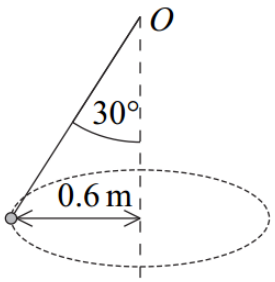
M2: Summations

Past Paper Questions 2006 - 2013

Name:

January 2006

2 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O. The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O.



(a) Show that the tension in the string is 22.6 N, correct to three significant figures. *(3 marks)*

(b) Find the speed of the particle. *(4 marks)*

June 2006

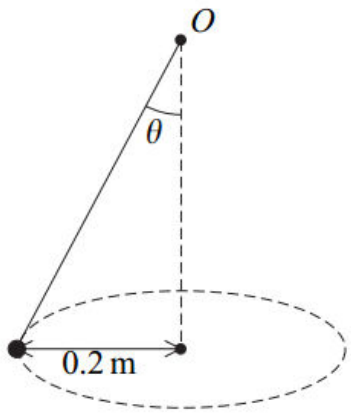
6 A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of $14\,\text{m}\,\text{s}^{-1}$. The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. *(4 marks)*

(b) The coefficient of friction between the car and the road is μ. Show that $\mu \geqslant 0.4$. *(3 marks)*

January 2007

6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

(c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

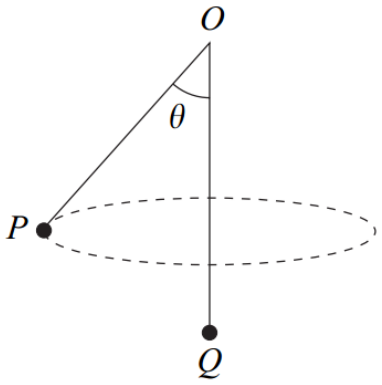
(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

June 2007

8 A particle, P, of mass 3 kg is attached to one end of a light inextensible string. The string passes through a smooth fixed ring, O, and a second particle, Q, of mass 5 kg is attached to the other end of the string. The particle Q hangs at rest vertically below the ring and the particle P moves with speed $4\,\text{m}\,\text{s}^{-1}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



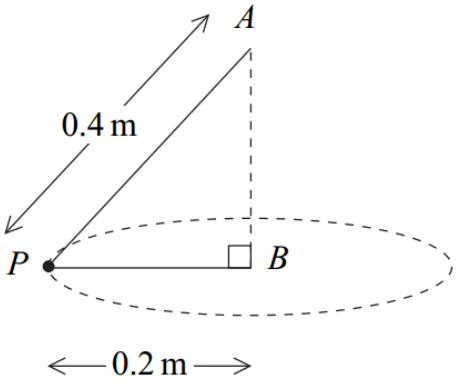
(a) Explain why the tension in the string is 49 N. *(2 marks)*

(b) Find θ. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

January 2008

5 Two light inextensible strings, of lengths 0.4 m and 0.2 m, each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the points A and B respectively. The point A is vertically above the point B. The particle moves in a horizontal circle, centre B and radius 0.2 m, at a speed of $2\,\text{m}\,\text{s}^{-1}$. The particle and strings are shown in the diagram.



(a) Calculate the magnitude of the acceleration of the particle. *(2 marks)*

(b) Show that the tension in string PA is 45.3 N, correct to three significant figures. *(4 marks)*

(c) Find the tension in string PB. *(3 marks)*

June 2008

5 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

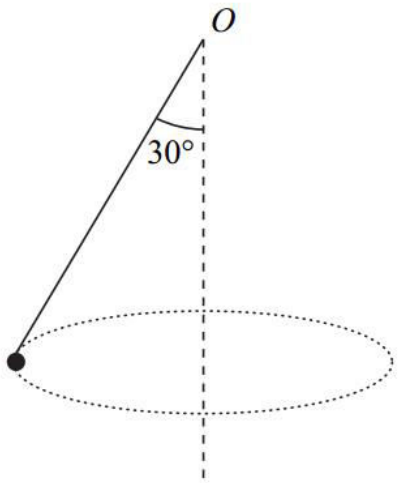
(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

January 2009

5 A particle, of mass 6 kg, is attached to one end of a light inextensible string. The other end of the string is attached to the fixed point O. The particle is set in motion, so that it moves in a horizontal circle at constant speed, with the string at an angle of 30° to the vertical. The centre of this circle is vertically below O.



