Decision 1: Minimum Spanning Trees

Past Paper Questions
2006 - 2013

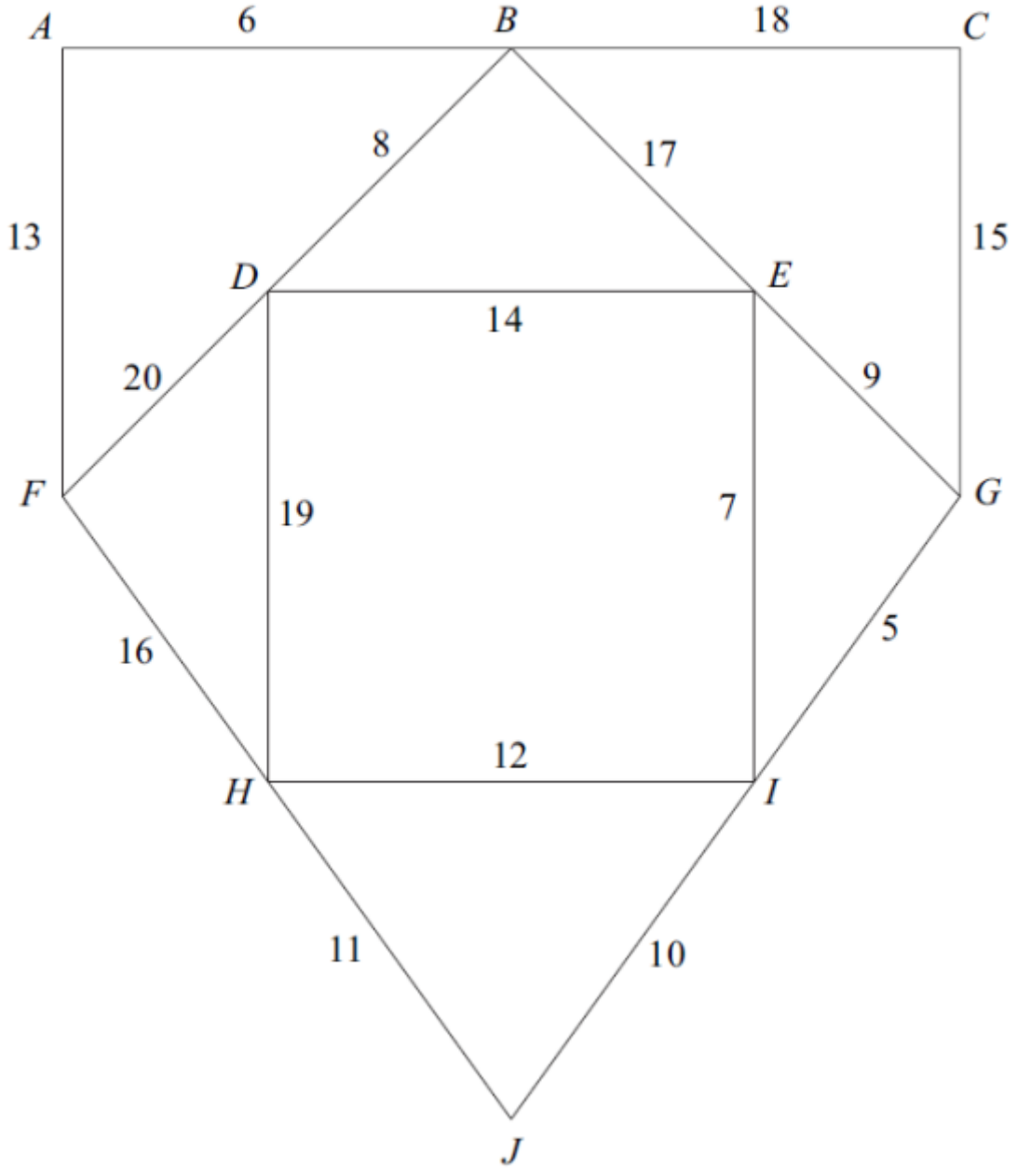
Name:

January 2006

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, . . ., J. The numbers on each edge represent the distances, in miles, between pairs of vertices.



(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

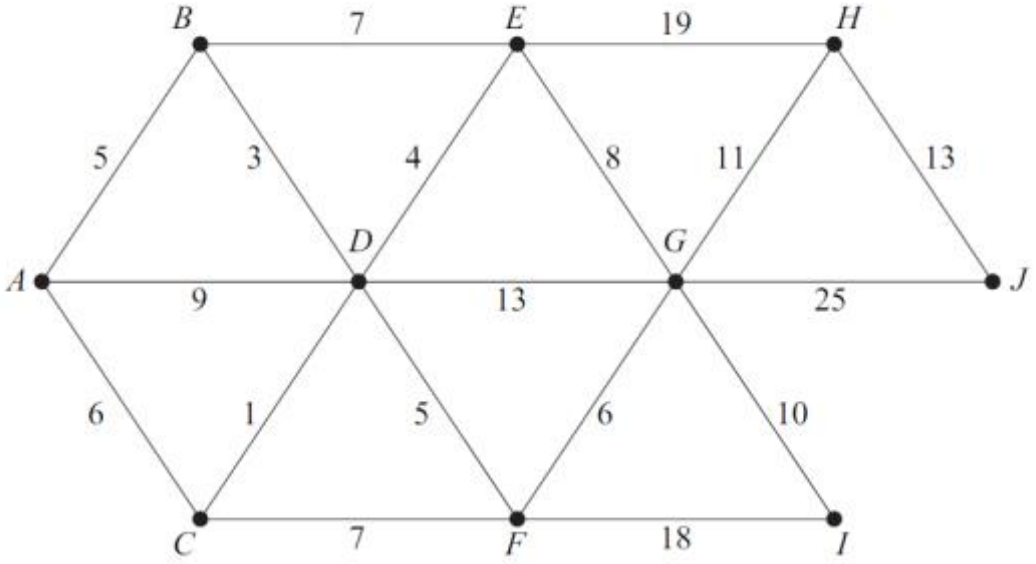
(ii) State the length of your spanning tree. *(1 mark)*

(iii) Draw your spanning tree. *(2 marks)*

June 2006

3 [**Figure 1**, printed on the insert, is provided for use in **part (b)** of this question.]

