8^x = 0.05$ , giving your answer to three decimal places. *(3 marks)*
- (b) An infinite geometric series has common ratio  $r$ . The sum to infinity of the series is five times the first term of the series.
- (i) Show that  $r = 0.8$ . *(3 marks)*
- (ii) Given that the first term of the series is 20, find the least value of  $n$  such that the  $n$ th term of the series is less than 1. *(3 marks)*

7 It is given that  $n$  satisfies the equation

$$2 \log_a n - \log_a (5n - 24) = \log_a 4$$

- (a) Show that  $n^2 - 20n + 96 = 0$ . *(3 marks)*
- (b) Hence find the possible values of  $n$ . *(2 marks)*

June 2006

5 (a) Given that

$$\log_a x = 2 \log_a 6 - \log_a 3$$

show that  $x = 12$ .

*(3 marks)*

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express  $y$  in terms of  $a$ , giving your answer in a form not involving logarithms.

*(3 marks)*

Question 6

- (c) (i) Use logarithms to solve the equation  $3^x = 13$ , giving your answer to four decimal places. *(3 marks)*
- (ii) The line  $y = k$  intersects the curve  $y = 27 - 3^x$  at the point where  $3^x = 13$ . Find the value of  $k$ . *(1 mark)*

January 2007

**9** (a) Solve the equation  $3 \log_a x = \log_a 8$ . *(2 marks)*

(b) Show that

$$3 \log_a 6 - \log_a 8 = \log_a 27$$
*(3 marks)*

(c) (i) The point  $P(3, p)$  lies on the curve  $y = 3 \log_{10} x - \log_{10} 8$ .

Show that  $p = \log_{10} \left( \frac{27}{8} \right)$ . *(2 marks)*

(ii) The point  $Q(6, q)$  also lies on the curve  $y = 3 \log_{10} x - \log_{10} 8$ .

Show that the gradient of the line  $PQ$  is  $\log_{10} 2$ . *(4 marks)*

June 2007

**6** (c) The line  $y = 21$  intersects the curve  $y = 3(2^x + 1)$  at the point  $P$ .

(i) Show that the  $x$ -coordinate of  $P$  satisfies the equation

$$2^x = 6$$
*(1 mark)*

(ii) Use logarithms to find the  $x$ -coordinate of  $P$ , giving your answer to three significant figures. *(3 marks)*

**8** (a) It is given that  $n$  satisfies the equation

$$\log_a n = \log_a 3 + \log_a (2n - 1)$$

Find the value of  $n$ . *(3 marks)*

(b) Given that  $\log_a x = 3$  and  $\log_a y - 3 \log_a 2 = 4$ :

(i) express  $x$  in terms of  $a$ ; *(1 mark)*

(ii) express  $xy$  in terms of  $a$ . *(4 marks)*

January 2008

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of  $x$ .

(1 mark)

(b) Given that

$$\log_a y = 2 \log_a 3 + \log_a 4 + 1$$

express  $y$  in terms of  $a$ , giving your answer in a form **not** involving logarithms.

(3 marks)

June 2008

5 (a) Write down the value of:

(i)  $\log_a 1$ ;

(1 mark)

(ii)  $\log_a a$ .

(1 mark)

(b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of  $x$ .

(3 marks)

January 2009

6 (a) Write each of the following in the form  $\log_a k$ , where  $k$  is an integer:

(i)  $\log_a 4 + \log_a 10$ ;

(1 mark)

(ii)  $\log_a 16 - \log_a 2$ ;

(1 mark)

(iii)  $3 \log_a 5$ .

(1 mark)

(b) Use logarithms to solve the equation  $(1.5)^{3x} = 7.5$ , giving your value of  $x$  to three decimal places.

(3 marks)

(c) Given that  $\log_2 p = m$  and  $\log_8 q = n$ , express  $pq$  in the form  $2^y$ , where  $y$  is an expression in  $m$  and  $n$ .

(3 marks)

June 2009

- 9 (a) (i)** Find the value of  $p$  for which  $\sqrt{125} = 5^p$ . (2 marks)
- (ii)** Hence solve the equation  $5^{2x} = \sqrt{125}$ . (1 mark)
- (b)** Use logarithms to solve the equation  $3^{2x-1} = 0.05$ , giving your value of  $x$  to four decimal places. (3 marks)
- (c)** It is given that
- $$\log_a x = 2(\log_a 3 + \log_a 2) - 1$$
- Express  $x$  in terms of  $a$ , giving your answer in a form not involving logarithms. (4 marks)

January 2010

- 3 (a)** Find the value of  $x$  in each of the following:
- (i)**  $\log_9 x = 0$ ; (1 mark)
- (ii)**  $\log_9 x = \frac{1}{2}$ . (1 mark)
- (b)** Given that
- $$2 \log_a n = \log_a 18 + \log_a (n - 4)$$
- find the possible values of  $n$ . (5 marks)

June 2010 Question 8

- (e) (i)** Given that
- $$\log_a k = 3 \log_a 2 + \log_a 5 - \log_a 4$$
- show that  $k = 10$ . (3 marks)
- (ii)** The line  $y = \frac{5}{4}$  crosses the curve  $y = 2^{4x-3}$  at the point  $P$ . Show that the  $x$ -coordinate of  $P$  is  $\frac{1}{4 \log_{10} 2}$ . (3 marks)

January 2011

- 8 (a)** Given that  $2 \log_k x - \log_k 5 = 1$ , express  $k$  in terms of  $x$ . Give your answer in a form not involving logarithms. (4 marks)
- (b)** Given that  $\log_a y = \frac{3}{2}$  and that  $\log_4 a = b + 2$ , show that  $y = 2^p$ , where  $p$  is an expression in terms of  $b$ . (3 marks)

January 2012

**7 (c)** Given that

$$\log_a(b^2) + 3 \log_a y = 3 + 2 \log_a \left(\frac{y}{a}\right)$$

express  $y$  in terms of  $a$  and  $b$ .

Give your answer in a form not involving logarithms.

*(5 marks)*

January 2013

**4** Given that

$$\log_a N - \log_a x = \frac{3}{2}$$

express  $x$  in terms of  $a$  and  $N$ , giving your answer in a form not involving logarithms.

*(3 marks)*