Decision 1: Matchings

Past Paper Questions
2006 - 2013

Name:

1 (a) Draw a bipartite graph representing the following adjacency matrix. *(2 marks)*

	U	V	W	X	Y	Z
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(b) Given that initially A is matched to W, B is matched to X, C is matched to V, and E is matched to Y, use the alternating path algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

1 Five people, A, B, C, D and E, are to be matched to five tasks, 1, 2, 3, 4 and 5. The table shows which tasks each person can do.

Person	Tasks
A	1, 3, 5
B	2, 4
C	2
D	4, 5
E	3, 5

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially A is matched to task 3, B to task 4, C to task 2 and E to task 5.

Use an alternating path from this initial matching to find a complete matching. *(4 marks)*

2 Five people A, B, C, D and E are to be matched to five tasks R, S, T, U and V.

The table shows the tasks that each person is able to undertake.

Person	Tasks
A	R, V
B	R, T
C	T, V
D	U, V
E	S, U

   (a) Show this information on a bipartite graph. *(2 marks)*

   (b) Initially, A is matched to task V, B to task R, C to task T, and E to task U.

       Demonstrate, by using an alternating path from this initial matching, how each person can be matched to a task. *(4 marks)*

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	0	1	0	1	1
D	0	0	0	1	0	0
E	0	1	0	0	0	1
F	0	0	0	1	1	0

   (a) Show this information on a bipartite graph. *(2 marks)*

   (b) At first F insists on being matched to task 4. Explain why, in this case, a complete matching is impossible. *(1 mark)*

   (c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

       Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

       From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*

1 Five people, A, B, C, D and E, are to be matched to five tasks, J, K, L, M and N. The table shows the tasks that each person is able to undertake.

Person	Task
A	J, N
B	J, L
C	L, N
D	M, N
E	K, M

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task N, B to task J, C to task L, and E to task M.

Complete the alternating path $D-M$ ..., from this initial matching, to demonstrate how each person can be matched to a task. *(3 marks)*

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6.

The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6
A	0	0	1	0	1	1
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	1	0	1	0	1	0
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task 3, B to task 4, C to task 2 and E to task 5. From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(5 marks)*

2 Six people, *A*, *B*, *C*, *D*, *E* and *F*, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.
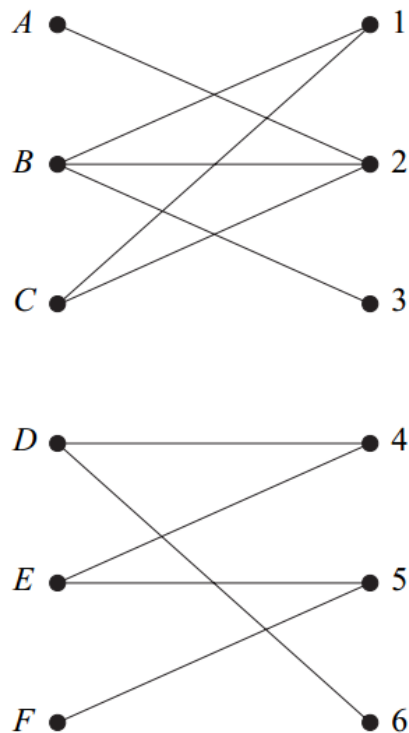
(a) Represent this information in an adjacency matrix. *(2 marks)*

(b) Initially, *B* is assigned to task 1, *C* to task 2, *D* to task 4, and *E* to task 5.

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated to a task. *(5 marks)*

1 (a) Draw a bipartite graph representing the following adjacency matrix.

	1	2	3	4	5	6
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(2 marks)

(b) Initially, *A* is matched to 3, *B* is matched to 4, *C* is matched to 2, and *E* is matched to 5. Use the maximum matching algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

1 Six girls, Alfonsa (A), Bianca (B), Claudia (C), Desiree (D), Erika (E) and Flavia (F), are going to a pizza restaurant. The restaurant provides a special menu of six different pizzas: Margherita (M), Neapolitana (N), Pepperoni (P), Romana (R), Stagioni (S) and Viennese (V).

The table shows the pizzas that each girl likes.

Girl	Pizza
Alfonsa (A)	Margherita (M), Pepperoni (P), Stagioni (S)
Bianca (B)	Neapolitana (N), Romana (R)
Claudia (C)	Neapolitana (N), Viennese (V)
Desiree (D)	Romana (R), Stagioni (S)
Erika (E)	Pepperoni (P), Stagioni (S), Viennese (V)
Flavia (F)	Romana (R)

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Each girl is to eat a different pizza. Initially, the waiter brings six different pizzas and gives Alfonsa the Pepperoni, Bianca the Romana, Claudia the Neapolitana and Erika the Stagioni. The other two pizzas are put in the middle of the table.