The diagram shows a network of roads. The number on each edge is the length, in kilometres, of the road.



(a) (i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

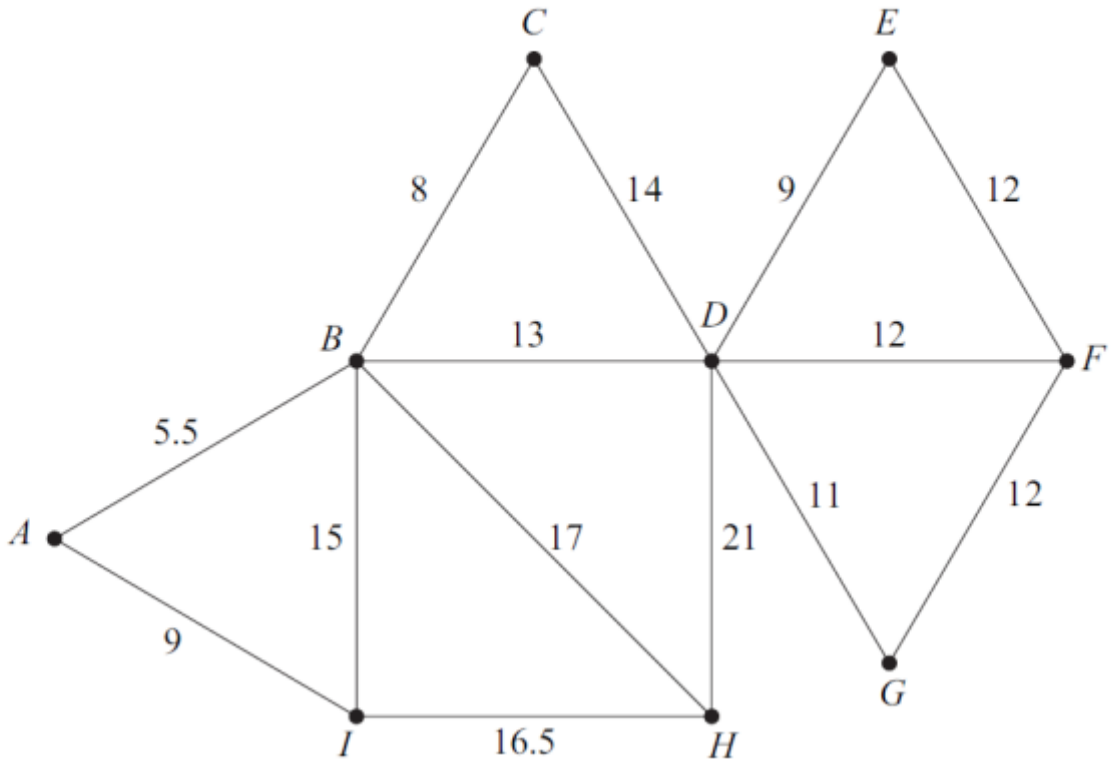
(ii) State the length of your minimum spanning tree. *(1 mark)*

(b) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest distance from A to J. *(6 marks)*

(ii) A new road, of length x km, is built connecting I to J. The minimum distance from A to J is reduced by using this new road. Find, and solve, an inequality for x. *(2 marks)*

January 2007

1 The following network shows the lengths, in miles, of roads connecting nine villages.



(a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(b) Find the length of your minimum spanning tree. *(1 mark)*

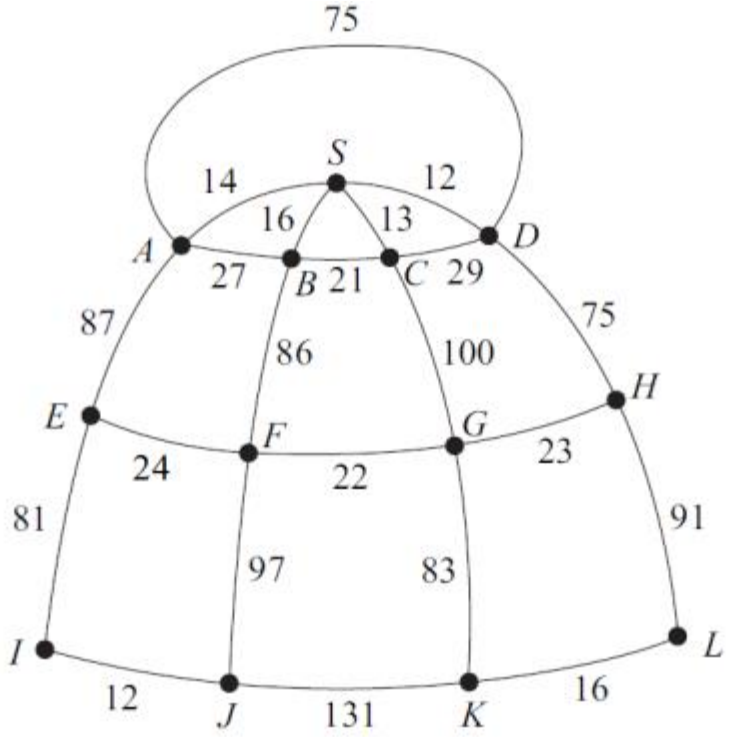
(c) Draw your minimum spanning tree. *(3 marks)*

(d) State the number of other spanning trees that are of the same length as your answer in part (a). *(1 mark)*

June 2007

4 The diagram shows the various ski-runs at a ski resort. There is a shop at S. The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points A, B,...,L and at the shop at S.

The number on each edge represents the distance, in metres, between two points.



Total of all edges = 1135

(a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points A, B,...,L and the shop at S.

