M2: Centre of Mass
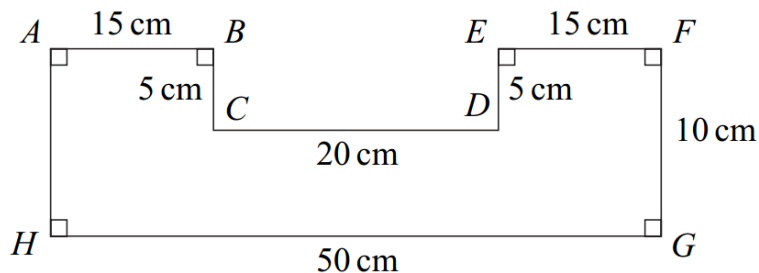
Past Paper Questions
2006 - 2013

Name:

4 The diagram shows a uniform lamina $ABCDEFGH$.

(a) Explain why the centre of mass is 25 cm from AH. *(1 mark)*

(b) Show that the centre of mass is 4.375 cm from HG. *(4 marks)*

(c) The lamina is freely suspended from A. Find the angle between AB and the vertical when the lamina is in equilibrium. *(4 marks)*

(d) Explain, briefly, how you have used the fact that the lamina is uniform. *(1 mark)*

2 A uniform lamina is in the shape of a rectangle $ABCD$ and a square $EFGH$, as shown in the diagram.

The length AB is 20 cm, the length BC is 30 cm, the length DE is 5 cm and the length EF is 10 cm.

The point P is the midpoint of AB and the point Q is the midpoint of HG.
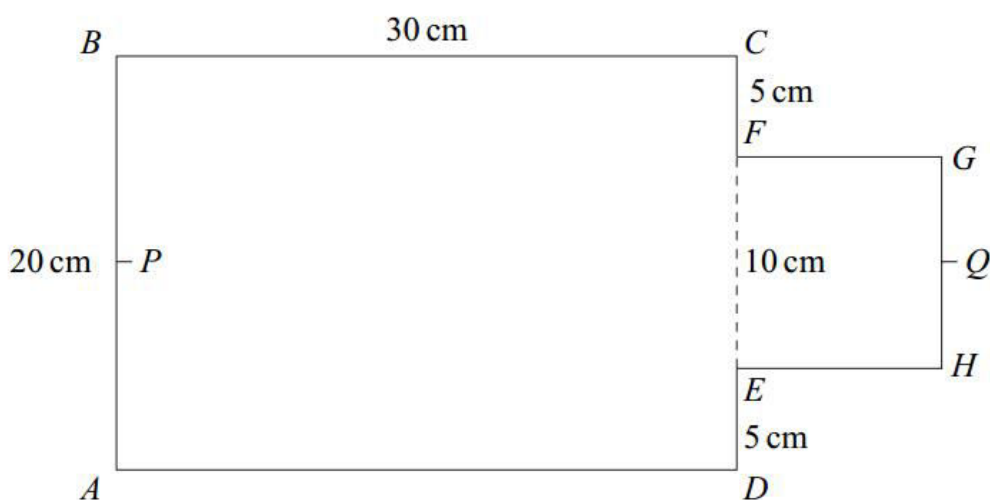
(a) Explain why the centre of mass of the lamina lies on PQ. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from AB. *(4 marks)*

(c) The lamina is freely suspended from A.

Find, to the nearest degree, the angle between AD and the vertical when the lamina is in equilibrium. *(4 marks)*

4 A uniform T-shaped lamina is formed by rigidly joining two rectangles *ABCH* and *DEFG*, as shown in the diagram.
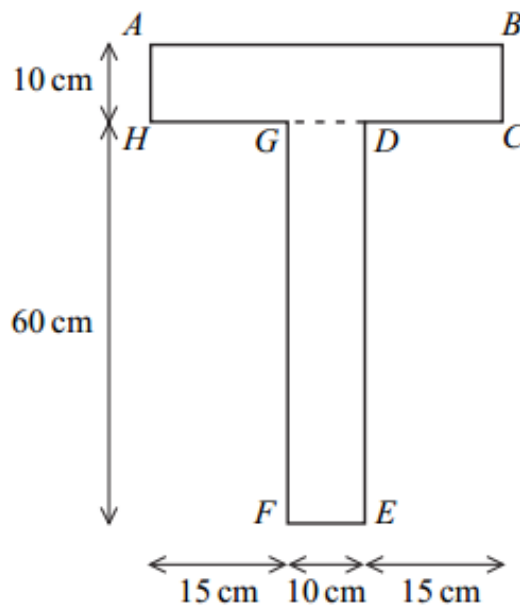
(a) Show that the centre of mass of the lamina is 26 cm from the edge *AB*. *(4 marks)*