The particle moves in a horizontal circle with an angular speed of 40 revolutions per minute.

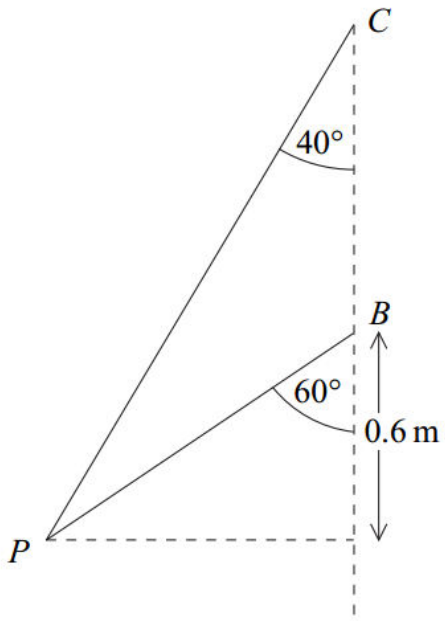
(a) Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) Show that the tension in the string is 67.9 N, correct to three significant figures. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

June 2009

4 Two light inextensible strings each have one end attached to a particle, P, of mass 6 kg. The other ends of the strings are attached to the fixed points B and C. The point C is vertically above the point B. The particle moves, at constant speed, in a horizontal circle, with centre 0.6 m below point B, with the strings inclined at 40° and 60° to the vertical, as shown in the diagram. Both strings are taut.



(a) As the particle moves in the horizontal circle, the tensions in the two strings are equal.

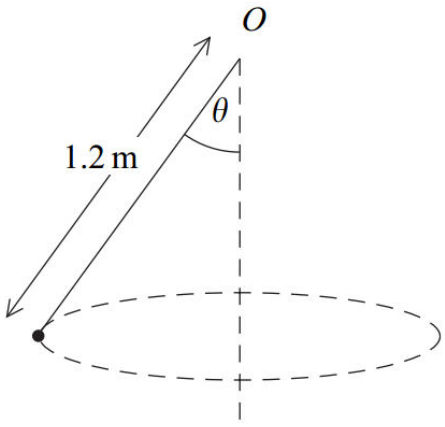
Show that the tension in the strings is 46.4 N, correct to three significant figures.

(4 marks)

(b) Find the speed of the particle. *(4 marks)*

January 2010

6 A particle, of mass 4 kg, is attached to one end of a light inextensible string of length 1.2 metres. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle at a constant speed. The angle between the string and the vertical is θ.



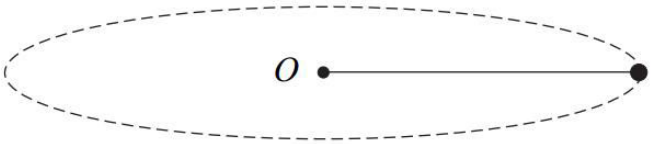
(a) Find the radius of the horizontal circle in terms of θ. *(1 mark)*

(b) The angular speed of the particle is 5 radians per second. Find θ. *(6 marks)*

June 2010

9 A particle, of mass 8 kg, is attached to one end of a length of elastic string. The particle is placed on a smooth horizontal surface. The other end of the elastic string is attached to a point O fixed on the horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 192 N.



The particle is set in motion on the horizontal surface so that it moves in a circle, centre O, with constant speed $3\,\text{m}\,\text{s}^{-1}$.

