M2: Hooke’s Law

Past Paper Questions
2006 - 2013

Name:

8 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point O.

(a) Find the length of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(b) The particle is pulled down and held at a point P, which is 1 metre vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 45 J. *(2 marks)*

(c) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\text{m s}^{-1}$ when it is x metres **below** $\boldsymbol{O}$.

(i) Show that, while the string is taut,

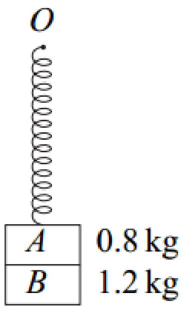
$$v^2 = 39.6x - 25x^2 - 14.6$$ *(7 marks)*

(ii) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(3 marks)*

January 2007

8 Two small blocks, A and B, of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O.

(a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. *(3 marks)*

(b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. *(2 marks)*

(c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

(i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0$$ *(5 marks)*

(ii) Find the value of x. *(2 marks)*

June 2007

6 An elastic string has one end attached to a point O, fixed on a horizontal table. The other end of the string is attached to a particle of mass 5 kilograms. The elastic string has natural length 2 metres and modulus of elasticity 200 newtons. The particle is pulled so that it is 2.5 metres from the point O and it is then released from rest on the table.

(a) Calculate the elastic potential energy when the particle is 2.5 m from the point O. *(2 marks)*

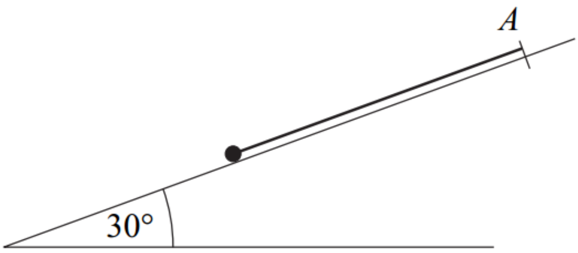
(b) If the table is smooth, show that the speed of the particle when the string becomes slack is $\sqrt{5}$ m s^{-1}. *(3 marks)*

(c) The table is, in fact, rough and the coefficient of friction between the particle and the table is 0.4.

Find the speed of the particle when the string becomes slack. *(7 marks)*

6 A light elastic string has one end attached to a point *A* fixed on a smooth plane inclined at 30° to the horizontal. The other end of the string is attached to a particle of mass 6 kg. The elastic string has natural length 4 metres and modulus of elasticity 300 newtons.

The particle is pulled down the plane in the direction of the line of greatest slope through *A*. The particle is released from rest when it is 5.5 metres from *A*.



(a) Calculate the elastic potential energy of the string when the particle is 5.5 metres from the point *A*. *(2 marks)*

(b) Show that the speed of the particle when the string becomes slack is $3.66\,\text{m}\,\text{s}^{-1}$, correct to three significant figures. *(5 marks)*

(c) Show that the particle will **not** reach point *A* in the subsequent motion. *(3 marks)*

June 2008

8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\dfrac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 0.9 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. *(2 marks)*

(iii) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\text{m s}^{-1}$ when it is x metres **above** $\boldsymbol{P}$.

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2$$ *(7 marks)*

(iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(2 marks)*

January 2009

9 A bungee jumper, of mass 80 kg, is attached to one end of a light elastic cord, of natural length 16 metres and modulus of elasticity 784 N. The other end of the cord is attached to a horizontal platform, which is at a height of 65 metres above the ground.

The bungee jumper steps off the platform at the point where the cord is attached and falls vertically. The bungee jumper can be modelled as a particle. Hooke's law can be assumed to apply throughout the motion and air resistance can be assumed to be negligible.

(a) Find the length of the cord when the acceleration of the bungee jumper is zero. *(3 marks)*

(b) The cord extends by x metres beyond its natural length before the bungee jumper first comes to rest.

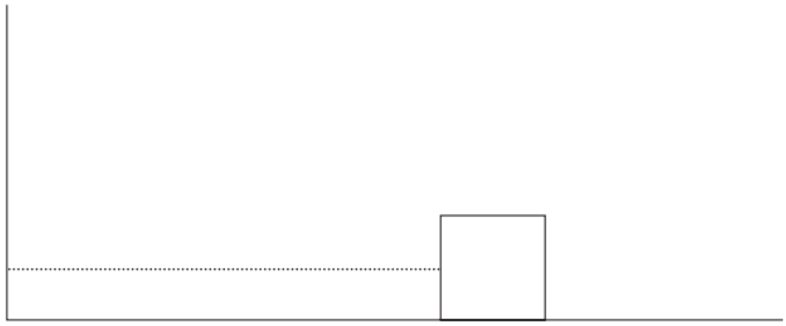
(i) Show that $x^2 - 32x - 512 = 0$. *(4 marks)*

(ii) Find the distance above the ground at which the bungee jumper first comes to rest. *(4 marks)*

June 2009

6 A block, of mass 5 kg, is attached to one end of a length of elastic string. The other end of the string is fixed to a vertical wall. The block is placed on a horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 180 N. The block is pulled so that it is 2 m from the wall and is then released from rest. Whilst taut, the string remains horizontal. It may be assumed that, after the string becomes slack, it does not interfere with the movement of the block.