(b) Explain why the centre of mass of the lamina is 5 cm from the edge *GF*. *(1 mark)*

(c) The point *X* is on the edge *AB* and is 7 cm from *A*, as shown in the diagram below.
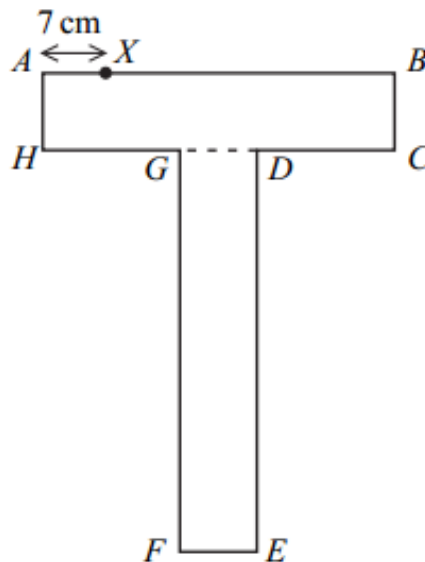
The lamina is freely suspended from *X* and hangs in equilibrium.

Find the angle between the edge *AB* and the vertical, giving your answer to the nearest degree. *(4 marks)*

3 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

Particle P has mass $1\,$kg and is attached at the point $(25, 10)$.
Particle Q has mass $4\,$kg and is attached at the point $(12, 7)$.
Particle R has mass $5\,$kg and is attached at the point $(4, 18)$.
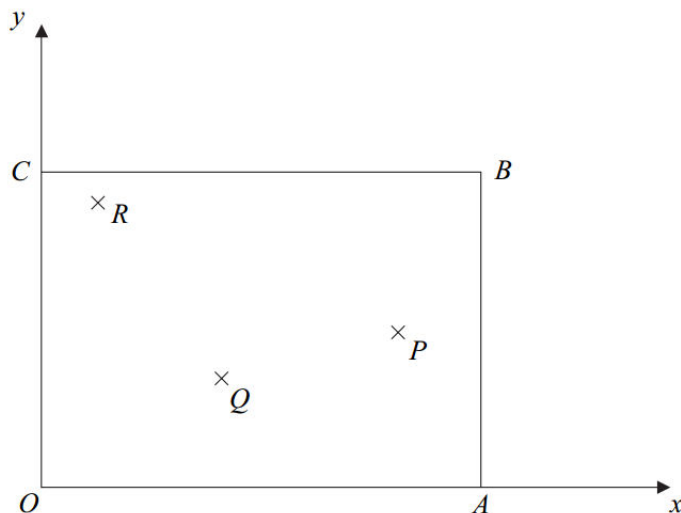
Find the coordinates of the centre of mass of the three particles. *(4 marks)*

4 A uniform rectangular lamina $ABCD$ has a mass of $8\,$kg. The side AB has length $20\,$cm, the side BC has length $10\,$cm, and P is the mid-point of AB.

A uniform circular lamina, of mass $2\,$kg and radius $5\,$cm, is fixed to the rectangular lamina to form a sign. The centre of the circular lamina is $5\,$cm from each of AB and BC, as shown in the diagram.
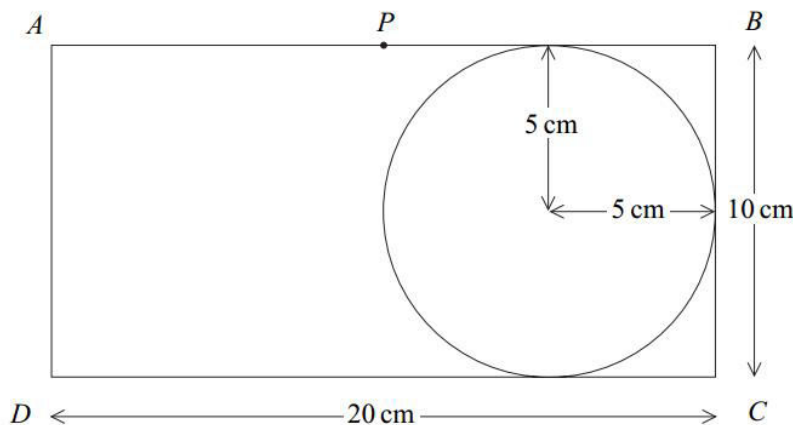
(a) Find the distance of the centre of mass of the sign from AD. *(3 marks)*

(b) Write down the distance of the centre of mass of the sign from AB. *(1 mark)*

(c) The sign is freely suspended from P.