Find the radius of the circle. *(8 marks)*

January 2011

5 (a) A shiny coin is on a rough horizontal turntable at a distance 0.8 m from its centre. The turntable rotates at a constant angular speed. The coefficient of friction between the shiny coin and the turntable is 0.3.

Find the maximum angular speed, in radians per second, at which the turntable can rotate if the shiny coin is not going to slide. *(4 marks)*

(b) The turntable is stopped and the shiny coin is removed. An old coin is placed on the turntable at a distance 0.15 m from its centre. The turntable is made to rotate at a constant angular speed of 45 revolutions per minute.

(i) Find the angular speed of the turntable in radians per second. *(2 marks)*

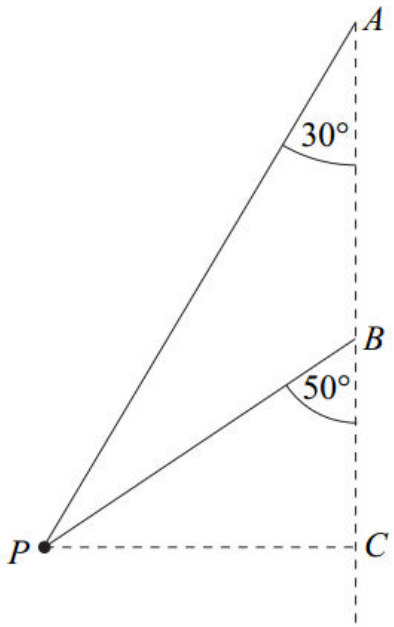
(ii) The old coin remains in the same position on the turntable.

Find the least value of the coefficient of friction between the old coin and the turntable needed to prevent the old coin from sliding. *(4 marks)*

7 Two light inextensible strings each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the fixed points A and B. The point A is vertically above the point B.

The particle moves at a constant speed in a horizontal circle. The centre, C, of this circle is directly below the point B. The two strings are inclined at 30° and 50° to the vertical, as shown in the diagram. Both strings are taut.

As the particle moves in the horizontal circle, the tension in the string BP is 20 N.



(a) Find the tension in the string AP. *(4 marks)*

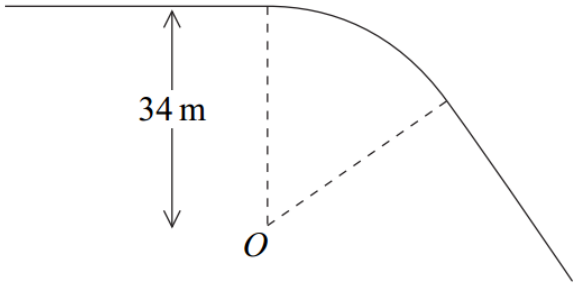
(b) The speed of the particle is $5\,\text{m}\,\text{s}^{-1}$.

Find the length of CP, the radius of the horizontal circle. *(4 marks)*

January 2012

5 A parcel is placed on a flat rough horizontal surface in a van. The van is travelling along a horizontal road. It travels around a bend of radius 34 m at a constant speed. The coefficient of friction between the parcel and the horizontal surface in the van is 0.85.

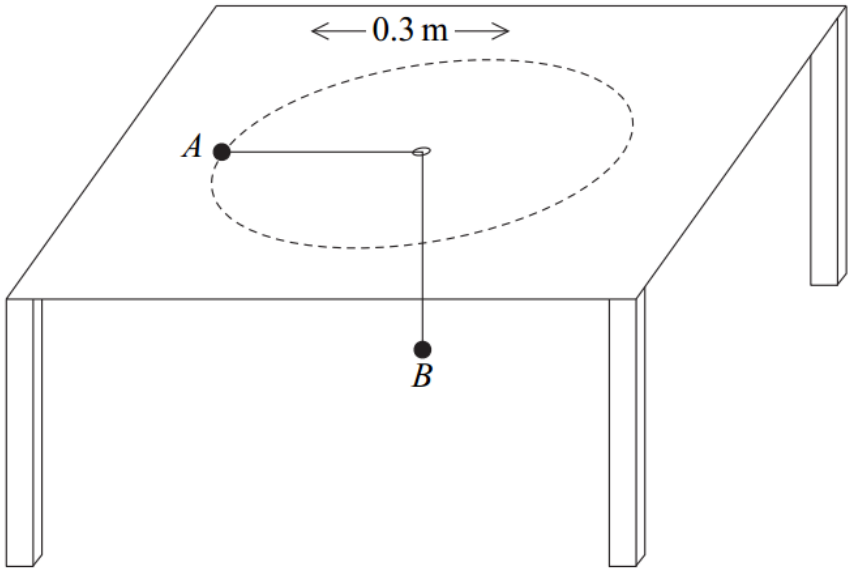
Model the parcel as a particle travelling around part of a circle of radius 34 m and centre O, as shown in the diagram.