(a) Calculate the elastic potential energy when the block is 2 m from the wall. *(2 marks)*

(b) If the horizontal surface is smooth, find the speed of the block when it hits the wall. *(3 marks)*

(c) The surface is in fact rough and the coefficient of friction between the block and the surface is μ.

Find μ if the block comes to rest just as it reaches the wall. *(7 marks)*

January 2010

8 A bungee jumper, of mass 49 kg, is attached to one end of a light elastic cord of natural length 22 metres and modulus of elasticity 1078 newtons. The other end of the cord is attached to a horizontal platform, which is at a height of 60 metres above the ground.

The bungee jumper steps off the platform at the point where the cord is attached, and falls vertically. The bungee jumper can be modelled as a particle. Assume that Hooke's Law applies whilst the cord is taut and that air resistance is negligible throughout the motion.

When the bungee jumper has fallen x metres, his speed is $v\,\text{m s}^{-1}$.

(a) By considering energy, show that, when x is greater than 22,

$$5v^2 = 318x - 5x^2 - 2420$$ *(6 marks)*

(b) Explain why x must be greater than 22 for the equation in part (a) to be valid. *(1 mark)*

(c) Find the maximum value of x. *(4 marks)*

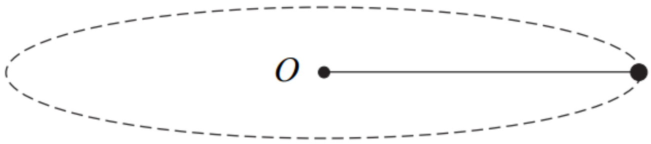
(d) (i) Show that the speed of the bungee jumper is a maximum when $x = 31.8$. *(3 marks)*

(ii) Hence find the maximum speed of the bungee jumper. *(2 marks)*

June 2010

9 A particle, of mass 8 kg, is attached to one end of a length of elastic string. The particle is placed on a smooth horizontal surface. The other end of the elastic string is attached to a point O fixed on the horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 192 N.



The particle is set in motion on the horizontal surface so that it moves in a circle, centre O, with constant speed 3 m s^{-1}.

Find the radius of the circle. *(8 marks)*

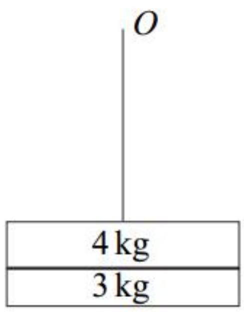
January 2011

7 (a) An elastic string has natural length l and modulus of elasticity λ. The string is stretched from length l to length $l + e$.

Show, by integration, that the work done in stretching the string is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A block, of mass 4 kg, is attached to one end of a light elastic string. The string has natural length 2 m and modulus of elasticity 196 N. The other end of the string is attached to a fixed point O.

(i) A second block, of mass 3 kg, is attached to the 4 kg block and the system hangs in equilibrium, as shown in the diagram.



Find the extension in the string. *(3 marks)*

(ii) The block of mass 3 kg becomes detached from the 4 kg block and falls to the ground. The 4 kg block now begins to move vertically upwards.

Find the extension of the string when the 4 kg block is next at rest. *(6 marks)*

(iii) Find the extension of the string when the speed of the 4 kg block is a maximum. *(3 marks)*

9 At a theme park, a light elastic rope is used to bring a carriage to rest at the end of a ride.

The carriage has mass 200 kg and is travelling at $8\,\text{m s}^{-1}$ when the elastic rope is attached to the carriage as it passes over a point O. The other end of the elastic rope is fixed to the point O. The carriage then moves along a horizontal surface until it is brought to rest. The elastic rope is then detached so that the carriage remains at rest.

The elastic rope has natural length 6 m and modulus of elasticity 1800 N. The rope, once taut, remains horizontal throughout the motion.

(a) Calculate the elastic potential energy of the rope when the carriage is 10 m from O. *(3 marks)*

(b) A student's simple model assumes that there are no resistance forces acting on the carriage so that it is brought to rest by the elastic rope alone.

Find the distance of the carriage from O when it is brought to rest. *(3 marks)*

(c) The student improves the model by also including a constant resistance force of 800 N which acts while the carriage is in motion.

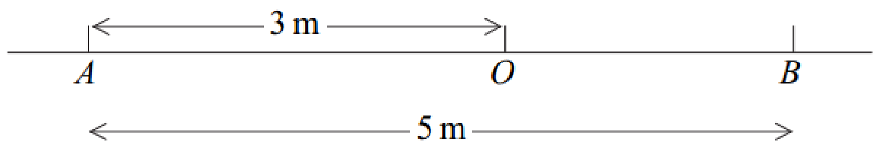
Find the distance of the carriage from O when it is brought to rest. *(8 marks)*

8 An elastic string has one end attached to a point O fixed on a rough horizontal surface. The other end of the string is attached to a particle of mass 2 kg. The elastic string has natural length 0.8 metres and modulus of elasticity 32 newtons.

