S1: Normal Distribution

Past Paper Questions
2006 - 2013

Name:

January 2006

7 (a) The weight, X grams, of soup in a carton may be modelled by a normal random variable with mean 406 and standard deviation 4.2.

Find the probability that the weight of soup in a carton:

(i) is less than 400 grams; *(3 marks)*

(ii) is between 402.5 grams and 407.5 grams. *(4 marks)*

(b) The weight, Y grams, of chopped tomatoes in a tin is a normal random variable with mean μ and standard deviation σ.

(i) Given that $P(Y < 310) = 0.975$, explain why:

$$310 - \mu = 1.96\sigma$$ *(3 marks)*

(ii) Given that $P(Y < 307.5) = 0.86$, find, to two decimal places, values for μ and σ. *(4 marks)*

June 2006

2 The heights of sunflowers may be assumed to be normally distributed with a mean of 185 cm and a standard deviation of 10 cm.

(a) Determine the probability that the height of a randomly selected sunflower:

(i) is less than 200 cm; *(3 marks)*

(ii) is more than 175 cm; *(3 marks)*

(iii) is between 175 cm and 200 cm. *(2 marks)*

(b) Determine the probability that the mean height of a random sample of 4 sunflowers is more than 190 cm. *(4 marks)*

January 2007

6 When Monica walks to work from home, she uses either route A or route B.

(a) Her journey time, X minutes, by route A may be assumed to be normally distributed with a mean of 37 and a standard deviation of 8.

Determine:

(i) $P(X < 45)$; *(3 marks)*

(ii) $P(30 < X < 45)$. *(3 marks)*

(b) Her journey time, Y minutes, by route B may be assumed to be normally distributed with a mean of 40 and a standard deviation of σ.

Given that $P(Y > 45) = 0.12$, calculate the value of σ. *(4 marks)*

(c) If Monica leaves home at 8.15 am to walk to work hoping to arrive by 9.00 am, state, with a reason, which route she should take. *(2 marks)*

(d) When Monica travels to work from home by car, her journey time, W minutes, has a mean of 18 and a standard deviation of 12.

Estimate the probability that, for a random sample of 36 journeys to work from home by car, Monica's mean time is more than 20 minutes. *(4 marks)*

(e) Indicate where, if anywhere, in this question you needed to make use of the Central Limit Theorem. *(1 mark)*

June 2007

7 (a) Electra is employed by E & G Ltd to install electricity meters in new houses on an estate. Her time, X minutes, to install a meter may be assumed to be normally distributed with a mean of 48 and a standard deviation of 20.

Determine:

(i) $P(X < 60)$; *(2 marks)*

(ii) $P(30 < X < 60)$; *(3 marks)*

(iii) the time, k minutes, such that $P(X < k) = 0.9$. *(4 marks)*

(b) Gazali is employed by E & G Ltd to install gas meters in the same new houses. His time, Y minutes, to install a meter has a mean of 37 and a standard deviation of 25.

(i) Explain why Y is unlikely to be normally distributed. *(2 marks)*

(ii) State why $\overline{Y}$, the mean of a random sample of 35 gas meter installations, is likely to be approximately normally distributed. *(1 mark)*

(iii) Determine $P(\overline{Y} > 40)$. *(4 marks)*

January 2008

1 In large-scale tree-felling operations, a machine cuts down trees, strips off the branches and then cuts the trunks into logs of length X metres for transporting to a sawmill.

It may be assumed that values of X are normally distributed with mean μ and standard deviation 0.16, where μ can be set to a specific value.

(a) Given that μ is set to 3.3, determine:

(i) $P(X < 3.5)$; *(3 marks)*

(ii) $P(X > 3.0)$; *(3 marks)*

(iii) $P(3.0 < X < 3.5)$. *(2 marks)*

(b) The sawmill now requires a batch of logs such that there is a probability of 0.025 that any given log will have a length less than 3.1 metres.

Determine, to two decimal places, the new value of μ. *(4 marks)*

June 2008

5 When a particular make of tennis ball is dropped from a vertical distance of 250 cm on to concrete, the height, X centimetres, to which it first bounces may be assumed to be normally distributed with a mean of 140 and a standard deviation of 2.5.

