M2: Variable acceleration and forces

Past Paper Questions
2006 - 2013

Name:

January 2006

3 A particle moves in a straight line and at time t has velocity v, where

$$v = 2t - 12e^{-t}, \quad t \geqslant 0$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) State the range of values of the acceleration of the particle. *(3 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

5 A particle moves such that at time t seconds its acceleration is given by

$$(2\cos t\mathbf{i} - 5\sin t\mathbf{j})\,\mathrm{m\,s^{-2}}$$

(a) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 0$. *(3 marks)*

(b) When $t = 0$, the velocity of the particle is $(2\mathbf{i} + 10\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find an expression for the velocity of the particle at time t. *(5 marks)*

June 2006

1 A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time t seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

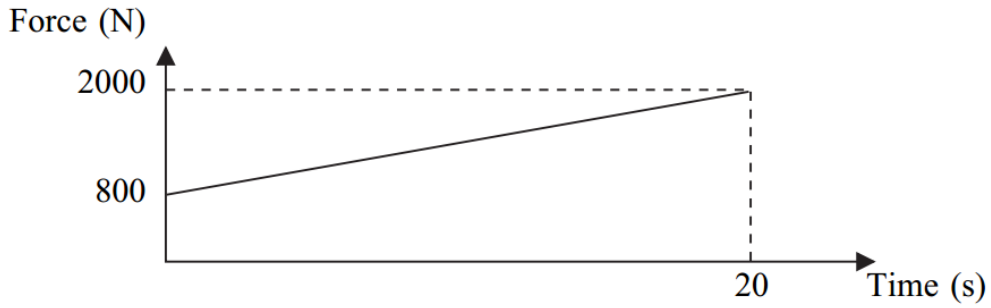
(b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

(d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

5 The graph shows a model for the resultant horizontal force on a car, which varies as it accelerates from rest for 20 seconds. The mass of the car is 1200 kg.



(a) The acceleration of the car at time t seconds is a m s^{-2}. Show that

$$a = \frac{2}{3} + \frac{t}{20}, \quad \text{for} \quad 0 \leqslant t \leqslant 20$$

(5 marks)

(b) Find an expression for the velocity of the car at time t. *(3 marks)*

(c) Find the distance travelled by the car in the 20 seconds. *(4 marks)*

(d) An alternative model assumes that the resultant force increases uniformly from 900 to 2100 newtons during the 20 seconds. Which term in your expression for the velocity would change as a result of this modification? Explain why. *(2 marks)*

January 2007

5 Tom is on a fairground ride.

Tom's position vector, **r** metres, at time t seconds is given by

$$\mathbf{r} = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (10 - 0.4t)\mathbf{k}$$

The perpendicular unit vectors **i** and **j** are in the horizontal plane and the unit vector **k** is directed vertically upwards.

(a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*

(ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*

(iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*

(b) Find an expression for Tom's velocity at time t. *(3 marks)*

(c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude. State the magnitude of the resultant force. *(5 marks)*

June 2007

3 A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j})\,\mathrm{m\,s^{-1}}$. The velocity of the particle at time t is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

January 2008

2 A particle moves in a straight line and at time t it has velocity v, where

$$v = 3t^2 - 2\sin 3t + 6$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. *(2 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

4 A particle moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively. At time t seconds, the position vector, $\mathbf{r}$ metres, of the particle is given by

$$\mathbf{r} = (t^3 - 3t^2 + 4)\mathbf{i} + (4t + t^2)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) The mass of the particle is 3 kg.

(i) Find an expression for $\mathbf{F}$ at time t. *(3 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 3$. *(2 marks)*

(c) Find the value of t when $\mathbf{F}$ acts due north. *(2 marks)*

June 2008

1 A particle moves in a straight line and at time t seconds has velocity $v\,\text{m s}^{-1}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

(c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

5 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

January 2009

1 A particle moves along a straight line. At time t, it has velocity v, where

$$v = 4t^3 - 8\sin 2t + 5$$

When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

3 A particle moves on a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the position vector of the particle is **r** metres, where

$$\mathbf{r} = \left(2e^{\frac{1}{2}t} - 8t + 5\right)\mathbf{i} + (t^2 - 6t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the speed of the particle when $t = 3$. *(2 marks)*

(ii) State the direction in which the particle is travelling when $t = 3$. *(1 mark)*

(c) Find the acceleration of the particle when $t = 3$. *(3 marks)*

(d) The mass of the particle is 7 kg.

Find the magnitude of the resultant force on the particle when $t = 3$. *(3 marks)*

June 2009

1 A particle moves under the action of a force, **F** newtons. At time t seconds, the velocity, $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, of the particle is given by

$$\mathbf{v} = (t^3 - 15t - 5)\mathbf{i} + (6t - t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 4 kg.

(i) Show that, at time t,

$$\mathbf{F} = (12t^2 - 60)\mathbf{i} + (24 - 8t)\mathbf{j}$$ *(2 marks)*

(ii) Find the magnitude of **F** when $t = 2$. *(4 marks)*

January 2010

4 A particle moves so that at time t seconds its velocity $\mathbf{v}\,\text{m s}^{-1}$ is given by

$$\mathbf{v} = (4t^3 - 12t + 3)\mathbf{i} + 5\mathbf{j} + 8t\mathbf{k}$$

(a) When $t = 0$, the position vector of the particle is $(-5\mathbf{i} + 6\mathbf{k})$ metres.

Find the position vector of the particle at time t. *(4 marks)*

(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) Find the magnitude of the acceleration of the particle at time t. Do not simplify your answer. *(2 marks)*

(d) Hence find the time at which the magnitude of the acceleration is a minimum. *(2 marks)*

(e) The particle is moving under the action of a single variable force $\mathbf{F}$ newtons. The mass of the particle is 7 kg.