Find the greatest speed at which the van can travel around the bend without causing the parcel to slide. *(6 marks)*

June 2012

5 Two particles, A and B, are connected by a light inextensible string which passes through a hole in a smooth horizontal table. The edges of the hole are also smooth. Particle A, of mass 1.4 kg, moves, on the table, with constant speed in a circle of radius 0.3 m around the hole. Particle B, of mass 2.1 kg, hangs in equilibrium under the table, as shown in the diagram.



(a) Find the angular speed of particle A. *(4 marks)*

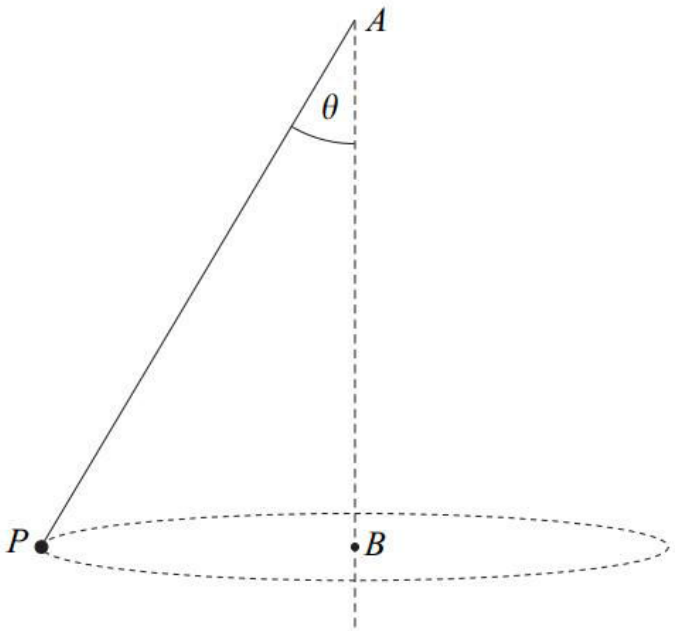
(b) Find the speed of particle A. *(2 marks)*

(c) Find the time taken for particle A to complete one full circle around the hole. *(2 marks)*

January 2013

6 A light inextensible string has one end attached to a particle, P, of mass 2 kg. The other end of the string is attached to the fixed point A. The point A is vertically above the point B. The particle moves at a constant speed in a horizontal circle of radius 0.8 m and centre B. The tension in the string is 34 N.

The string is inclined at an angle θ to the vertical, as shown in the diagram.



(a) Find the angle θ. *(3 marks)*

(b) Find the speed of the particle. *(3 marks)*

(c) Find the time taken for the particle to make one complete revolution. *(2 marks)*

June 2013

5 Tom is travelling on a train which is moving at a constant speed of 15 m s^{-1} on a horizontal track. Tom has placed his mobile phone on a rough horizontal table. The coefficient of friction between the phone and the table is 0.2.

The train moves round a bend of constant radius. The phone does not slide as the train travels round the bend.

Model the phone as a particle moving round part of a circle, with centre O and radius r metres.

Find the least possible value of r. *(4 marks)*