(a) Determine:

(i) $P(X < 145)$; *(3 marks)*

(ii) $P(138 < X < 142)$. *(4 marks)*

(b) Determine, to one decimal place, the maximum height exceeded by 85% of first bounces. *(4 marks)*

(c) Determine the probability that, for a random sample of 4 first bounces, the mean height is greater than 139 cm. *(4 marks)*

January 2009

3 UPVC facia board is supplied in lengths labelled as 5 metres. The actual length, X metres, of a board may be modelled by a normal distribution with a mean of 5.08 and a standard deviation of 0.05.

(a) Determine:

(i) $P(X < 5)$; *(3 marks)*

(ii) $P(5 < X < 5.10)$. *(2 marks)*

(b) Determine the probability that the mean length of a random sample of 4 boards:

(i) exceeds 5.05 metres; *(4 marks)*

(ii) is exactly 5 metres. *(1 mark)*

(c) Assuming that the value of the standard deviation remains unchanged, determine the mean length necessary to ensure that only 1 per cent of boards have lengths less than 5 metres. *(4 marks)*

June 2009

3 The weight, X grams, of talcum powder in a tin may be modelled by a normal distribution with mean 253 and standard deviation σ.

(a) Given that $\sigma = 5$, determine:

(i) $P(X < 250)$; *(3 marks)*

(ii) $P(245 < X < 250)$; *(2 marks)*

(iii) $P(X = 245)$. *(1 mark)*

(b) Assuming that the value of the mean remains unchanged, determine the value of σ necessary to ensure that 98% of tins contain more than 245 grams of talcum powder. *(4 marks)*

January 2010

1 Draught excluder for doors and windows is sold in rolls of nominal length 10 metres.

The actual length, X metres, of draught excluder on a roll may be modelled by a normal distribution with mean 10.2 and standard deviation 0.15.

(a) Determine:

(i) $P(X < 10.5)$; *(3 marks)*

(ii) $P(10.0 < X < 10.5)$. *(3 marks)*

(b) A customer randomly selects six 10-metre rolls of the draught excluder.

Calculate the probability that all six rolls selected contain more than 10 metres of draught excluder. *(3 marks)*

June 2010

3 Each day, Margot completes the crossword in her local morning newspaper. Her completion times, X minutes, can be modelled by a normal random variable with a mean of 65 and a standard deviation of 20.

(a) Determine:

(i) $P(X < 90)$;

(ii) $P(X > 60)$. *(5 marks)*

(b) Given that Margot's completion times are independent from day to day, determine the probability that, during a particular period of 6 days:

(i) she completes one of the six crosswords in exactly 60 minutes; *(1 mark)*

(ii) she completes each crossword in less than 60 minutes; *(3 marks)*

(iii) her mean completion time is less than 60 minutes. *(4 marks)*

January 2011

6 The volume of shampoo, V millilitres, delivered by a machine into bottles may be modelled by a normal random variable with mean μ and standard deviation σ.

(a) Given that $\mu = 412$ and $\sigma = 8$, determine:

(i) $\mathrm{P}(V < 400)$; *(3 marks)*

(ii) $\mathrm{P}(V > 420)$; *(2 marks)*

(iii) $\mathrm{P}(V = 410)$. *(1 mark)*

(b) A new quality control specification requires that the values of μ and σ are changed so that

$$\mathrm{P}(V < 400) = 0.05 \quad \text{and} \quad \mathrm{P}(V > 420) = 0.01$$

(i) Show, with the aid of a suitable sketch, or otherwise, that

$$400 - \mu = -1.6449\sigma \quad \text{and} \quad 420 - \mu = 2.3263\sigma$$ *(3 marks)*

(ii) Hence calculate values for μ and σ. *(3 marks)*

June 2011

The diameter, D millimetres, of an American pool ball may be modelled by a normal random variable with mean 57.15 and standard deviation 0.04.

(a) Determine:

(i) $\mathrm{P}(D < 57.2)$; *(3 marks)*

(ii) $\mathrm{P}(57.1 < D < 57.2)$. *(2 marks)*

(b) A box contains 16 of these pool balls. Given that the balls may be regarded as a random sample, determine the probability that:

(i) all 16 balls have diameters less than 57.2 mm; *(2 marks)*

(ii) the mean diameter of the 16 balls is greater than 57.16 mm. *(4 marks)*

January 2012

3 During June 2011, the volume, X litres, of unleaded petrol purchased per visit at a supermarket's filling station by private-car customers could be modelled by a normal distribution with a mean of 32 and a standard deviation of 10.