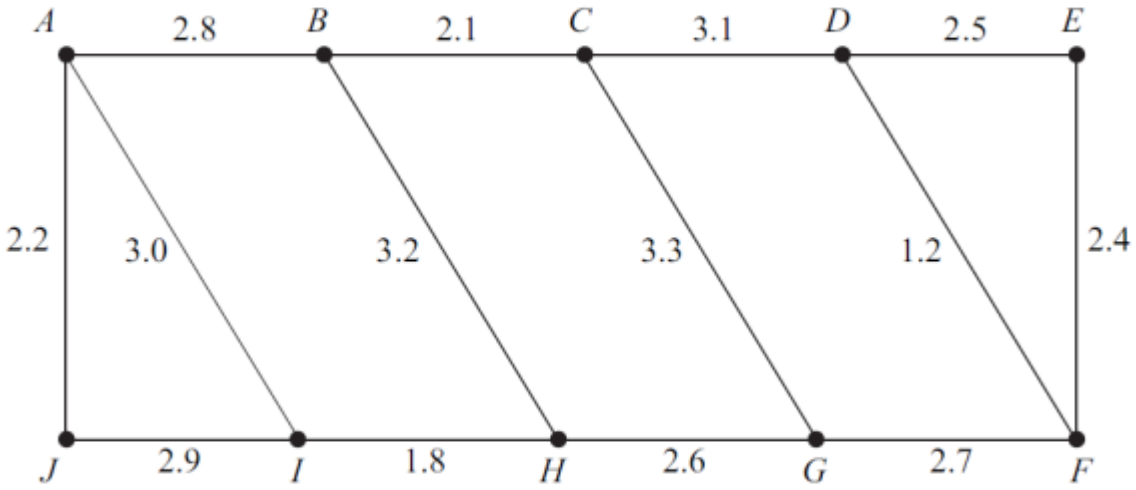
(i) Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(3 marks)*

(iv) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree. *(2 marks)*

3 The diagram shows 10 bus stops, A, B, C, ..., J, in Geneva. The number on each edge represents the distance, in kilometres, between adjacent bus stops.



The city council is to connect these bus stops to a computer system which will display waiting times for buses at each of the 10 stops. Cabling is to be laid between some of the bus stops.

(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 bus stops. *(5 marks)*

(b) State the minimum length of cabling needed. *(1 mark)*

(c) Draw your minimum spanning tree. *(2 marks)*

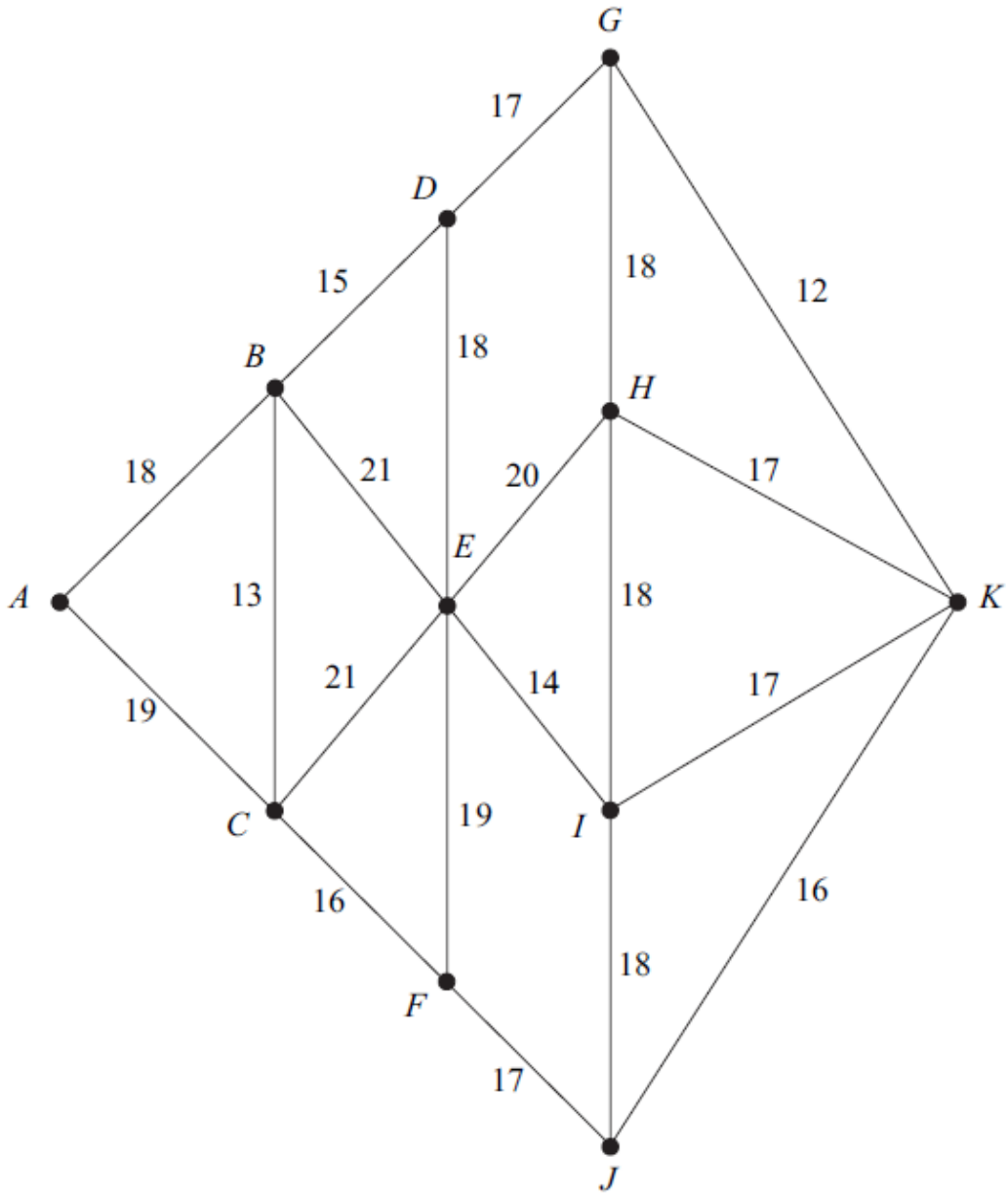
(d) If Prim's algorithm, starting from A, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(2 marks)*

June 2008

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 11 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 11 vertices, A, B, ..., K. The number on each edge represents the distance, in miles, between a pair of vertices.



