Core 4: Parametric Equations

Past Paper Questions
2006 - 2013

Name:

January 2006

2 A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x - 3)(y - 1) + 8 = 0$$ *(3 marks)*

January 2007

1 A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. *(2 marks)*

(ii) Hence find $\frac{dy}{dx}$ in terms of t. *(2 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

June 2007

6 A curve is given by the parametric equations

$$x = \cos\theta \qquad y = \sin 2\theta$$

(a) (i) Find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$. *(2 marks)*

(ii) Find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

where k is an integer. *(4 marks)*

January 2008

5 A curve is defined by the parametric equations $x = 2t + \frac{1}{t^2}$, $y = 2t - \frac{1}{t^2}$.

(a) At the point P on the curve, $t = \frac{1}{2}$.

(i) Find the coordinates of P. *(2 marks)*

(ii) Find an equation of the tangent to the curve at P. *(5 marks)*

(b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where k is an integer. *(3 marks)*

June 2008

2 A curve is defined, for $t \neq 0$, by the parametric equations

$$x = 4t + 3, \quad y = \frac{1}{2t} - 1$$

At the point P on the curve, $t = \frac{1}{2}$.

(a) Find the gradient of the curve at the point P. *(4 marks)*

(b) Find an equation of the normal to the curve at the point P. *(3 marks)*

(c) Find a cartesian equation of the curve. *(3 marks)*

June 2009

2 A curve is defined by the parametric equations

$$x = \frac{1}{t}, \quad y = t + \frac{1}{2t}$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(4 marks)*

(b) Find an equation of the normal to the curve at the point where $t = 1$. *(4 marks)*

(c) Show that the cartesian equation of the curve can be written in the form

$$x^2 - 2xy + k = 0$$

where k is an integer. *(3 marks)*

January 2010

6	(a)	(i)	Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $\sin\theta$ and $\cos\theta$.	*(2 marks)*
		(ii)	Given that $0 < \theta < \frac{\pi}{2}$ and $\cos\theta = \frac{3}{5}$, show that $\sin 2\theta = \frac{24}{25}$ and find the value of $\cos 2\theta$.	*(2 marks)*
	(b)		A curve has parametric equations $$x = 3\sin 2\theta, \quad y = 4\cos 2\theta$$	
		(i)	Find $\frac{dy}{dx}$ in terms of θ.	*(3 marks)*
		(ii)	At the point P on the curve, $\cos\theta = \frac{3}{5}$ and $0 < \theta < \frac{\pi}{2}$. Find an equation of the tangent to the curve at the point P.	*(3 marks)*

June 2010

2		A curve is defined by the parametric equations $$x = 1 - 3t, \qquad y = 1 + 2t^3$$	
	(a)	Find $\frac{dy}{dx}$ in terms of t.	*(3 marks)*
	(b)	Find an equation of the normal to the curve at the point where $t = 1$.	*(4 marks)*
	(c)	Find a cartesian equation of the curve.	*(2 marks)*

January 2011

4		A curve is defined by the parametric equations $$x = 3e^t, \quad y = e^{2t} - e^{-2t}$$	
	(a) (i)	Find the gradient of the curve at the point where $t = 0$.	*(3 marks)*
	(ii)	Find an equation of the tangent to the curve at the point where $t = 0$.	*(1 mark)*
	(b)	Show that the cartesian equation of the curve can be written in the form $$y = \frac{x^2}{k} - \frac{k}{x^2}$$ where k is an integer.	*(2 marks)*

June 2011

4 (a) A curve is defined by the parametric equations $x = 3\cos 2\theta$, $y = 2\cos\theta$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{1}{k\cos\theta}$, where k is an integer. *(4 marks)*

(ii) Find an equation of the normal to the curve at the point where $\theta = \dfrac{\pi}{3}$. *(4 marks)*

January 2012

5 A curve is defined by the parametric equations

$$x = 8t^2 - t, \qquad y = \frac{3}{t}$$

(a) Show that the cartesian equation of the curve can be written as $xy^2 + 3y = k$, stating the value of the integer k. *(2 marks)*

(b) (i) Find an equation of the tangent to the curve at the point P, where $t = \frac{1}{4}$. *(7 marks)*

(ii) Verify that the tangent at P intersects the curve when $x = \frac{3}{2}$. *(2 marks)*

June 2012

5 A curve is defined by the parametric equations

$$x = 2\cos\theta, \qquad y = 3\sin 2\theta$$

(a) (i) Show that

$$\frac{dy}{dx} = a\sin\theta + b\operatorname{cosec}\theta$$

where a and b are integers. *(4 marks)*

(ii) Find the gradient of the normal to the curve at the point where $\theta = \dfrac{\pi}{6}$. *(2 marks)*

(b) Show that the cartesian equation of the curve can be expressed as

$$y^2 = px^2(4 - x^2)$$

where p is a rational number. *(3 marks)*

January 2013

4 (b) Show that $x = t + \frac{2}{t}$, $y = t - \frac{2}{t}$ are parametric equations of the curve $x^2 - y^2 = 8$.

(2 marks)

June 2013

4 A curve is defined by the parametric equations $x = 8e^{-2t} - 4$, $y = 2e^{2t} + 4$.

(a) Find $\frac{dy}{dx}$ in terms of t. *(3 marks)*

(b) The point P, where $t = \ln 2$, lies on the curve.

(i) Find the gradient of the curve at P. *(1 mark)*

(ii) Find the coordinates of P. *(2 marks)*

(iii) The normal at P crosses the x-axis at the point Q. Find the coordinates of Q.

(3 marks)

(c) Find the Cartesian equation of the curve in the form $xy + 4y - 4x = k$, where k is an integer. *(3 marks)*