Find the angle between AD and the vertical when the sign is in equilibrium. *(4 marks)*

(d) Explain how you have used the fact that each lamina is uniform in your solution to this question. *(1 mark)*

2 A piece of modern art is modelled as a uniform lamina and three particles. The diagram shows the lamina, the three particles A, B and C, and the x- and y-axes.
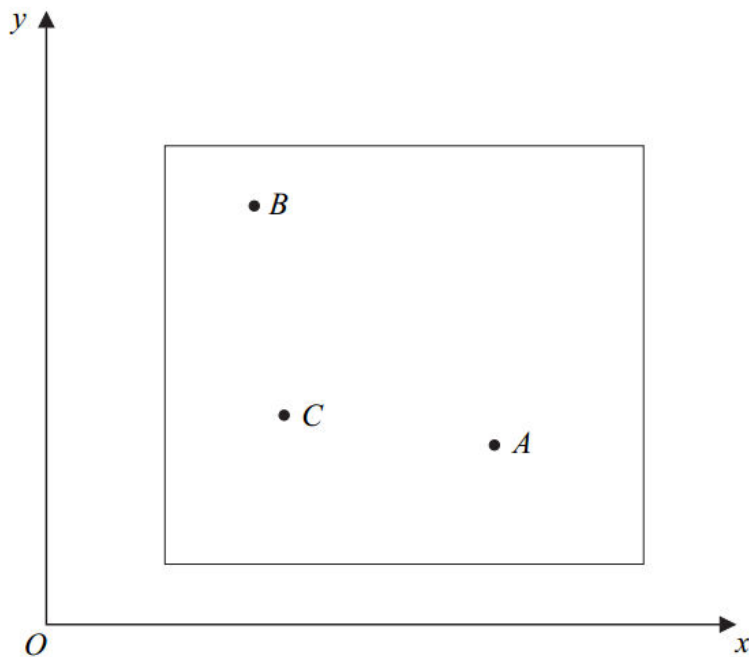
The lamina, which is fixed in the x–y plane, has mass 10 kg and its centre of mass is at the point $(12, 9)$.

The three particles are attached to the lamina.
Particle A has mass 3 kg and is at the point $(15, 6)$.
Particle B has mass 1 kg and is at the point $(7, 14)$.
Particle C has mass 6 kg and is at the point $(8, 7)$.

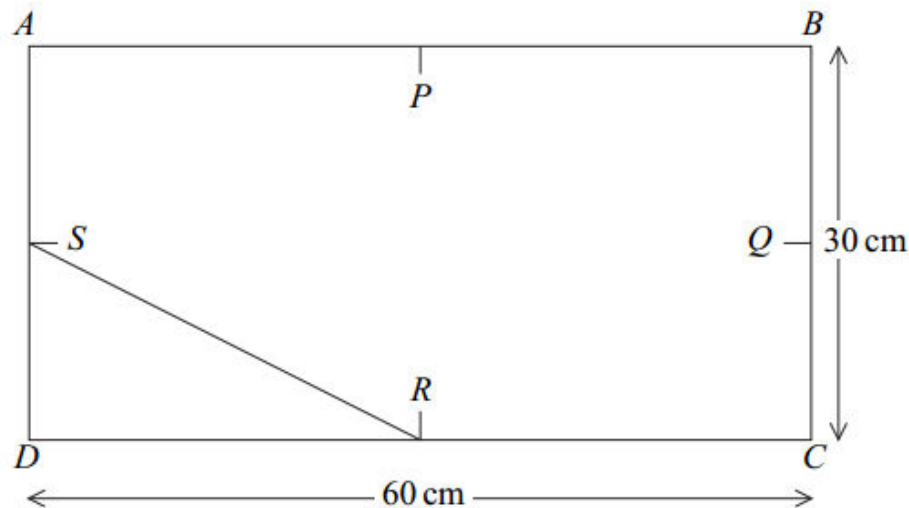Find the coordinates of the centre of mass of the piece of modern art. *(6 marks)*