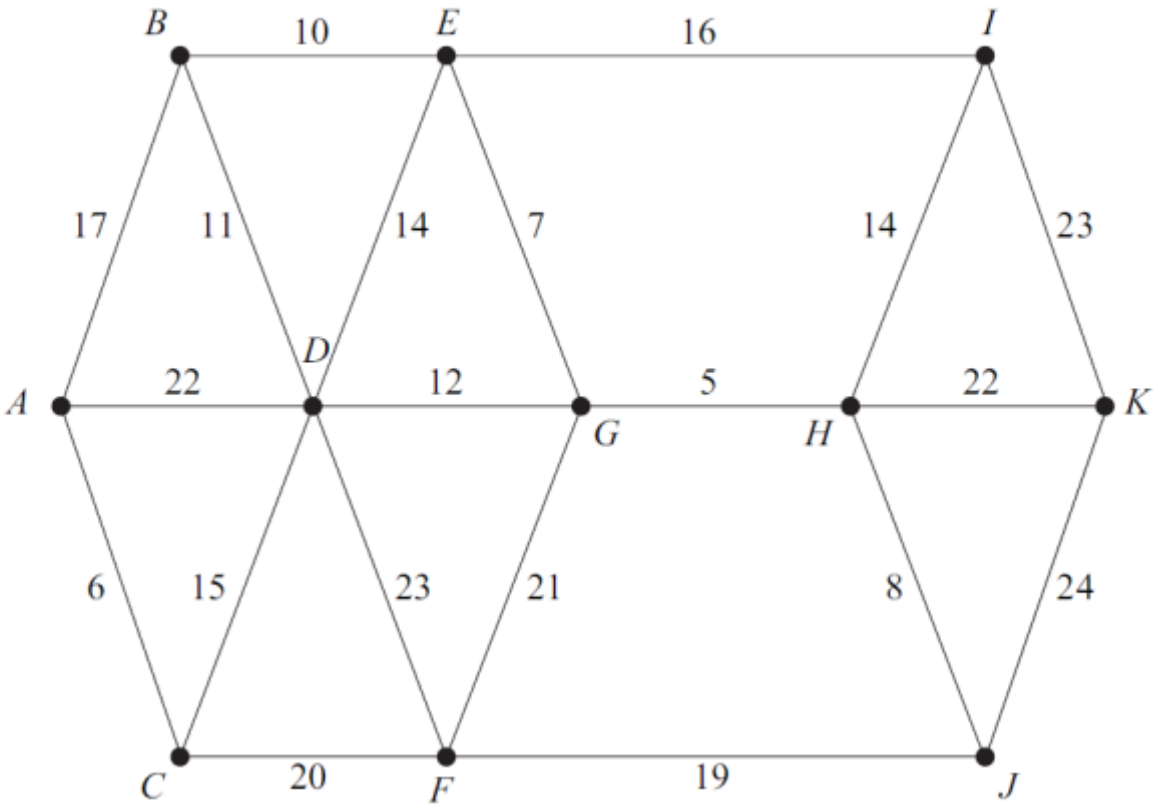
(i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(ii) Find the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

January 2009

1 The following network shows the lengths, in miles, of roads connecting 11 villages, A, B, ..., K.



(a) Starting from G and showing your working at each stage, use Prim's algorithm to find a minimum spanning tree for the network. *(6 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

6 A connected graph G has five vertices and has eight edges with lengths 8, 10, 10, 11, 13, 17, 17 and 18.

(a) Find the minimum length of a minimum spanning tree for G. *(2 marks)*

(b) Find the maximum length of a minimum spanning tree for G. *(2 marks)*

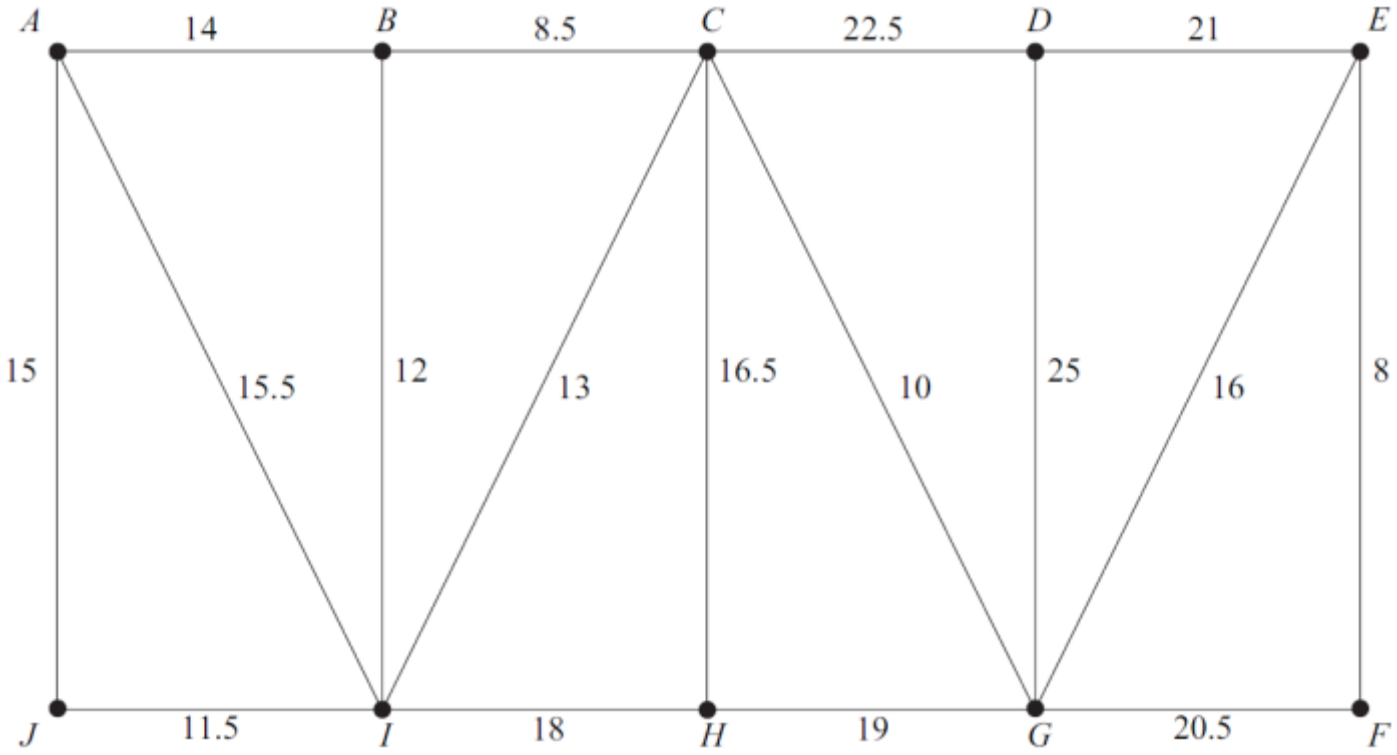
(c) Draw a sketch to show a possible graph G when the length of the minimum spanning tree is 53. *(3 marks)*

June 2009

3 (a) (i) State the number of edges in a minimum spanning tree for a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, ..., J. The number on each edge represents the distance between a pair of adjacent vertices.