(a) Determine:

(i) $P(X < 40)$;

(ii) $P(X > 25)$;

(iii) $P(25 < X < 40)$. *(7 marks)*

(b) Given that during June 2011 unleaded petrol cost £1.34 per litre, calculate the probability that the unleaded petrol bill for a visit during June 2011 by a private-car customer exceeded £65. *(3 marks)*

(c) Give **two** reasons, in context, why the model $N(32, 10^2)$ is unlikely to be valid for a visit by **any** customer purchasing fuel at this filling station during June 2011. *(2 marks)*

June 2012

5 A general store sells lawn fertiliser in 2.5 kg bags, 5 kg bags and 10 kg bags.

(a) The actual weight, W kilograms, of fertiliser in a 2.5 kg bag may be modelled by a normal random variable with mean 2.75 and standard deviation 0.15.

Determine the probability that the weight of fertiliser in a 2.5 kg bag is:

(i) less than 2.8 kg;

(ii) more than 2.5 kg. *(5 marks)*

(b) The actual weight, X kilograms, of fertiliser in a 5 kg bag may be modelled by a normal random variable with mean 5.25 and standard deviation 0.20.

(i) Show that $P(5.1 < X < 5.3) = 0.372$, correct to three decimal places. *(2 marks)*

(ii) A random sample of **four** 5 kg bags is selected. Calculate the probability that none of the four bags contains between 5.1 kg and 5.3 kg of fertiliser. *(2 marks)*

(c) The actual weight, Y kilograms, of fertiliser in a 10 kg bag may be modelled by a normal random variable with mean 10.75 and standard deviation 0.50.

A random sample of **six** 10 kg bags is selected. Calculate the probability that the mean weight of fertiliser in the six bags is less than 10.5 kg. *(4 marks)*

2 The volume of *Everwhite* toothpaste in a pump-action dispenser may be modelled by a normal distribution with a mean of 106 ml and a standard deviation of 2.5 ml.

Determine the probability that the volume of *Everwhite* in a randomly selected dispenser is:

(a) less than 110 ml; *(3 marks)*

(b) more than 100 ml; *(2 marks)*

(c) between 104 ml and 108 ml; *(3 marks)*

(d) **not** exactly 106 ml. *(1 mark)*

7 A machine, which cuts bread dough for loaves, can be adjusted to cut dough to any specified set weight. For any set weight, μ grams, the actual weights of cut dough are known to be approximately normally distributed with a mean of μ grams and a fixed standard deviation of σ grams.

It is also known that the machine cuts dough to within 10 grams of any set weight.

(a) Estimate, with justification, a value for σ. *(2 marks)*

(b) The machine is set to cut dough to a weight of 415 grams.

As a training exercise, Sunita, the quality control manager, asked Dev, a recently employed trainee, to record the weight of each of a random sample of 15 such pieces of dough selected from the machine's output. She then asked him to calculate the mean and the standard deviation of his 15 recorded weights.

Dev subsequently reported to Sunita that, for his sample, the mean was 391 grams and the standard deviation was 95.5 grams.

Advise Sunita on whether or not **each** of Dev's values is likely to be correct. Give numerical support for your answers. *(3 marks)*

(c) Maria, an experienced quality control officer, recorded the weight, y grams, of each of a random sample of 10 pieces of dough selected from the machine's output when it was set to cut dough to a weight of 820 grams. Her summarised results were as follows.

$$\sum y = 8210.0 \quad \text{and} \quad \sum (y - \bar{y})^2 = 110.00$$

Explain, with numerical justifications, why **both** of these values are likely to be correct. *(4 marks)*

2 The weight, X grams, of the contents of a tin of baked beans can be modelled by a normal random variable with a mean of 421 and a standard deviation of 2.5.

(a) Find:

(i) $P(X = 421)$;

(ii) $P(X < 425)$;

(iii) $P(418 < X < 424)$. *(6 marks)*

(b) Determine the value of x such that $P(X < x) = 0.98$. *(3 marks)*

(c) The weight, Y grams, of the contents of a tin of ravioli can be modelled by a normal random variable with a mean of μ and a standard deviation of 3.0.

Find the value of μ such that $P(Y < 410) = 0.01$. *(4 marks)*