3 A uniform circular lamina, of radius 4 cm and mass 0.4 kg, has a centre O, and AB is a diameter. To create a medal, a smaller uniform circular lamina, of radius 2 cm and mass 0.1 kg, is attached so that the centre of the smaller lamina is at the point A, as shown in the diagram.
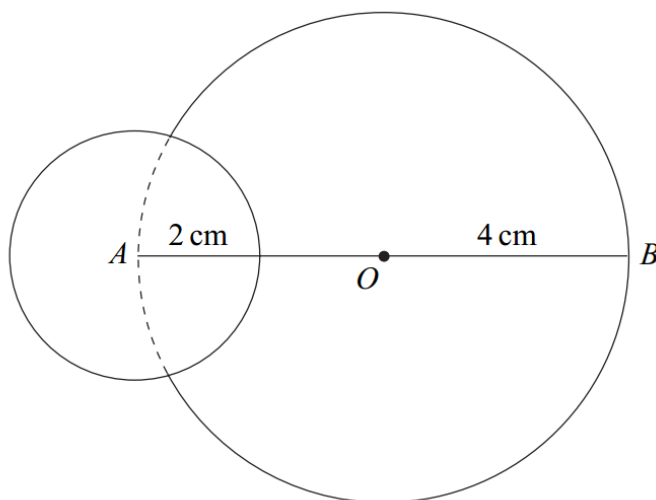
(a) Explain why the centre of mass of the medal is on the line AB. *(1 mark)*

(b) Find the distance of the centre of mass of the medal from the point B. *(3 marks)*

4   A uniform rectangular lamina $ABCD$ has a mass of 5 kg. The side AB has length 60 cm and the side BC has length 30 cm. The points P, Q, R and S are the mid-points of the sides, as shown in the diagram below.

A uniform triangular lamina SRD, of mass 4 kg, is fixed to the rectangular lamina to form a shop sign. The centre of mass of the triangular lamina SRD is 10 cm from the side AD and 5 cm from the side DC.

(a)   Find the distance of the centre of mass of the shop sign from AD.   *(3 marks)*

(b)   Find the distance of the centre of mass of the shop sign from AB.   *(3 marks)*

**(c)**   The shop sign is freely suspended from P.

Find the angle between AB and the horizontal when the shop sign is in equilibrium.
    (4 marks)

(d)   To ensure that the side AB is horizontal when the shop sign is freely suspended from point P, a particle of mass m kg is attached to the shop sign at point B.

Calculate m.   *(3 marks)*

(e)   Explain how you have used the fact that the rectangular lamina $ABCD$ is uniform in your solution to this question.   *(1 mark)*

2   The diagram shows four particles, A, B, C and D, which are fixed in a horizontal plane which contains the x- and y-axes, as shown.

Particle A has mass 2 kg and is attached at the point $(9, 6)$.
Particle B has mass 3 kg and is attached at the point $(2, 4)$.
Particle C has mass 8 kg and is attached at the point $(3, 8)$.
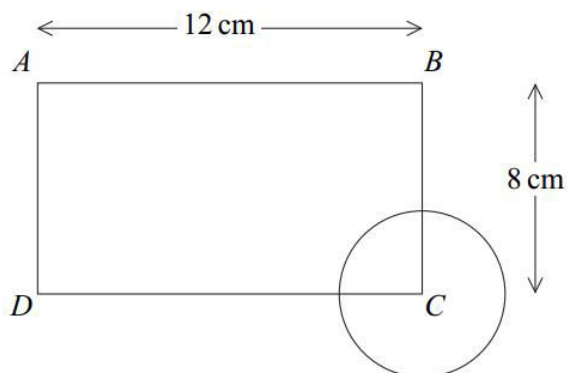Particle D has mass 7 kg and is attached at the point $(6, 11)$.
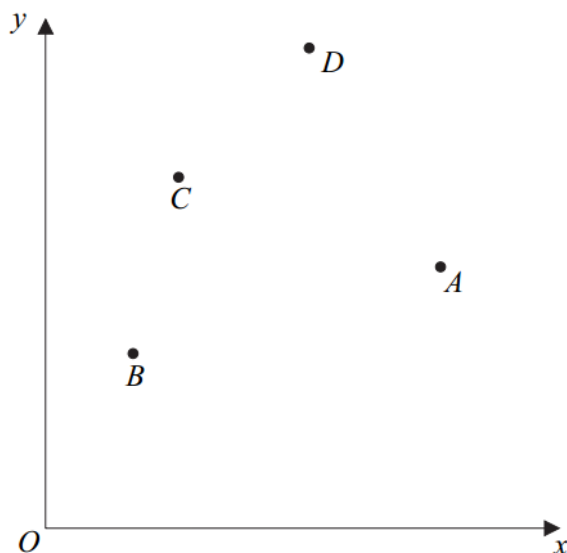
Find the coordinates of the centre of mass of the four particles.   *(5 marks)*

3   A uniform rectangular lamina $ABCD$, of mass 1.6 kg, has side AB of length 12 cm and side BC of length 8 cm.

To create a logo, a uniform circular lamina, of mass 0.4 kg, is attached. The centre of the circular lamina is at the point C, as shown in the diagram.