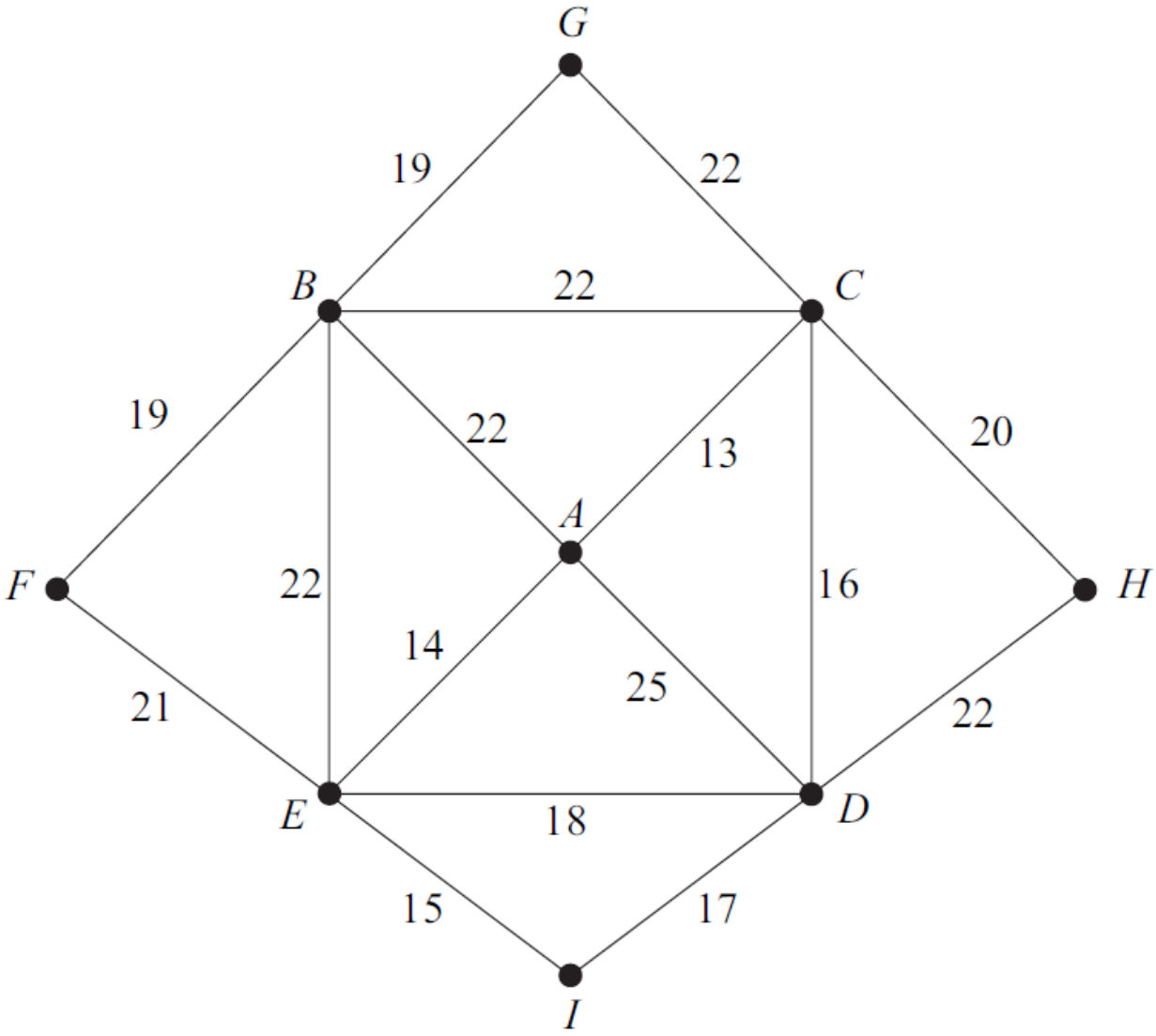
(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

4 In Paris, there is a park where there are statues of famous people; there are many visitors each day to this park. Lighting is to be installed at nine places, A, B, …, I, in the park. The places have to be connected either directly or indirectly by cabling, to be laid alongside the paths, as shown in the diagram.

The diagram shows the length of each path, in metres, connecting adjacent places.