From this initial matching, use the maximum matching algorithm to obtain a complete matching so that every girl gets a pizza that she likes. List your complete matching. *(5 marks)*

**1 (a)** Draw a bipartite graph representing the following adjacency matrix.

	1	2	3	4	5
A	1	0	0	0	1
B	0	1	1	1	0
C	0	1	1	1	0
D	1	0	0	0	1
E	1	0	0	0	1

(2 marks)

(b) If A, B, C, D and E represent five people and 1, 2, 3, 4 and 5 represent five tasks to which they are to be assigned, explain why a complete matching is impossible. *(2 marks)*

1   Six people, *A*, *B*, *C*, *D*, *E* and *F*, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.
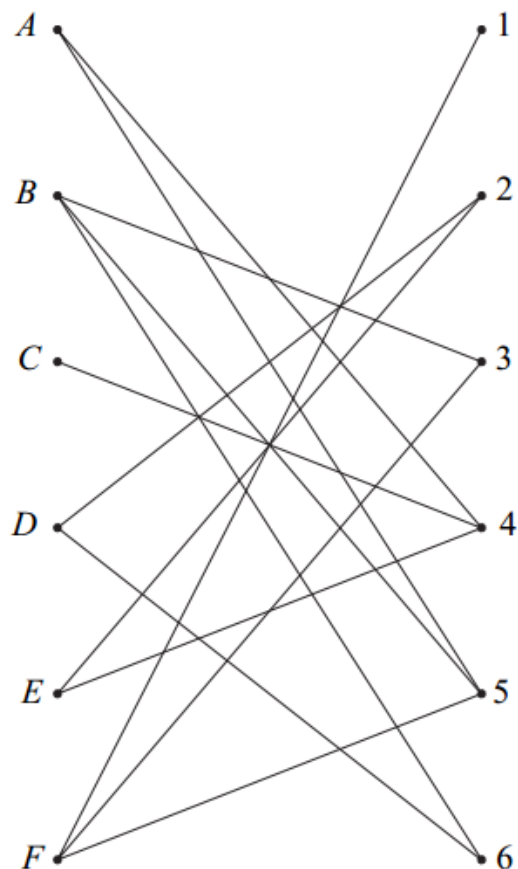
(a)   Represent this information in an adjacency matrix.   *(2 marks)*

(b)   Initially, *B* is assigned to task 5, *D* to task 2, *E* to task 4 and *F* to task 3.

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated to a task.   *(5 marks)*

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the tasks that each of the people is able to undertake.

	1	2	3	4	5	6
A	1	0	1	0	0	0
B	1	1	0	0	0	1
C	0	1	0	1	0	0
D	0	1	0	0	1	0
E	0	0	1	0	1	0
F	0	0	0	0	1	0

(a) Represent this information in a bipartite graph. *(2 marks)*

(b) Initially, A is assigned to task 3, B to task 2, C to task 4 and D to task 5.

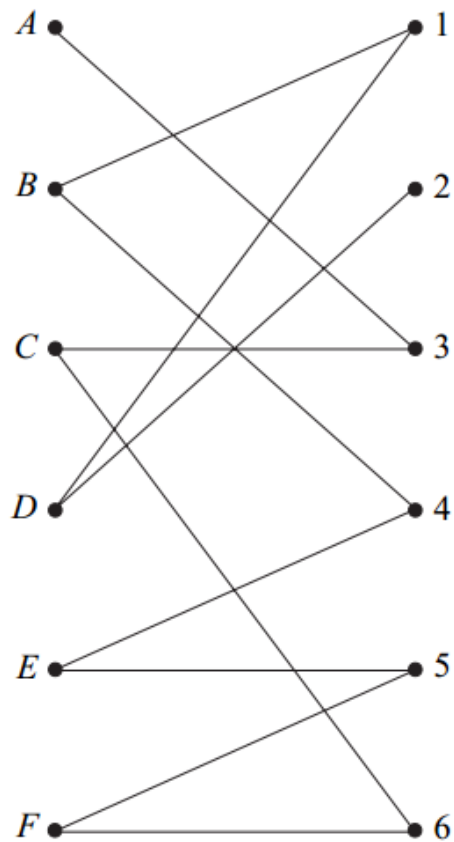Use an algorithm from this initial matching to find a maximum matching, listing your alternating paths. *(5 marks)*

2 (a) Draw a bipartite graph representing the following adjacency matrix.

	1	2	3	4	5	6
A	1	1	0	0	1	1
B	0	1	0	0	1	1
C	1	0	0	0	1	1
D	0	1	1	1	0	1
E	0	0	0	0	1	1
F	0	0	0	0	0	1

(2 marks)

(b) Given that A, B, C, D, E and F represent six people and that 1, 2, 3, 4, 5 and 6 represent six tasks to which they may be assigned, explain why a complete matching is impossible. *(3 marks)*

1   Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.

A 1

B 2

C 3

D 4

E 5

F 6

(a)   Represent this information in an adjacency matrix.   *(2 marks)*

(b)   Initially, B is assigned to task 4, C to task 3, D to task 1, E to task 5 and F to task 6.

By using an algorithm from this initial matching, find a complete matching.

(3 marks)

1 (a) Draw a bipartite graph to represent the following adjacency matrix.

	1	2	3	4	5
A	0	0	0	0	1
B	0	0	0	1	1
C	0	1	0	1	1
D	0	0	0	1	1
E	1	1	1	0	1

(2 marks)

(b) If *A*, *B*, *C*, *D* and *E* represent five people and 1, 2, 3, 4 and 5 represent five tasks to which they are to be assigned, explain why a complete matching is impossible.

(2 marks)

1 Six people, Andy, Bob, Colin, Dev, Eric and Faisal, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following table shows the tasks that each person is able to undertake.

Person	Task
Andy	1, 3
Bob	1, 4
Colin	2, 3
Dev	4, 5, 6
Eric	2, 5, 6
Faisal	1, 3

(a) Represent this information on a bipartite graph. *(2 marks)*

(b) Initially, Bob is allocated to task 1, Colin to task 3, Dev to task 5 and Eric to task 2.

Demonstrate, by using an alternating path algorithm from this initial matching, how each person can be allocated to a different task. *(5 marks)*