(a)   Find the distance of the centre of mass of the logo:

  (i)   from the line AB;   *(3 marks)*

 **(ii)** from the line AD.   *(3 marks)*

(b)   The logo is suspended in equilibrium, with AB horizontal, by two vertical strings. One string is attached at the point A and the other string is attached at the point B.

Find the tension in each of the two strings.   *(5 marks)*

4   The diagram shows a uniform lamina which is in the shape of two identical rectangles $AXGH$ and $YBCD$ and a square $XYEF$, arranged as shown.

The length of AX is 10 cm, the length of XY is 10 cm and the length of AH is 30 cm.
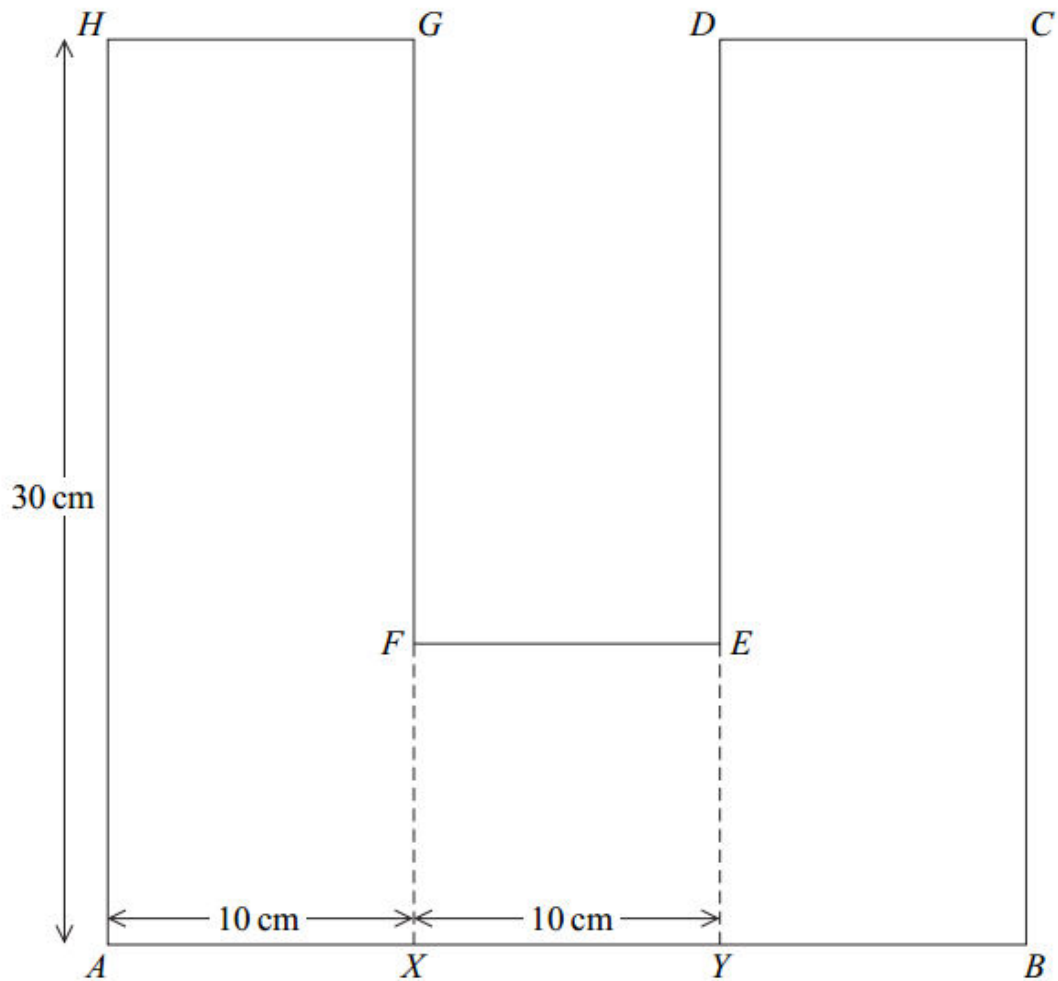
(a)   Explain why the centre of mass of the lamina is 15 cm from AH.   *(1 mark)*

(b)   Find the distance of the centre of mass of the lamina from AB.   *(3 marks)*

(c)   The lamina is freely suspended from the point H.

Find, to the nearest degree, the angle between HG and the horizontal when the lamina is in equilibrium.   *(4 marks)*