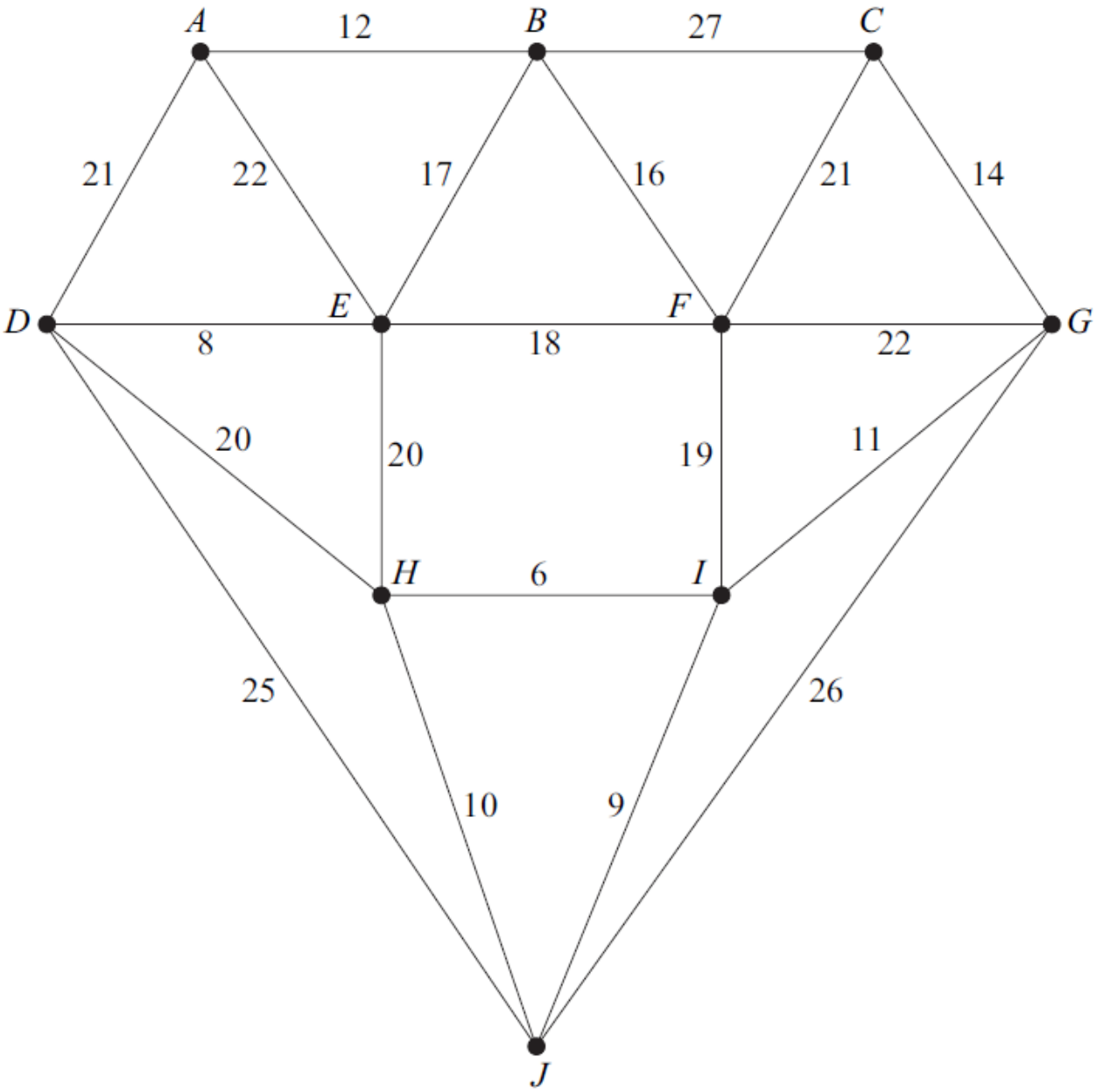
Total length of paths = 307 metres

(a) (i) Use Prim's algorithm, starting from A, to find the minimum length of cabling required. *(5 marks)*

(ii) State this minimum length. *(1 mark)*

(iii) Draw the minimum spanning tree. *(2 marks)*

3 The network shows 10 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



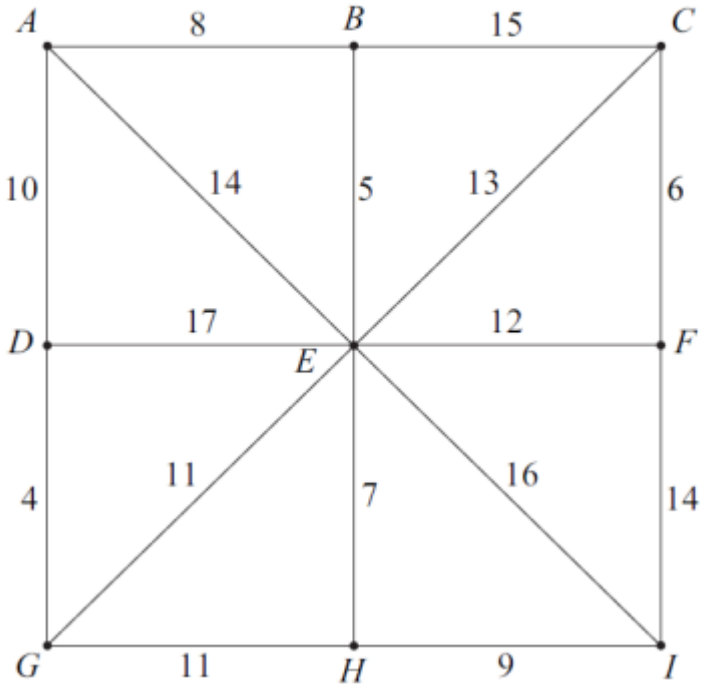
(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 towns. *(6 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(d) If Prim's algorithm, starting at B, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(1 mark)*

3 The following network shows the lengths, in miles, of roads connecting nine villages, $A, B, \ldots, I$.



(a) (i) Use Prim's algorithm starting from E, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

(b) On a particular day, village B is cut off, so its connecting roads cannot be used.

Find the length of a minimum spanning tree for the remaining eight villages. *(2 marks)*

June 2011

3 A group of eight friends, A, B, C, D, E, F, G and H, keep in touch by sending text messages. The cost, in pence, of sending a message between each pair of friends is shown in the following table.

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
A	–	15	10	12	16	11	14	17
B	15	–	15	14	15	16	16	15
C	10	15	–	11	10	12	14	9
D	12	14	11	–	11	12	14	12
E	16	15	10	11	–	13	15	14
F	11	16	12	12	13	–	14	8
G	14	16	14	14	15	14	–	13
H	17	15	9	12	14	8	13	–

One of the group wishes to pass on a piece of news to all the other friends, either by a direct text or by the message being passed on from friend to friend, at the minimum total cost.