Find the minimum magnitude of $\mathbf{F}$. *(2 marks)*

June 2010

1 A particle moves along a straight line through the origin. At time t, the displacement, s, of the particle from the origin is given by

$$s = 5t^2 + 3\cos 4t$$

Find the velocity of the particle at time t. *(3 marks)*

4 A particle has mass 200 kg and moves on a smooth horizontal plane. A single horizontal force, $\left(400\cos\left(\frac{\pi}{2}t\right)\mathbf{i} + 600t^2\,\mathbf{j}\right)$ newtons, acts on the particle at time t seconds.

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) When $t = 4$, the velocity of the particle is $(-3\mathbf{i} + 56\mathbf{j})\,\text{m s}^{-1}$.

Find the velocity of the particle at time t. *(5 marks)*

(c) Find t when the particle is moving due west. *(3 marks)*

(d) Find the speed of the particle when it is moving due west. *(2 marks)*

January 2011

1 The velocity of a particle at time t seconds is $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (4 + 3t^2)\mathbf{i} + (12 - 8t)\mathbf{j}$$

(a) When $t = 0$, the particle is at the point with position vector $(5\mathbf{i} - 7\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(4 marks)*

(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) The particle has mass 2 kg.

Find the magnitude of the force acting on the particle when $t = 1$. *(4 marks)*

June 2011

3 A particle moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively. At time t seconds, the velocity of the particle, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, is given by

$$\mathbf{v} = 4\mathrm{e}^{-2t}\,\mathbf{i} + (6t - 3t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 5 kg.

(i) Find an expression for the force $\mathbf{F}$ acting on the particle at time t. *(2 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 0$. *(2 marks)*

(c) Find the value of t when $\mathbf{F}$ acts due west. *(2 marks)*

(d) When $t = 0$, the particle is at the point with position vector $(6\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

January 2012

2 A particle, of mass 50 kg, moves on a smooth horizontal plane. A single horizontal force

$$[(300t - 60t^2)\,\mathbf{i} + 100\mathrm{e}^{-2t}\,\mathbf{j}] \text{ newtons}$$

acts on the particle at time t seconds.

The vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) When $t = 0$, the velocity of the particle is $(7\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find the velocity of the particle at time t. *(4 marks)*

(c) Calculate the speed of the particle when $t = 1$. *(4 marks)*

June 2012

2 A particle moves in a straight line. At time t seconds, it has velocity $v\,\text{m s}^{-1}$, where

$$v = 6t^2 - 2e^{-4t} + 8$$

and $t \geqslant 0$.

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) Find the acceleration of the particle when $t = 0.5$. *(2 marks)*

(b) The particle has mass 4 kg.

Find the magnitude of the force acting on the particle when $t = 0.5$. *(1 mark)*

(c) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

4 A particle moves on a horizontal plane, in which the unit vectors **i** and **j** are perpendicular.

At time t, the particle's position vector, **r**, is given by

$$\mathbf{r} = 4\cos 3t\mathbf{i} - 4\sin 3t\mathbf{j}$$

(a) Prove that the particle is moving on a circle, which has its centre at the origin. *(2 marks)*

(b) Find an expression for the velocity of the particle at time t. *(2 marks)*

(c) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(d) The acceleration of the particle can be written as

$$\mathbf{a} = k\mathbf{r}$$

where k is a constant.

Find the value of k. *(2 marks)*

(e) State the direction of the acceleration of the particle. *(1 mark)*

January 2013

2 A particle moves in a horizontal plane. The vectors **i** and **j** are perpendicular unit vectors in the horizontal plane. At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m s}^{-1}$, is given by

$$\mathbf{v} = 12\cos\left(\frac{\pi}{3}t\right)\mathbf{i} - 9t^2\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The particle, which has mass 4 kg, moves under the action of a single force, **F** newtons.

(i) Find an expression for the force **F** in terms of t. *(2 marks)*

(ii) Find the magnitude of **F** when $t = 3$. *(2 marks)*

(c) When $t = 3$, the particle is at the point with position vector $(4\mathbf{i} - 2\mathbf{j})$ m.

Find the position vector, **r** metres, of the particle at time t. *(5 marks)*

5 A particle, of mass 12 kg, is moving along a straight horizontal line. At time t seconds, the particle has speed $v\,\text{m s}^{-1}$. As the particle moves, it experiences a resistance force of magnitude $4v^{\frac{1}{3}}$. No other horizontal force acts on the particle.

The initial speed of the particle is $8\,\text{m s}^{-1}$.

(a) Show that

$$v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}}$$ *(6 marks)*

(b) Find the value of t when the particle comes to rest. *(1 mark)*

June 2013

1 A particle, of mass 3 kg, moves along a straight line. At time t seconds, the displacement, s metres, of the particle from the origin is given by

$$s = 8t^3 + 15$$

(a) Find the velocity of the particle at time t. *(2 marks)*

(b) Find the magnitude of the resultant force acting on the particle when $t = 2$. *(4 marks)*

3 A particle, of mass 10 kg, moves on a smooth horizontal plane. At time t seconds, the acceleration of the particle is given by

$$\{(40t + 3t^2)\,\mathbf{i} + 20\text{e}^{-4t}\,\mathbf{j}\}\,\text{m s}^{-2}$$

where the vectors **i** and **j** are perpendicular unit vectors.

(a) At time $t = 1$, the velocity of the particle is $(6\mathbf{i} - 5\text{e}^{-4}\mathbf{j})\,\text{m s}^{-1}$.

Find the velocity of the particle at time t. *(5 marks)*

(b) Calculate the initial speed of the particle. *(3 marks)*