The particle is pulled so that it is at the point A, on the surface, 3 metres from the point O.

(a) Calculate the elastic potential energy when the particle is at the point A. *(3 marks)*

(b) The particle is released from rest at the point A and moves in a straight line towards O. The particle is next at rest at the point B. The distance AB is 5 metres.



Find the frictional force acting on the particle as it moves along the surface. *(6 marks)*

(c) Show that the particle does not remain at rest at the point B. *(2 marks)*

(d) The particle next comes to rest at a point C with the string slack.

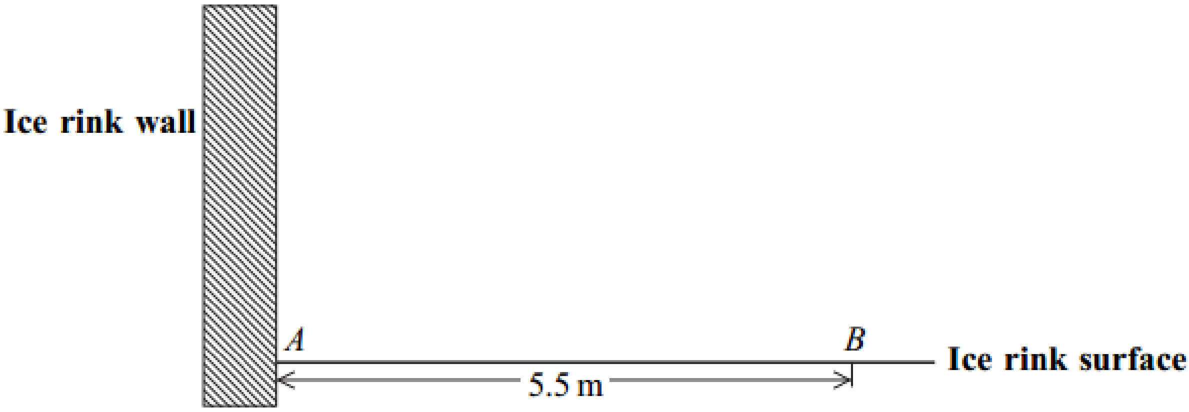
Find the distance BC. *(2 marks)*

(e) Hence, or otherwise, find the total distance travelled by the particle after it is released from the point A. *(1 mark)*

8 Zoë carries out an experiment with a block, which she places on the horizontal surface of an ice rink. She attaches one end of a light elastic string to a fixed point, A, on a vertical wall at the edge of the ice rink at the height of the surface of the ice rink.

The block, of mass 0.4 kg, is attached to the other end of the string. The string has natural length 5 m and modulus of elasticity 120 N.

The block is modelled as a particle which is placed on the surface of the ice rink at a point B, where AB is perpendicular to the wall and of length 5.5 m.



The block is set into motion at the point B with speed $9\,\text{m}\,\text{s}^{-1}$ directly towards the point A. The string remains horizontal throughout the motion.

(a) Initially, Zoë assumes that the surface of the ice rink is smooth.

Using this assumption, find the speed of the block when it reaches the point A.
(4 marks)

(b) Zoë now assumes that friction acts on the block. The coefficient of friction between the block and the surface of the ice rink is μ.

(i) Find, in terms of g and μ, the speed of the block when it reaches the point A.
(6 marks)

(ii) The block rebounds from the wall in the direction of the point B. The speed of the block immediately after the rebound is half of the speed with which it hit the wall.

Find μ if the block comes to rest just as it reaches the point B. *(6 marks)*

January 2013

8 (a) An elastic string has natural length l and modulus of elasticity λ. The string is stretched from length l to length $l + e$.

Show, by integration, that the work done in stretching the string is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is attached to a fixed point O.

The string has natural length 1.6 m and modulus of elasticity 392 N.

(i) Find the extension of the string when the particle hangs in equilibrium. *(2 marks)*

(ii) The particle is pulled down to a point A, which is 2.2 m below the point O.

Calculate the elastic potential energy in the string. *(3 marks)*

(iii) The particle is released when it is at rest at the point A.

Calculate the distance of the particle from the point A when its speed first reaches $0.8\,\text{m s}^{-1}$. *(5 marks)*

June 2013

9 Two particles, A and B, are connected by a light elastic string that passes through a hole at a point O in a rough horizontal table. The edges of the hole are smooth. Particle A has a mass of 8 kg and particle B has a mass of 3 kg.

The elastic string has natural length 3 metres and modulus of elasticity 60 newtons.

Initially, particle A is held 3.5 metres from the point O on the surface of the table and particle B is held at a point 2 metres vertically below O.

The coefficient of friction between the table and particle A is 0.4 .

The two particles are released from rest.

(a) (i) Show that initially particle A moves towards the hole in the table. *(3 marks)*

(ii) Show that initially particle B also moves towards the hole in the table. *(2 marks)*

(b) Calculate the **initial** elastic potential energy in the string. *(2 marks)*

(c) Particle A comes permanently to rest when it has moved 0.46 metres, at which time particle B is still moving upwards.

Calculate the distance that particle B has moved when it is at rest for the first time. *(7 marks)*