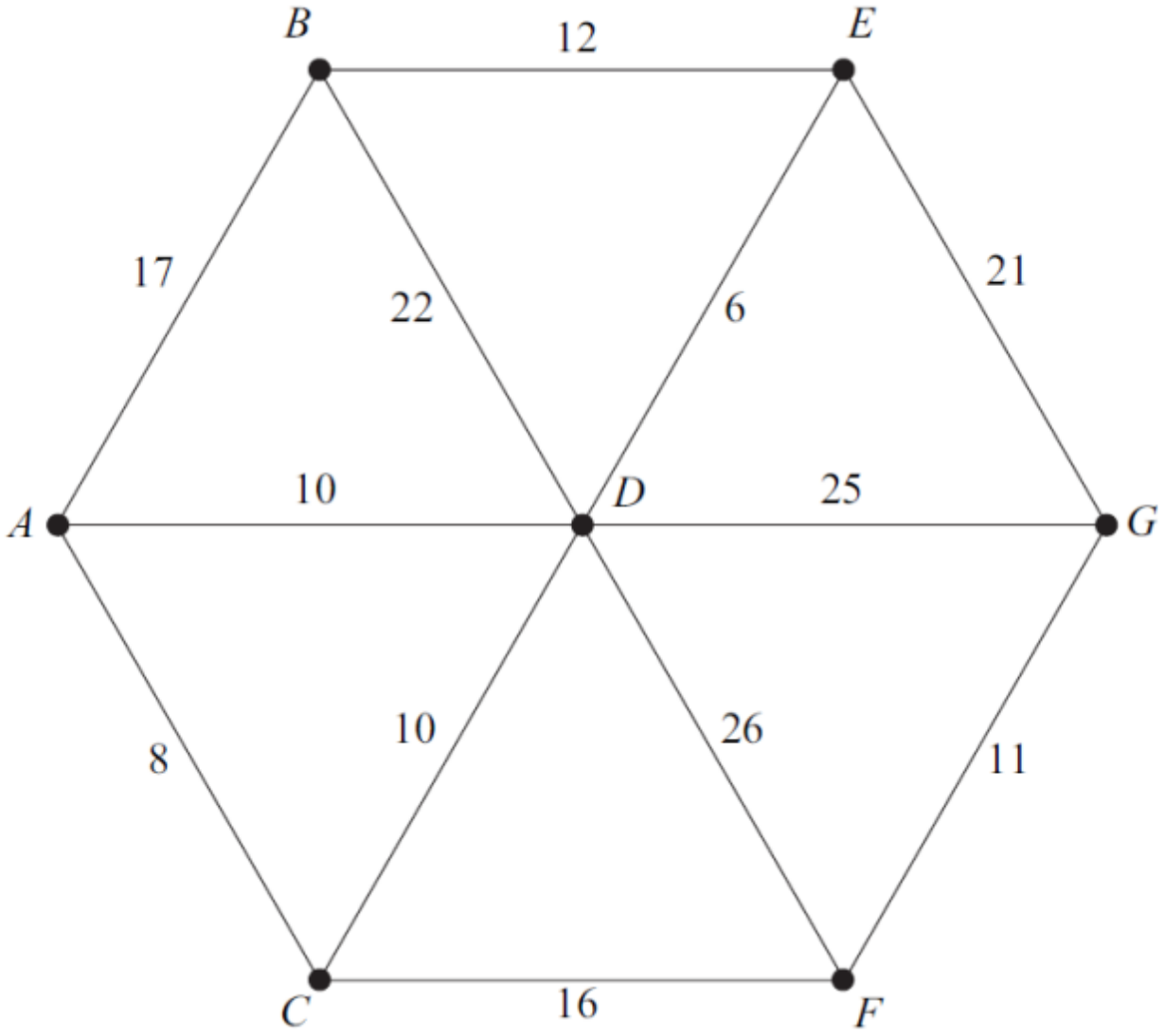
(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the table. *(4 marks)*

(ii) Draw your minimum spanning tree. *(2 marks)*

(iii) Find the minimum total cost. *(1 mark)*

(b) Person H leaves the group. Find the new minimum total cost. *(2 marks)*

3 The following network shows the roads connecting seven villages, A, B, C, ..., G. The number on each edge represents the length, in miles, between a pair of villages.



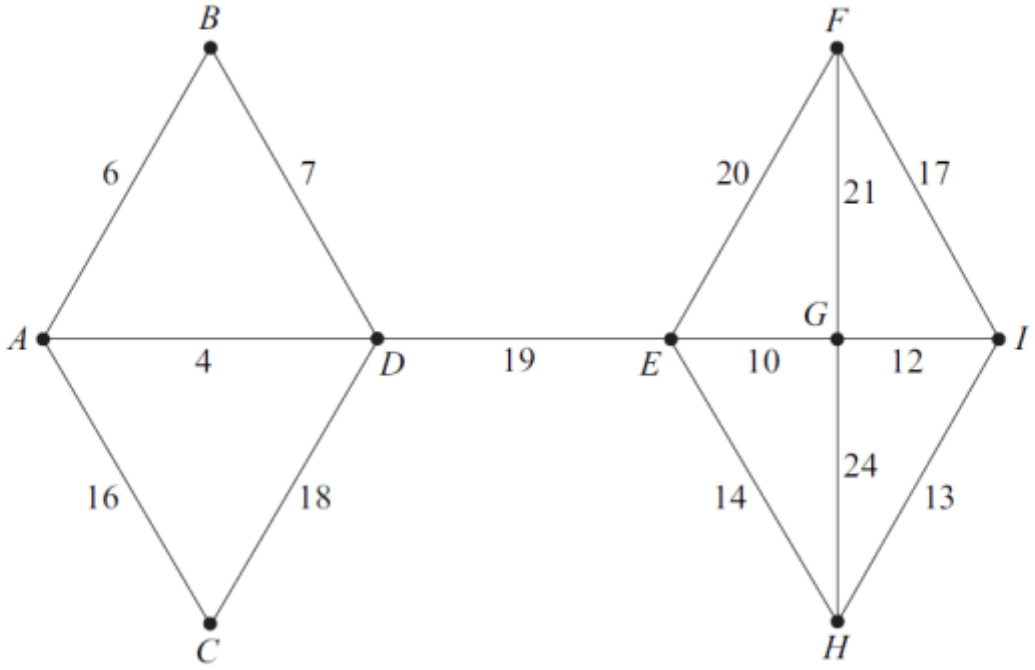
(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. *(5 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) There are two minimum spanning trees for this network. Draw both of these minimum spanning trees. *(3 marks)*

June 2012

3 The following network shows the lengths, in miles, of roads connecting nine villages, $A, B, \ldots, I$.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

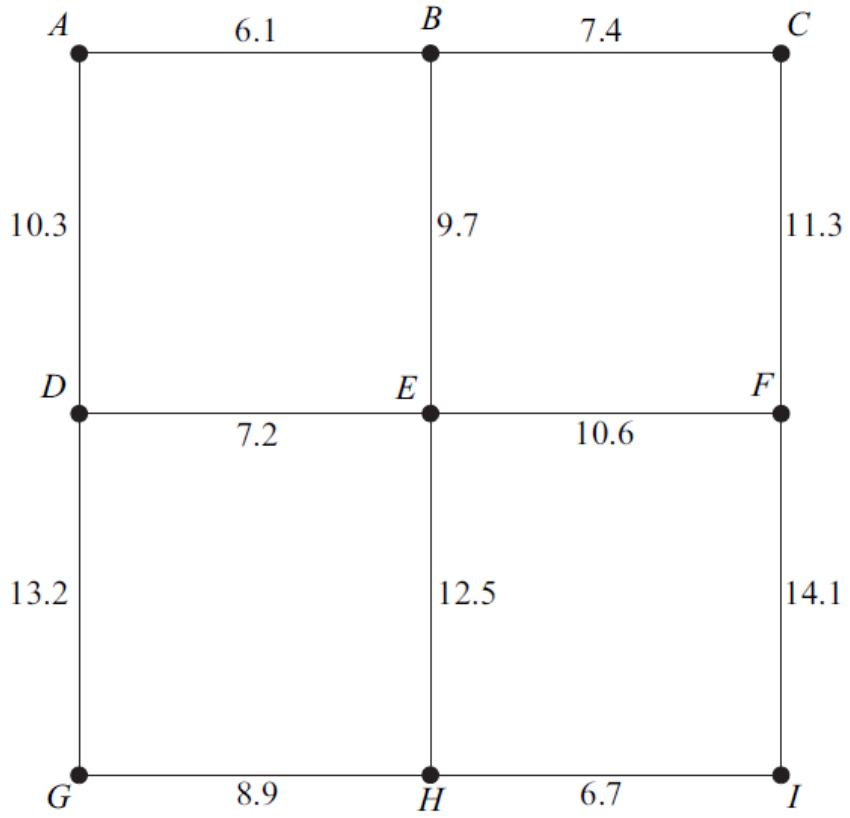
(b) Prim's algorithm from different starting points produces the same minimum spanning tree for this network. State the final edge that would complete the minimum spanning tree using Prim's algorithm:

(i) starting from D; *(1 mark)*

(ii) starting from H. *(1 mark)*

4 The following network shows the lengths, in miles, of roads connecting nine villages, A, B, …, I.

A programme of resurfacing some roads is undertaken to ensure that each village can access all other villages along a resurfaced road, while keeping the amount of road to be resurfaced to a minimum.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network.

(ii) State the length of your minimum spanning tree.

(iii) Draw your minimum spanning tree. *(7 marks)*

(b) Given that Prim's algorithm is used with different start vertices, state the final edge to be added to the minimum spanning tree if:

(i) the start vertex is E;

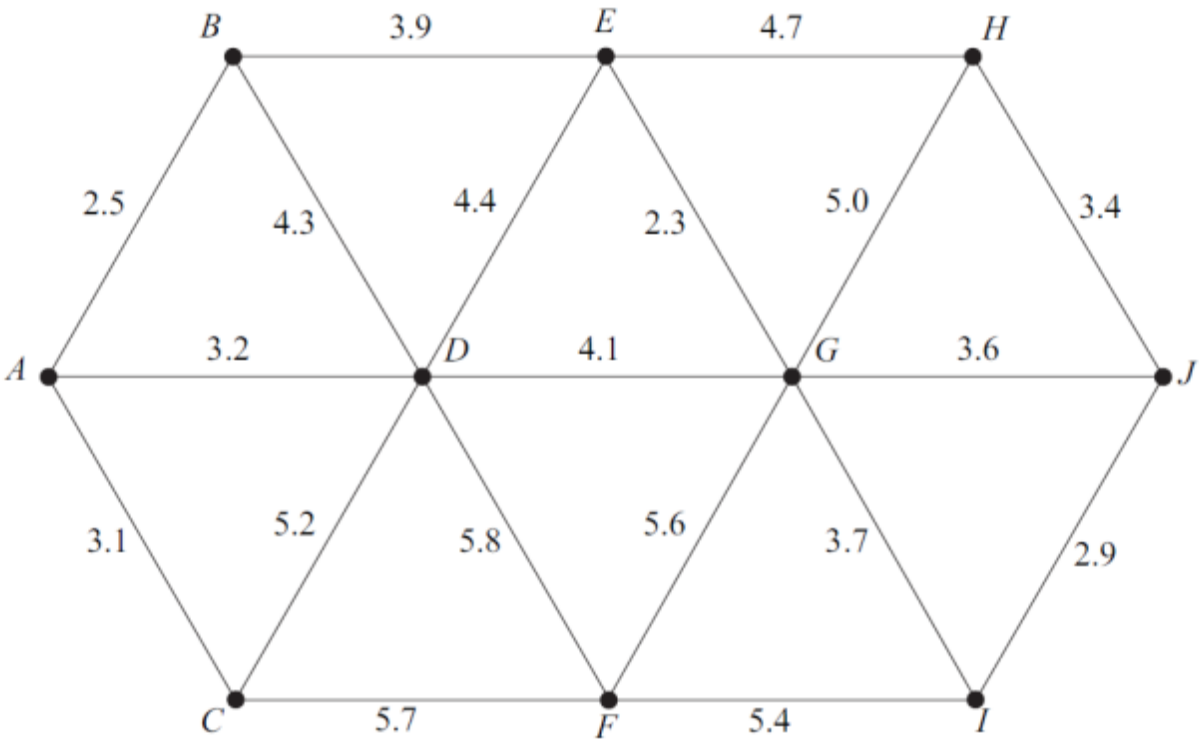
(ii) the start vertex is G. *(2 marks)*

(c) Given that Kruskal's algorithm is used to find the minimum spanning tree, state which edge would be:

(i) the first to be included in the tree;

(ii) the last to be included in the tree. *(2 marks)*

3 The following network shows the lengths, in miles, of roads connecting ten villages, $A, B, C, \ldots, J$.



(a) (i) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the network.

(ii) Find the length of your minimum spanning tree.

(iii) Draw your minimum spanning tree. *(7 marks)*

(b) Prim's algorithm from different starting points produces the same minimum spanning tree. State the final edge that would be added to complete the minimum spanning tree if the starting point were:

(i) A;

(ii) F. *(2 marks)*